



प्रतिलोम त्रिकोणमितीय फलन

Class-1

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# प्रतिलोम त्रिकोणमितीय फलन

$\sin x = y \rightarrow$  त्रिकोणमितीय फलन

$x = \sin^{-1}(y) \rightarrow$  प्रतिलोम त्रिकोणमितीय फलन

$\sin^{-1} x$

$\cos^{-1} x$

$\tan^{-1} x$

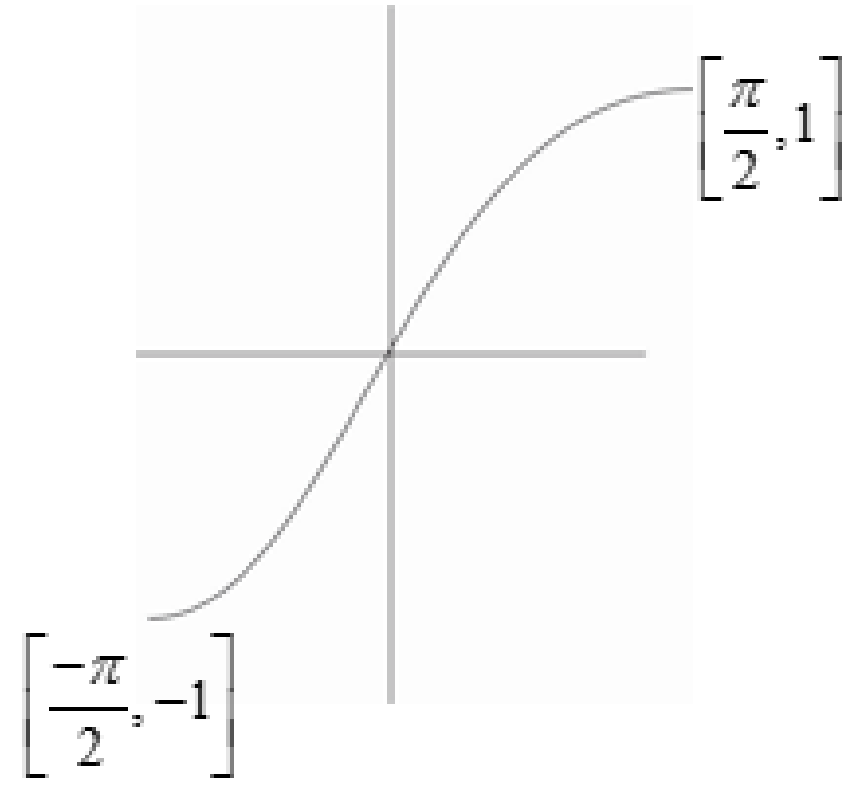
$\cot^{-1} x$

$\sec^{-1} x$

$\operatorname{cosec}^{-1} x$

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$\rightarrow \sin^{-1} x$       प्रान्त  $[-1, 1]$       सह प्रान्त  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



**Note:-**  $\sin^{-1} x \rightarrow (\sin x)^{-1}$  सत्य नहीं है।

Q.1  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left[\sin\frac{\pi}{3}\right] = \frac{\pi}{3}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$  का मुख्य मान  $\frac{\pi}{3}$  है।

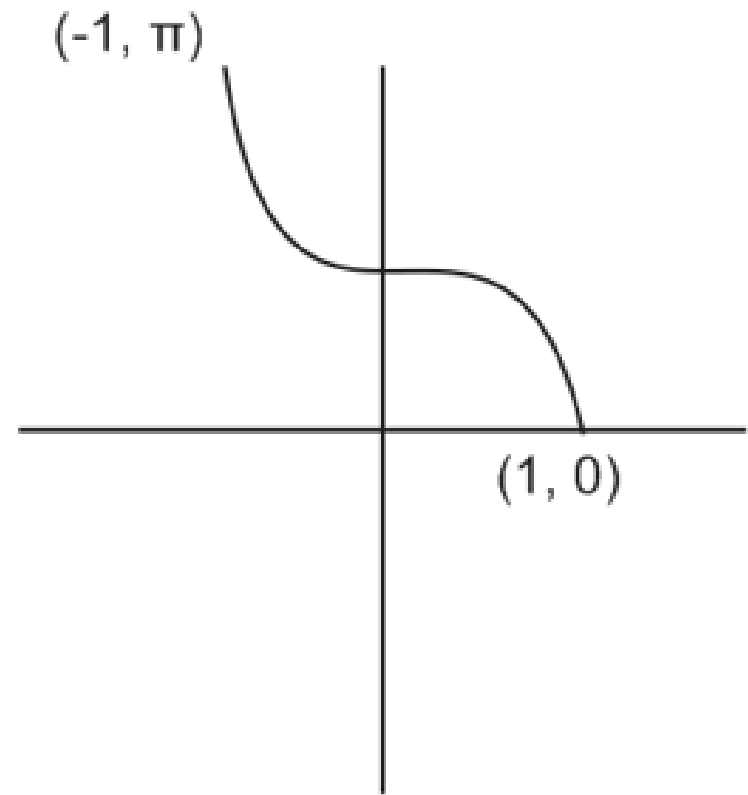
Q.2  $\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left[-\sin\frac{\pi}{6}\right] = -\sin^{-1}\left[\sin\frac{\pi}{6}\right] = -\frac{\pi}{6}$

$\cos^{-1}(x)$  का ग्राफ  $\rightarrow$

$\cos^{-1}(x)$

प्रान्त  
[-1, 1]

सह प्रान्त  
[0,  $\pi$ ]



उदाहरण:—

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$  का मुख्य मान

$$\cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$$

फलन	प्रान्त	सह प्रन्त	मुख्य मान
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$\left(-1, -\frac{\pi}{2}\right) \text{ to } \left(1, \frac{\pi}{2}\right)$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$	$(-1, \pi) \text{ to } (1, 0)$
$\tan^{-1} x$	वास्तविक संख्या	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$\left(-\infty, \frac{-\pi}{2}\right) \text{ to } \left(\infty, \frac{\pi}{2}\right)$

(प्रतिलोम त्रिकोणमितीय फलनों का गुण)

Type -1

$$\sin^{-1}(\sin \theta) = \theta \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cos^{-1}(\cos \theta) = \theta \quad (0, \pi)$$

$$\tan^{-1}(\tan \theta) = \theta \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cot^{-1}(\cot \theta) = \theta \quad [0, \pi]$$

$$\sec^{-1}(\sec \theta) = \theta \quad [0, \pi] - \left(\frac{\pi}{2}\right)$$

$$\operatorname{cosec}^{-1}(\operatorname{cosec} \theta) = \theta \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$



Type -2

$$\sin(\sin^{-1} \theta) = \theta$$

$$\cos(\cos^{-1} \theta) = \theta$$

$$\tan(\tan^{-1} \theta) = \theta$$

$$\cot(\cot^{-1} \theta) = \theta$$

$$\sec(\sec^{-1} \theta) = \theta$$

$$\operatorname{cosec}(\operatorname{cosec}^{-1} \theta) = \theta$$

## Type -3

$$\sin^{-1}(-x) = -\sin^{-1} x$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

$$\tan^{-1}(-x) = -\tan^{-1} x$$

$$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$$

$$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$$

Type -4 Prove That / सिद्ध कीजिए

$$\sin^{-1} \left( \frac{1}{x} \right) = \operatorname{cosec}^{-1}(x)$$

$$\cos^{-1} \left( \frac{1}{x} \right) = \sec^{-1}(x)$$

$$\tan^{-1} \left( \frac{1}{x} \right) = \cot^{-1}(x)$$

$$\cot^{-1} \left( \frac{1}{x} \right) = \tan^{-1}(x)$$

$$\sec^{-1} \left( \frac{1}{x} \right) = \cos^{-1}(x)$$

$$\operatorname{cosec}^{-1} \left( \frac{1}{x} \right) = \sin^{-1}(x)$$

Type -5 Prove That/सिद्ध कीजिए

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

$$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$$

## Type -6

$$1. \sin^{-1}x + \sin^{-1}y = \sin^{-1} [x\sqrt{1-y^2} + y\sqrt{1-x^2}]$$

$$2. \sin^{-1}x - \sin^{-1}y = \sin^{-1} [x\sqrt{1-y^2} - y\sqrt{1-x^2}]$$

$$3. \cos^{-1}x + \cos^{-1}y = \cos^{-1}[xy - \sqrt{1-x^2}\sqrt{1-y^2}]$$

$$4. \cos^{-1}x - \cos^{-1}y = \cos^{-1}[xy + \sqrt{1-x^2}\sqrt{1-y^2}]$$

$$5. \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left[ \frac{x+y}{1-xy} \right]$$

$$6. \tan^{-1} x - \tan^{-1} y = \tan^{-1} \left[ \frac{x-y}{1+xy} \right]$$

$$7. \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1} \left[ \frac{x+y+z-xyz}{1-xy-yz-zx} \right]$$

## Type-7

$$(1) 2 \sin^{-1} x = \sin^{-1} [2x \sqrt{1 - x^2}]$$

$$(2) 2 \cos^{-1} x = \cos^{-1} [2x^2 - 1]$$

$$(3) 2 \tan^{-1} x = \tan^{-1} \left[ \frac{2x}{1-x^2} \right]$$

$$(4) 2 \tan^{-1} x = \sin^{-1} \left[ \frac{2x}{1+x^2} \right]$$

$$(5) 2 \tan^{-1} x = \cos^{-1} \left[ \frac{1-x^2}{1+x^2} \right]$$

Type- 8

$$(1) 3 \sin^{-1} x = \sin^{-1} [3x - 4x^3]$$

$$(2) 3 \cos^{-1} x = \cos^{-1} [4x^3 - 3x]$$

$$(3) 3 \tan^{-1} x = \tan^{-1} \left[ \frac{3x - x^3}{1 - 3x^2} \right]$$



Type- 9

$$(1) \sin^{-1}(x) = \cos^{-1} \sqrt{1 - x^2}$$

$$(2) \sin^{-1}(x) = \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right)$$

$$(3) \cos^{-1}(x) = \sin^{-1} \sqrt{1 - x^2}$$

$$(4) \cos^{-1}(x) = \tan^{-1} \left( \frac{\sqrt{1-x^2}}{x} \right)$$

$$(5) \tan^{-1}(x) = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$(6) \tan^{-1}(x) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$$