SAFALTA CLASS™

An Initiative by अमर उजाला

KINEMATICS

C_2

① $x = t^2 - 2t + 1$, Calculate Velocity at $t = 3$ sec.

$$v = \frac{dx}{dt}$$

$$= \frac{d}{dt}(t^2 - 2t + 1)$$

$$= 2t - 2$$

$$= 6 - 2$$

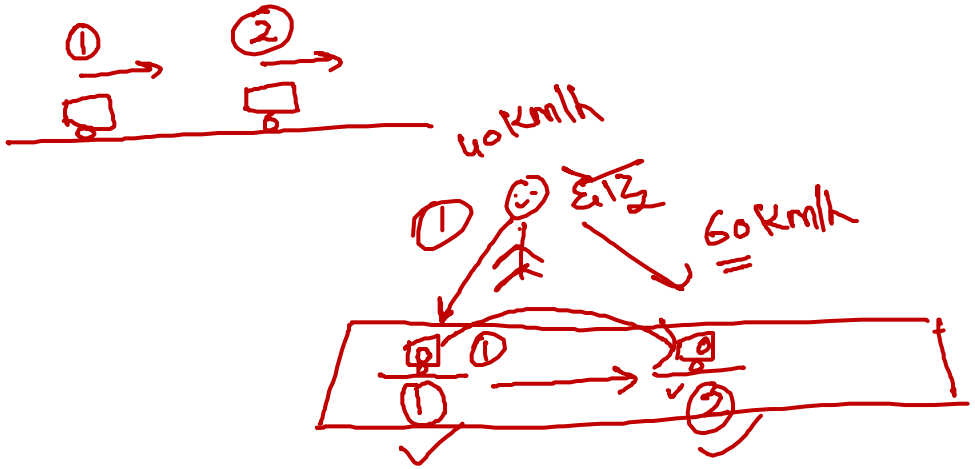
$$v = 4 \text{ m/s}$$

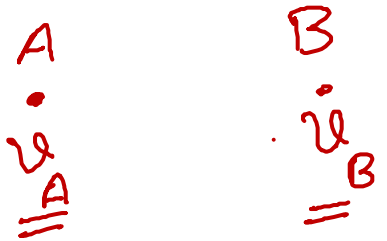
Relative Velocity (आपेक्षिक गति)

- Relative velocity of one object with respect to another object is the time rate of change of relative position of one object with respect to another object.

- Relative velocity of object A with respect to object B

$$V_{AB} = V_A - V_B$$

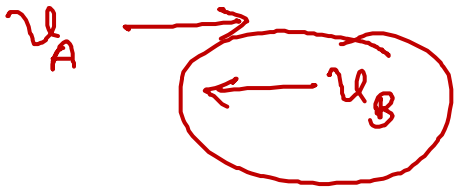




$$v_{AB} = v_A - v_B$$

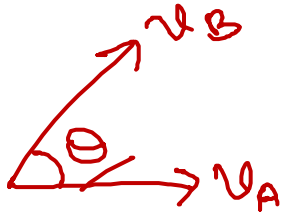
- When two objects are moving in the same direction, then

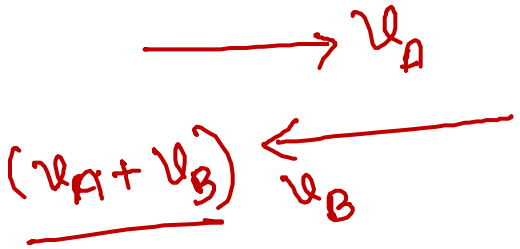
$$VAB = VA - VB$$



- When two objects are moving in opposite direction, then

$$VAB = VA + VB$$

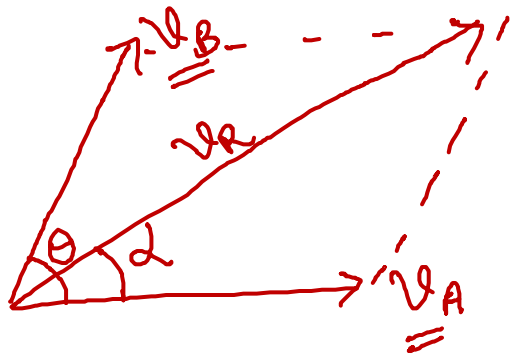




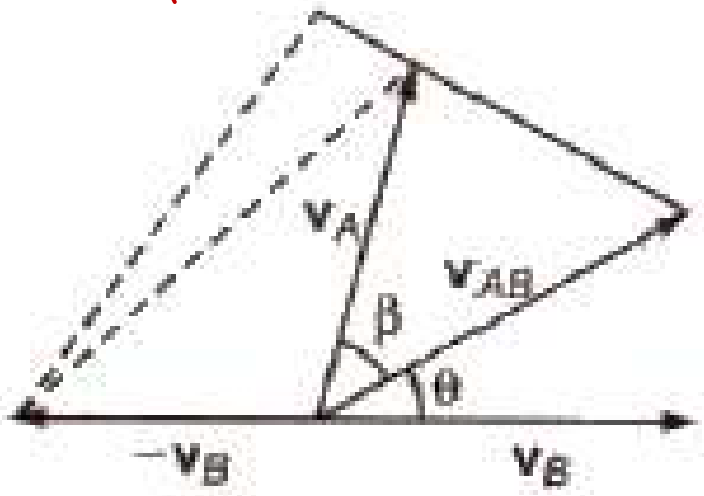
- When two objects are moving at an angle, then

$$\left\{ \tan\alpha = \frac{v_2 \sin\theta}{v_1 - v_2 \cos\theta} \right\}$$

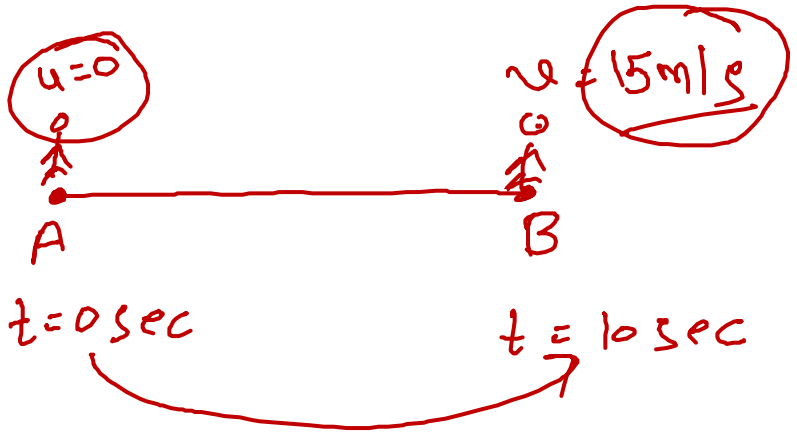
- and $\tan\beta = v_B \sin\theta / v_A - v_B \cos\theta$



$$v_R = \sqrt{v_1^2 + v_2^2 + 2 v_1 v_2 \cos\theta}$$



$$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2 v_A v_B \cos\theta}$$



$$\frac{m/s}{s} = m/s^2$$

$$\vec{a} = \frac{\text{change in velocity}}{\text{Time Taken}}$$

$$= \frac{v_{final} - v_{initial}}{\Delta t}$$

$$= \frac{15 - 0}{10} = 1.5\ m/s^2$$

$v_f > v_i \rightarrow$ Acct.

$v_f < v_i \Rightarrow$ Retardation (Neg. Acc.)

Acceleration (ವೇಗವರ್ಧನೆ)

- **Acceleration** (Vector): ANY change in velocity
  - Speeding up (final velocity is a larger magnitude than the initial velocity)
  - Slowing down (final velocity is a smaller magnitude than the initial velocity)
  - Changing directions (the direction of the vector is changing)

- **Average Acceleration**: the rate at which velocity is changing

- Units = $^{m}/_{s^2}$ = $m \cdot s^{-2}$

$[LT^{-2}]$

$$a = \frac{\Delta v}{t} = \frac{v_2 - v_1}{t}$$

- त्वरण (वेक्टर): वेग में कोई परिवर्तन

- तेजी (अंतिम वेग प्रारंभिक वेग की तुलना में एक बड़ा परिमाण है)

- धीमा होना (अंतिम वेग प्रारंभिक वेग की तुलना में एक छोटा परिमाण है)
बदलती दिशाएं (वेक्टर की दिशा बदल रही है)

- औसत त्वरण: जिस दर पर वेग बदल रहा है

$$a = \frac{\Delta v}{t} = \frac{v_2 - v_1}{t}$$

- इकाइयाँ = m / s2 = m · s-2

Velocity & Acceleration Sign Chart

v
Speed = 20 m/s
Constant

$\vec{a}$ → zero
Nonzero
∞
Constant

		VELOCITY: +	VELOCITY: −
ACCELERATION	+	Moving forward; Speeding up	Moving backward; Slowing down
ACCELERATION	−	Moving forward; Slowing down	Moving backward; Speeding up

Acceleration due to Gravity

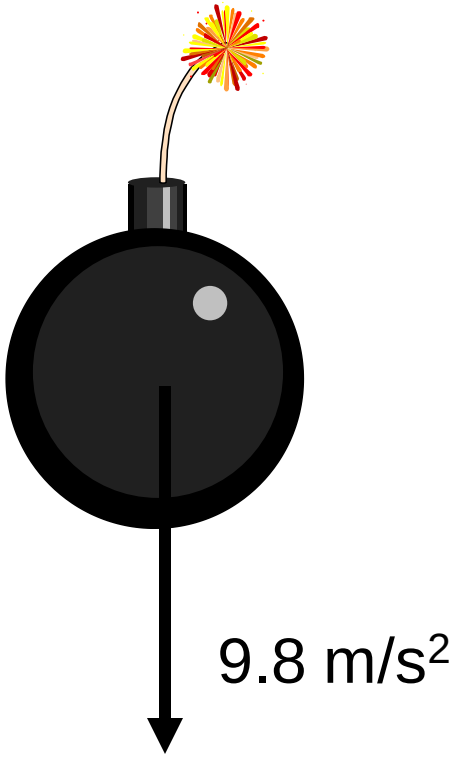
Near the surface of the Earth, all objects accelerate at the same rate (ignoring air resistance).

$\vec{g}$

9.8 m/s²

g → (-ve)

$\boldsymbol{a} = -\boldsymbol{g} = -9.8\ m/s^2$



This acceleration vector is the same on the way up, at the top, and on the way down!

Interpretation: Velocity decreases by 9.8 m/s each second, meaning velocity is becoming less positive or more negative. Less positive means slowing down while going up. More negative means speeding up while going down.

*

① $t = 10\ sec$ → $t = 20\ sec$

$v_i = 10\ m/s$ → $v_f = 30\ m/s$

$$\vec{a} = \frac{30 - 10}{10} = \frac{20}{10} = 2\ m/s^2$$

Av. $\vec{a} \Rightarrow \vec{a} = \frac{v_f - v_i}{\Delta t} = \frac{\Delta v}{\Delta t}$

Inst $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

$$\Rightarrow \boxed{\vec{a} = \frac{dv}{dt}}$$ Inst. Acc

② S = Disp. (meter), t = Time (sec)

$$S = 2t^2 + t + 2$$

$\vec{a} = ?$

$$v = \frac{dS}{dt} = (4t + 1)$$

$$\vec{a} = \frac{dv}{dt} = 4\ m/s^2$$

(2) $v(t) = 2t^2 + 2t + 2$

$\Rightarrow$ Calculate Acceleration (त्वरण) at $t = 2$ sec.

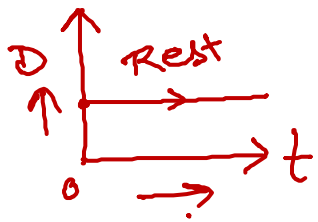
$\Rightarrow$ $\vec{a} = \frac{dv}{dt} = \frac{d}{dt}(2t^2 + 2t + 2)$

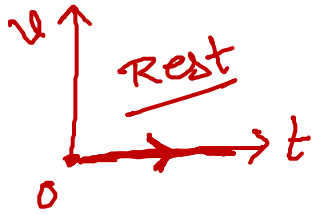
$$\boxed{\vec{a} = 4t + 2}$$

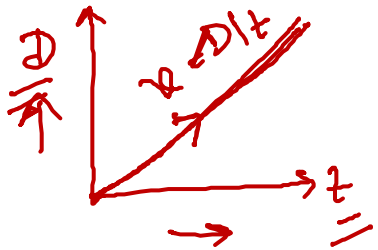
$$\vec{a} = 4 \times 2 + 2$$

$$\Rightarrow \boxed{\vec{a} = 10 \text{ m/s}^2}$$

GRAPH







10 sec → 10m

10 sec = 10m

Motion of object	Distance-Time graph	Speed-Time graph	Acceleration-Time graph
At rest वस्तु विराम में	d / m t / s gradient = 0	v / ms^{-1} t / s speed = 0 gradient = 0	a / ms^{-2} t / s acceleration = 0
At constant speed or uniform speed एकसमान चाल	d / m t / s gradient = constant, k	v / ms^{-1} k 10m/s t / s speed = constant, k gradient = 0	a / ms^{-2} t / s acceleration = 0

$\vec{a}$ = 10m/s2

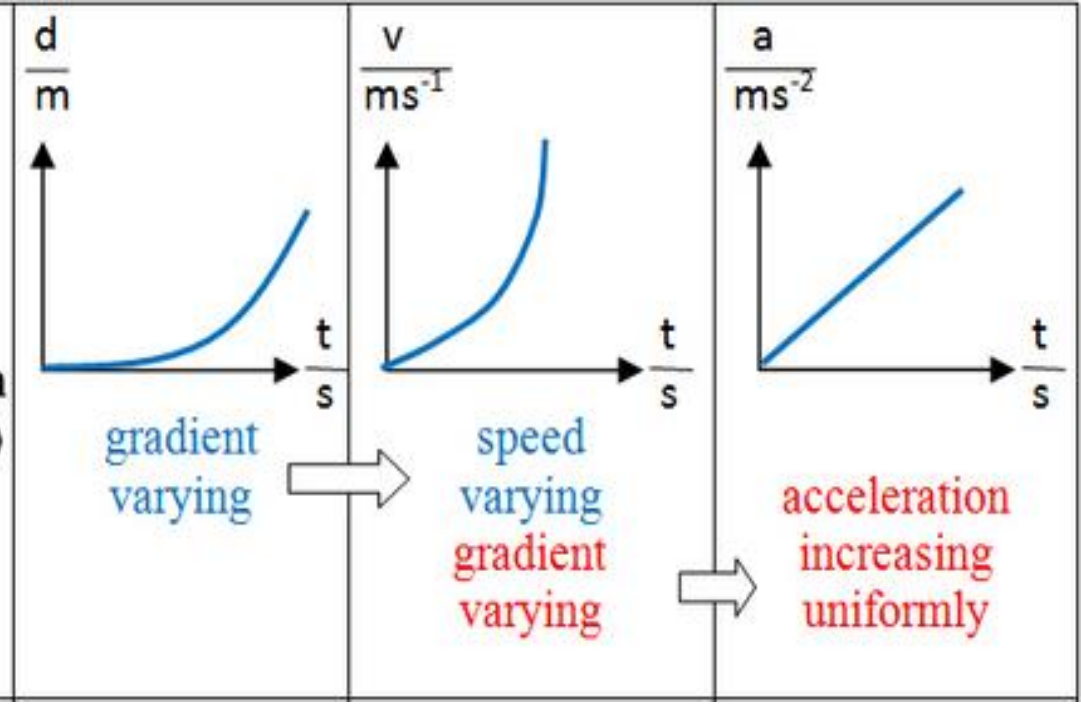
10 m/s

एकसमान त्वरण

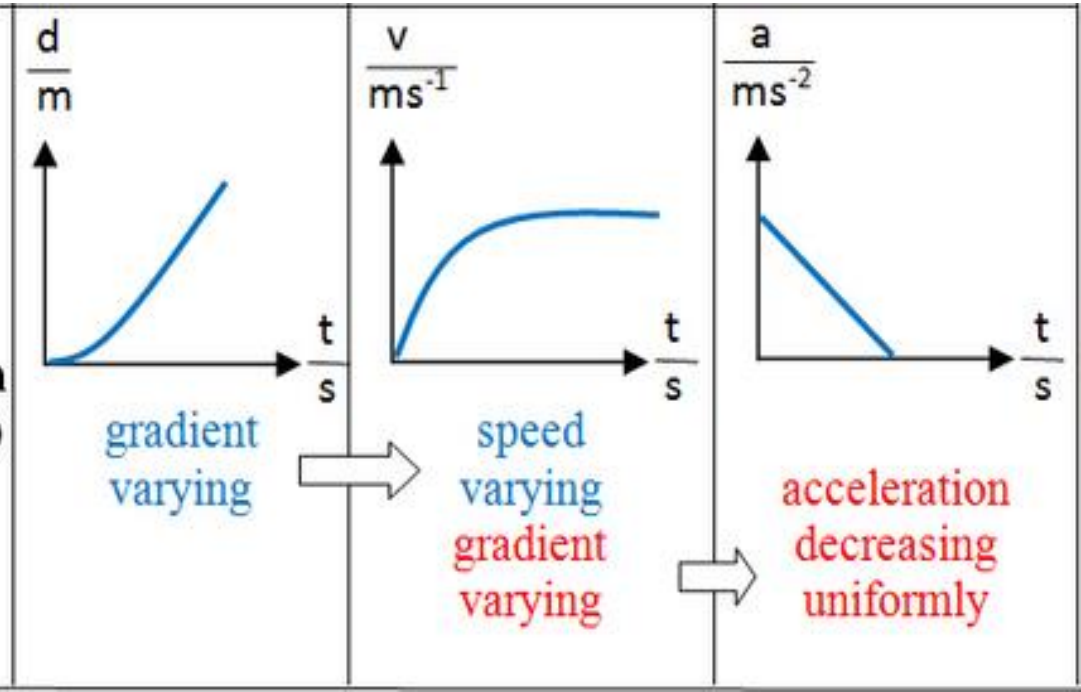
Uniform acceleration

	d / m vs t / s	v / ms^{-1} vs t / s	a / ms^{-2} vs t / s
	gradient varying	speed increasing uniformly	gradient = constant, k
			acceleration = constant, k

Increasing acceleration (non-uniform acceleration)

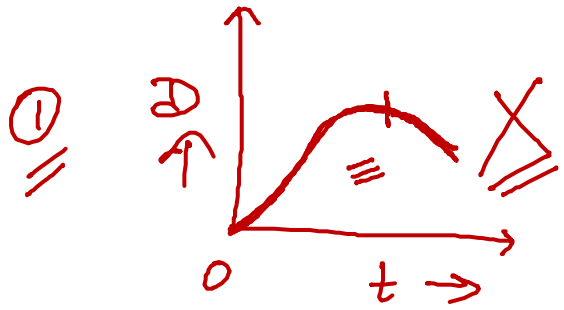


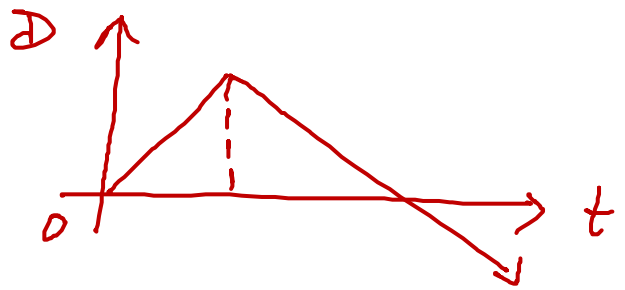
Decreasing acceleration (non-uniform acceleration)

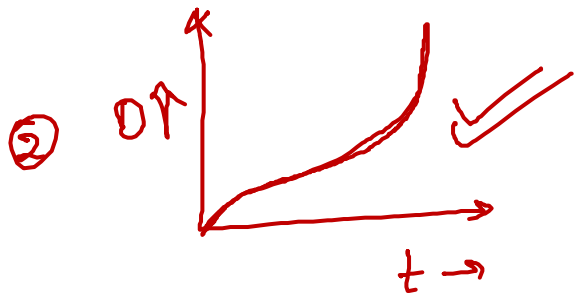


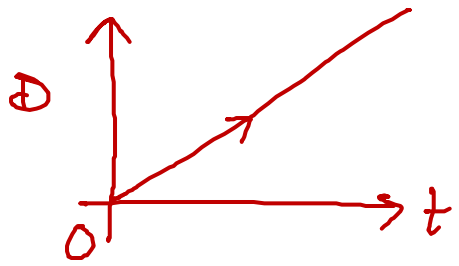
Distance - Time

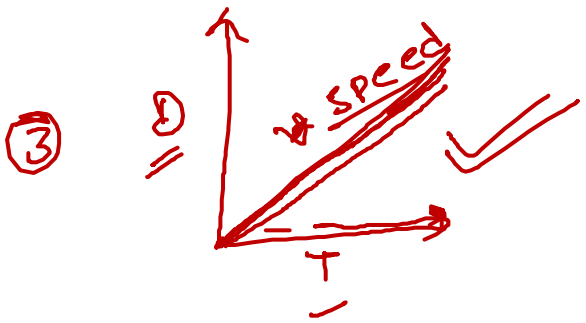
दूरी - समय

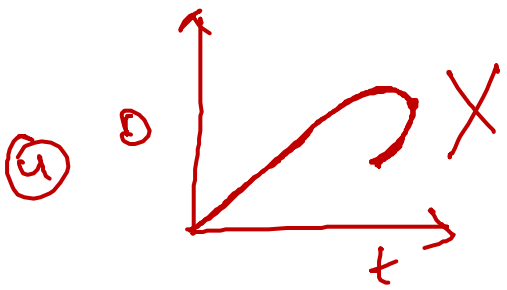


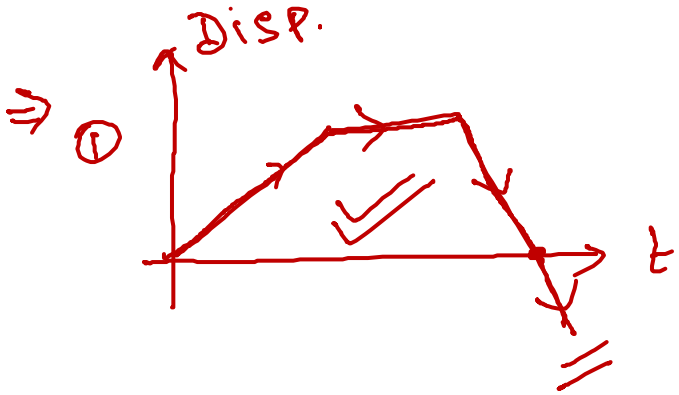






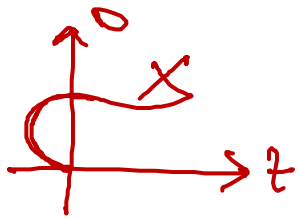


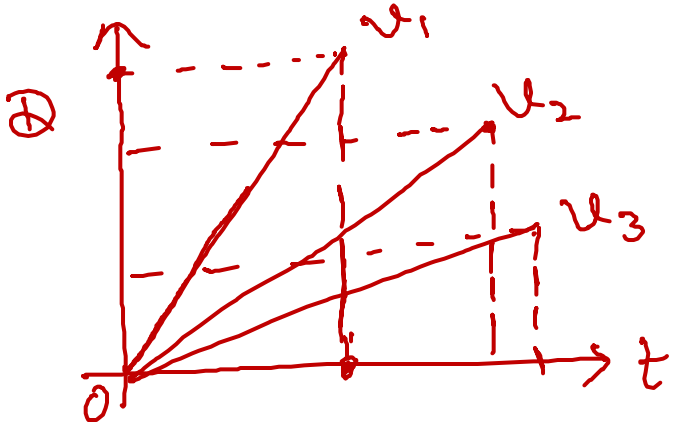




Displacement Time (विस्थापन समय)

(+ve) (−ve) (0)



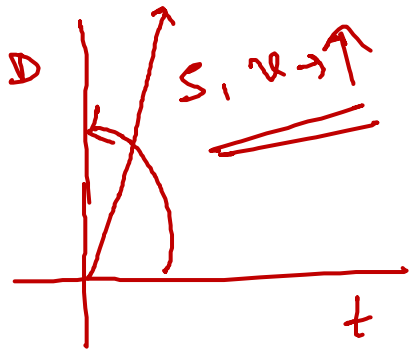


(i) $v_1 = v_2 = v_3$

(ii) $v_1 < v_2 < v_3$

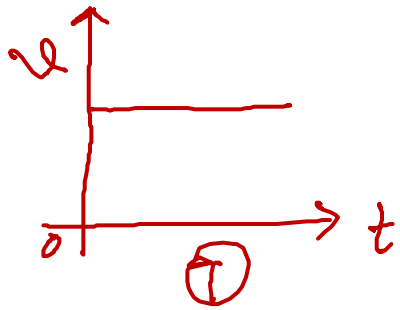
(iii) $v_1 > v_2 > v_3$ ✓✓

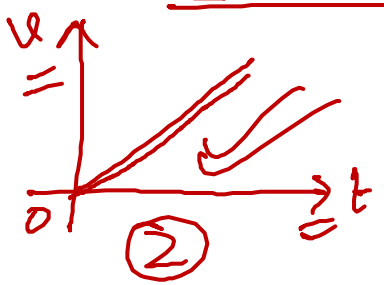
(iv) NOT



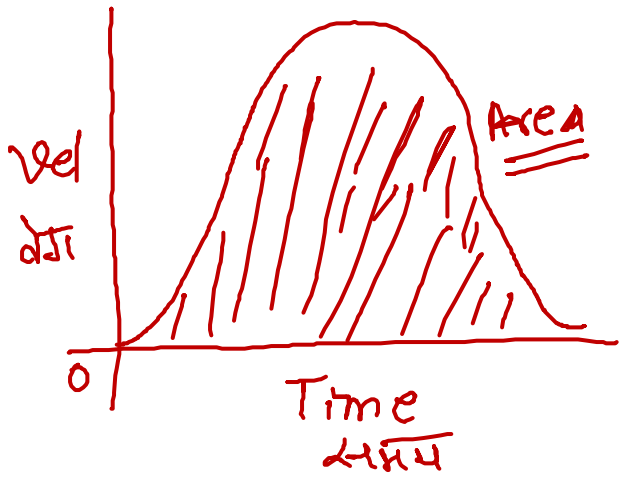
① Uniform Acc. (एकसमान त्वरण) , u, $\xrightarrow{t}$ v

$v = u + at$



① 

② 

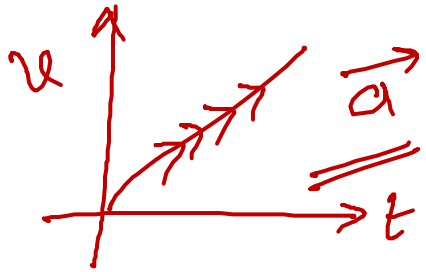
① 

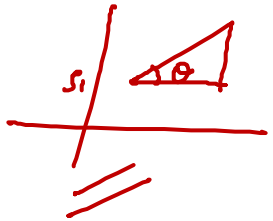
① Acceleration ज्ञात करना (Slope ढलान)

② Distance दूरी

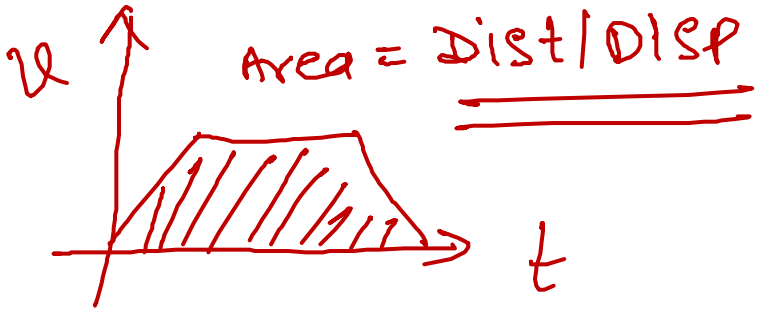
③ momentum

④ Unif. Accel. समत्वरण ज्ञात करना



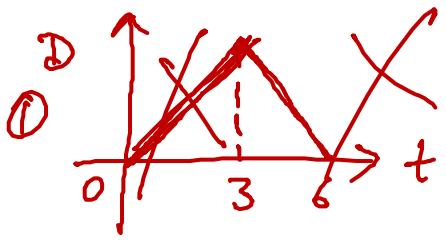


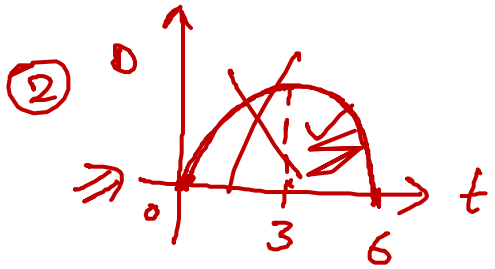
$\Rightarrow$

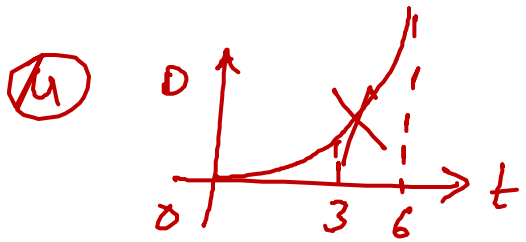


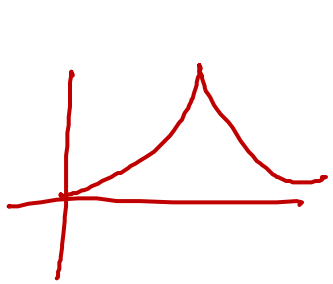
① Rest से चली है → Unif. Acc. ✓ → t → 3 sec तक → Rest में.

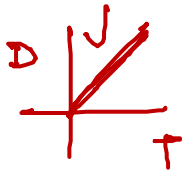
Unif Deacc. ⇒ t → 3 sec

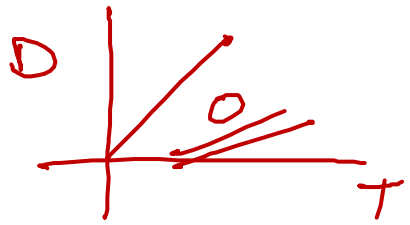


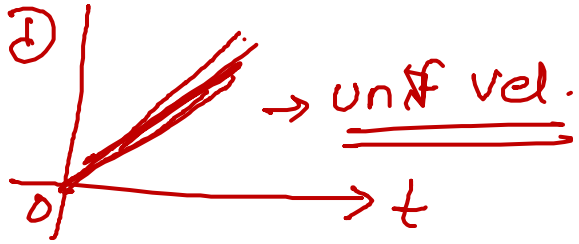


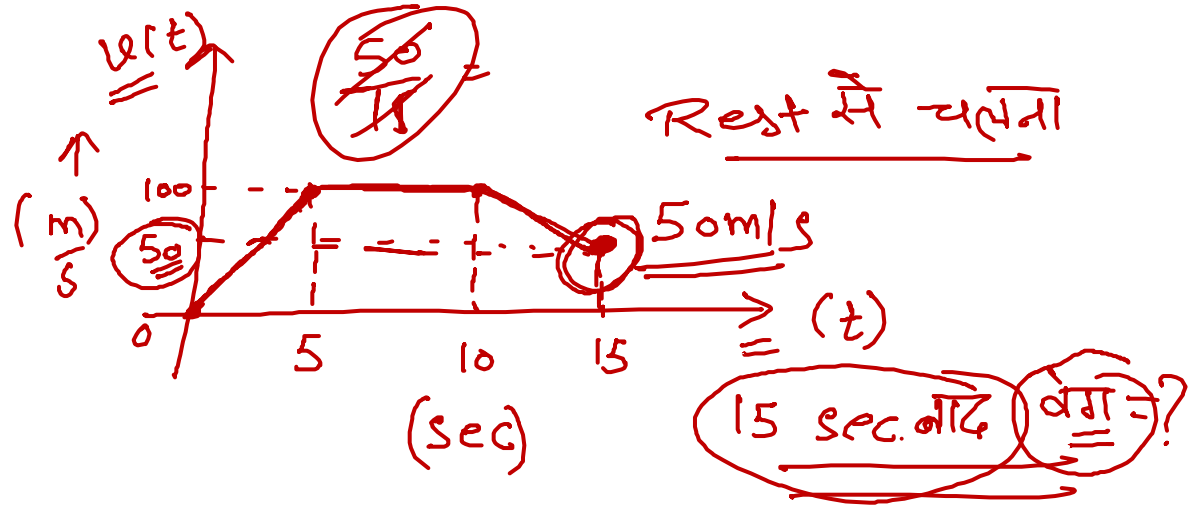










① 

Rest से चलना

15 sec. बाद वेग = ?

② n वें सेकेण्ड में चली गई दूरी,

Distance covered in n^{th} second,

$$S_n = u + \frac{1}{2}a(2n-1)$$

① Straight Line motion

5 वें sec → 14 m

8 वें sec → 20 m

15 वें sec → D ?

$S_n = u + \frac{1}{2} a(2n-1)$

5th $S_5 = u + \frac{1}{2} a(2\times5-1)$

$14 = u + \frac{1}{2} a(9)$

$28 = 2u + 9a$ —— (1)

8th $S_8 = u + \frac{1}{2} a(2\times8-1)$

$20 = u + \frac{1}{2} a(15)$

$40 = 2u + 15a$ —— (2)

$\Rightarrow 15a - 9a = 12$

$6a = 12$

$a = 2\ m/s^2$

$28 = 2u + 18$

$2u = 10$

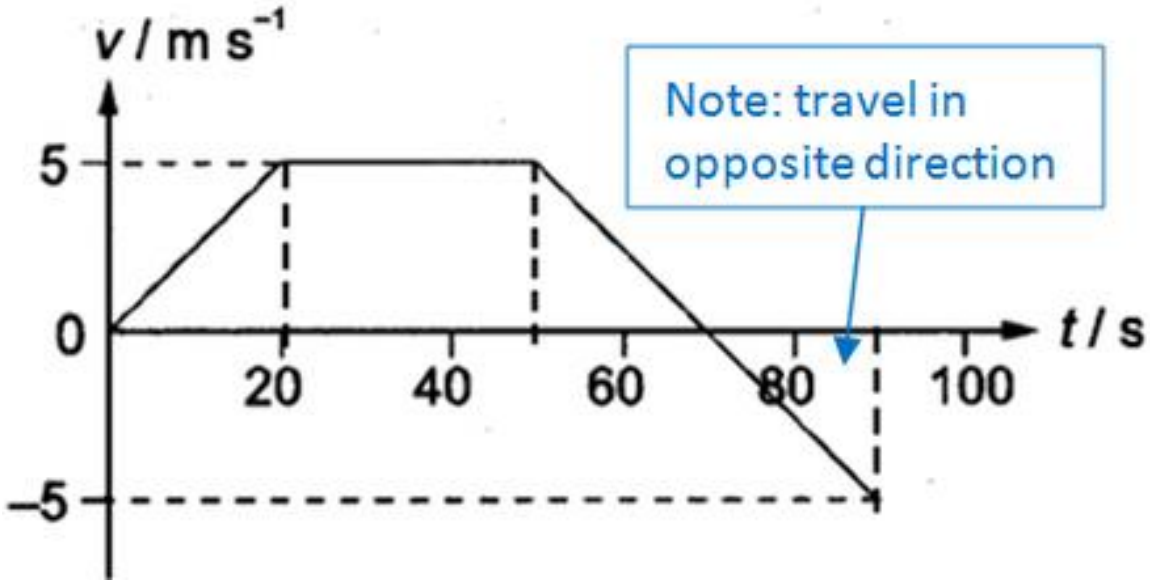
$u = 5\ m/s$

$S_{15} = 5 + \frac{1}{2} \times 2 (2\times15-1)$

$= 34$

A girl starts from rest and travels along a straight line. The diagram below shows the velocity-time graph of the girl from 0 s to 90 s.

H.W.



(a) Describe the motion of the girl from 0 s to 40 s.

(b) Find the average velocity of the girl in the first 70 s.

(c) Draw the acceleration-time graph of the girl from 0 s to 90 s.

(d) Find the displacement of the girl from the starting point to the position at 90 s.

Kinematics Formula Summary

For 1-D motion with *constant* acceleration:

- $v_f = v_0 + at$
- $\bar{v} = (v_0 + v_f)/2$
- $\Delta x = v_0 t + \frac{1}{2} a t^2$
- $v_f^2 - v_0^2 = 2a\Delta x$

Prob.

- A particle moves according to the equation $x=10t^2$ (x in meters, t in seconds).Find the **average velocity** for the time interval from 2s to 3s.

- Ans: 50m/s

Projectile Motion ⇒ प्रक्षेप्य का पथ ⇒ परवलयाकार Parabolic

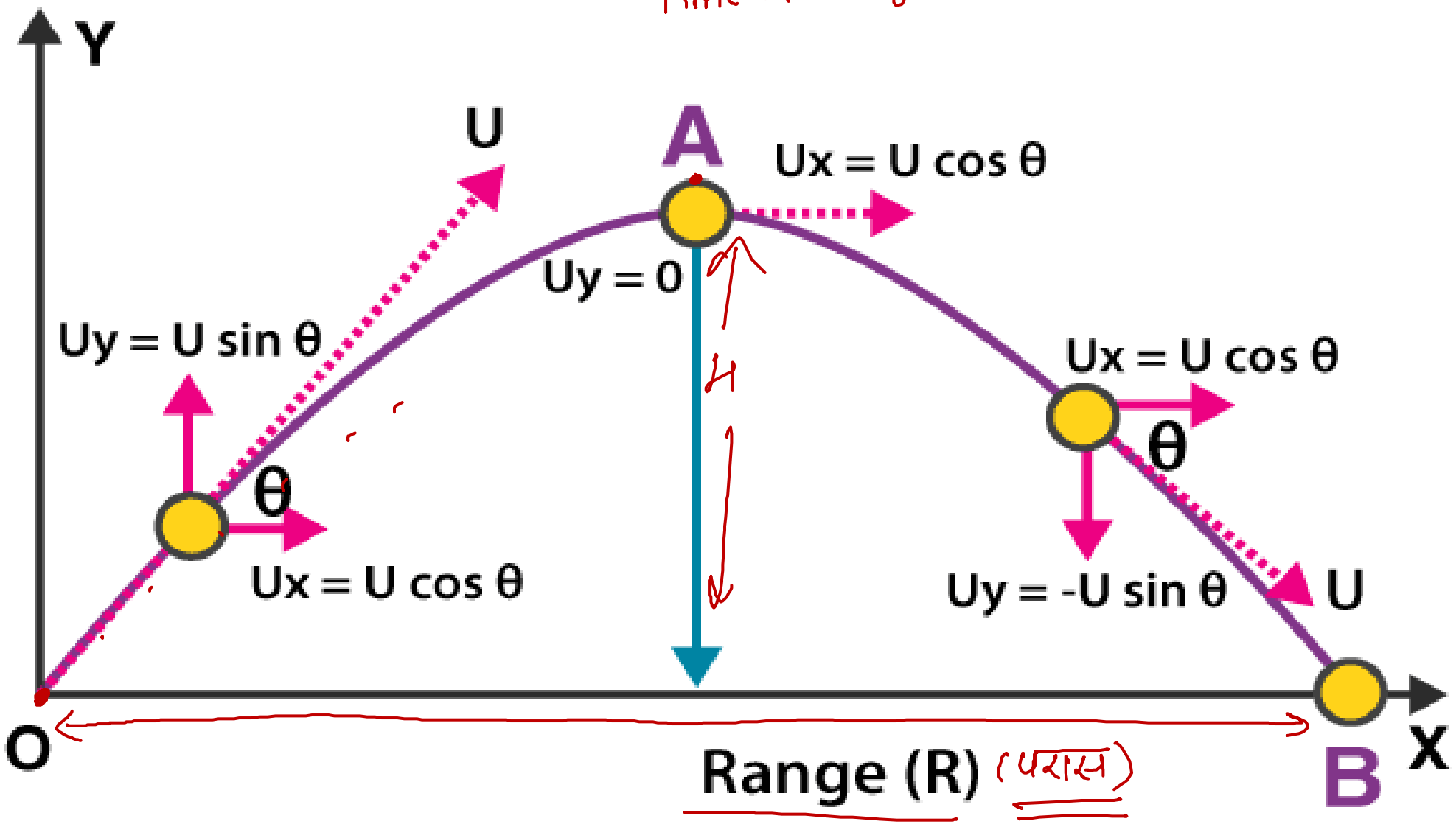
- **In a Projectile Motion, there are two simultaneous independent rectilinear motions:**

1. **Along the x-axis:** uniform velocity, responsible for the **horizontal** (forward) **motion** of the particle.

2. **Along y-axis:** uniform acceleration, responsible for the **vertical** (downwards) **motion** of the particle.

PROJECTILE MOTION

Time of flight : ?



1. $v = u - gt$

2. $s = ut - \frac{1}{2}gt^2$

3. $v^2 = u^2 - 2gs$

u = Initial velocity | g = Acceleration due to gravity

t = Time | s = Displacement | v = Final velocity

- Total Time of Flight: Resultant displacement (s) = 0 in Vertical direction. Therefore, by using the Equation of motion:

- $gt2 = 2(uyt - sy)$ [Here, $uy = u \sin \theta$ and $sy = 0$]

- i.e. $gt2 = 2t \times u \sin \theta$

- Therefore, the total time of flight (t):

- Total Time of Flight(t)= $\boxed{T_f = \frac{2u \sin\theta}{g}}$

- Horizontal Range: Horizontal Range (OA) = Horizontal component of velocity (ux) × Total Flight Time (t)

- $R = u \cos\theta \times \frac{2u \times \sin\theta}{g}$

- Therefore, in a projectile motion the Horizontal Range is given by (R):

- Horizontal Range(R)= $\frac{u^2 \sin 2\theta}{g}$ $= \quad 45^\circ$

- Maximum Height: It is the highest point of the trajectory (point A). When the ball is at point A, the vertical component of the velocity will be zero. i.e. $0 = (u \sin \theta)2 - 2g\ Hmax$ [$s = Hmax$, $v = 0$ and $u = u \sin \theta$]

- Therefore, in projectile motion, the Maximum Height is given by (Hmax):

- Maximum Height (Hmax)= $\frac{u^2 \sin^2 \theta}{2g}$

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