APPLICATION OF DERIVATIVE

Tangent and Normal

(1) slope of tangent $= \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = m = \tan\psi$

(2) slope of Normal $= -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}} = -\frac{1}{m}$

(3) Equation of tangent at (x_1, y_1)

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} (x - x_1)$$

(4) Equation of Normal at (x_1, y_1)

$$y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}} (x - x_1)$$

(5) If tangent $\parallel$ to x-axis then

$$\frac{dy}{dx} = 0$$

(6) If tangent $\parallel$ to y-axis then

$$\frac{dx}{dy} = 0$$

(7) Angle b/w two tangent

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

(8) Length of tangent $= \left|\frac{y\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\frac{dy}{dx}}\right| = \left|\frac{y\sqrt{1+m^2}}{m}\right|$

(9) Length of Normal $= \left|y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right| = \left|y\sqrt{1+m^2}\right|$

(10) Length of Subtangent $= \left|\frac{y}{\frac{dy}{dx}}\right| = \left|\frac{y}{m}\right|$

(11) Length of SubNormal $= \left|y\left(\frac{dy}{dx}\right)\right| = |y\,m|$

Maxima and Minima

Second derivative test

Let $y=f(x)$ be a function defined on $[a,b]$

Step 1 Find $\frac{dy}{dx}=f'(x)$

II Put $f'(x)=0$ and find the value of x let c_1, c_2 .... be the value of x.

III Find $f''(x)$ at given value

(i) If $f''(c_1)<0$ then function is maximum

(ii) If $f''(c_2)>0$ then function is minimum

(IV) then find max and min value.

Increasing and decreasing

① A function $f(x)$ is increasing function if $f(x)$ is increase as x is increase i.e $x_1>x_2$ $\Rightarrow f(x_1)>f(x_2)$ $\{(x_1,x_2)\in$ Domain of $f(x)\}$

Hence $f'(x)>0$

② A function $y=f(x)$ is a decreasing function if $f(x)$ is decrease as x increase i.e $x_1>x_2 \Rightarrow f(x_1)<f(x_2)$ Hence $f'(x)<0$

(i) If $f'(x)>0 \ \forall \ x\in(a,b)$ then $f(x)$ is increasing on (a,b)

(ii) If $f'(x)<0 \ \forall \ x\in(a,b)$ then $f(x)$ is decreasing on (a,b)

Rate of change of quantities

① Rate of change of length $=\frac{dl}{dt}$ m/sec

② Rate of change of Area $=\frac{dA}{dt}$ m^2/sec

③ Rate of change of volume $=\frac{dV}{dt}$ m^3/sec.

④ Rate of change of velocity $=\frac{dv}{dt}$ m/sec^2

Definite Integral

① $\int_a^b f(x)\,dx = [F(x)]_a^b \Rightarrow F(b) - F(a)$

properties of definite integral

① $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \int_a^b f(u)\,du$

② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

③ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

④ $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

or $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx.$

⑤ $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$

⑥ $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(-x) = f(x) \\ 0, & \text{if } f(-x) = -f(x) \end{cases}$

$\cancel{⑦ \ \frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = h'(x)}$

⑦ $\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = h'(x)\,f(h(x)) - g'(x)\,f(g(x))$

[Leibnitz's Rule]

Some Important result

① $\int_0^{\pi/2} \log\sin x\,dx = \int_0^{\pi/2} \log\cos x\,dx = -\frac{\pi}{2}\log 2$

② $\int_0^{\pi/2} \log(\tan x)\,dx = \int_0^{\pi/2} \log(\cot x)\,dx = 0$

③ $\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx = \frac{(n-1)(n-3)\cdots 2}{n(n-2)\cdots 3\cdot 1}$ when n is odd

$= \frac{(n-1)(n-3)\cdots 1}{n(n-2)\cdots 2}\left(\frac{\pi}{2}\right)$ when n is even

④ $\int_0^{\pi/2} \sin^m x\cos^n x\,dx = \frac{\{(m-1)(m-3)\cdots 1\}\{(n-1)(n-3)\cdots 2\cdot 1\}}{(m+n)(m+n-2)}$ (when m, n both odd.)

$= \frac{\{(m-1)(m-3)\cdots 1\}\{(n-1)(n-3)\cdots 2\cdot 1\}}{(m+n)(m+n-2)}\left(\frac{\pi}{2}\right)$ when m and n even

Differential Equation

<u>Order of a D.E</u> $\Rightarrow$ The order of d.e is the order of the highest derivative involved in its ~~eqn~~ expression.

e.g $\frac{d^3y}{dx^3} + x\frac{dy}{dx} + xy\frac{d^2y}{dx^2} + y = 0$ Degree 3

<u>Degree of Differential eqn</u> $\Rightarrow$ The degree of a d.e is the degree of the highest order derivative when the differential coefficient are made free from redicals and fraction

e.g $\frac{d^3y}{dx^3} - 6\left(\frac{dy}{dx}\right)^2 - 4y = 0$ Degree $\Rightarrow$ 1

<u>Different form of first order and first degree D.E</u>

(1) <u>Variable seporable D.E</u>

$f(x)\,dx = g(y)\,dy$

Integrate it on both sides, we get

$\int f(x)\,dx = \int g(y)\,dy + C$

(2) <u>Homogeneous D.E</u>

Method $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ Put $\frac{y}{x} = t \Rightarrow y = tx$

$\frac{dy}{dx} = t + x\frac{dt}{dx}$

$t + x\frac{dt}{dx} = f(t)$

$x\frac{dt}{dx} = f(t) - t$

$\int \frac{dx}{x} = \int \frac{dt}{f(t) - t}$ on solving we get Answer

(3) <u>Linear D.E</u>

$\frac{dy}{dx} + Py = Q$ is known as D.E

First we calculate $I.F = e^{\int P dx}$

Sol. $\boxed{y.\,I.F = \int (Q \times I.F)\,dx + C}$