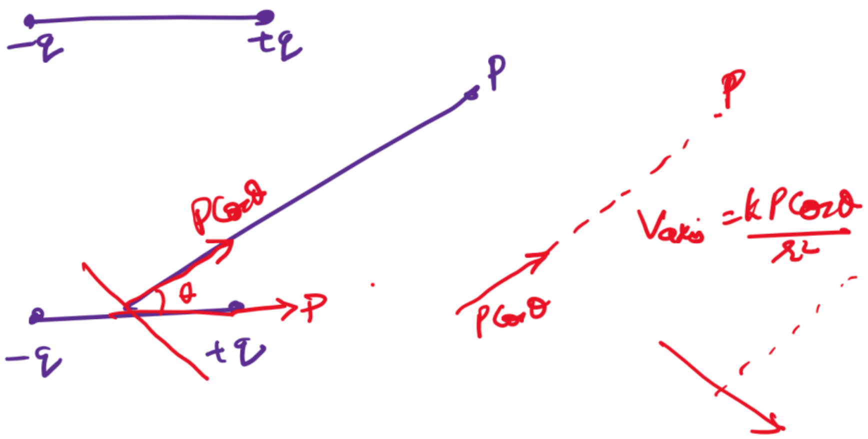
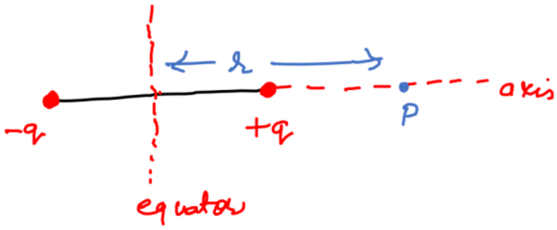


Electric Dipole

Potential due to Dipole :-

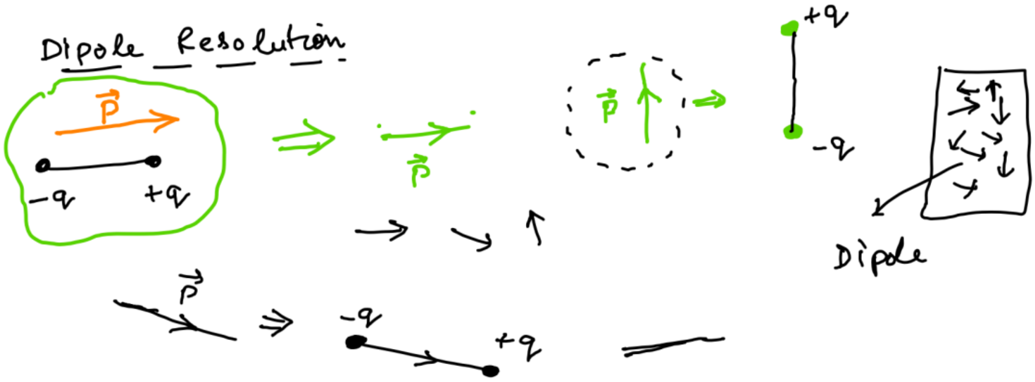


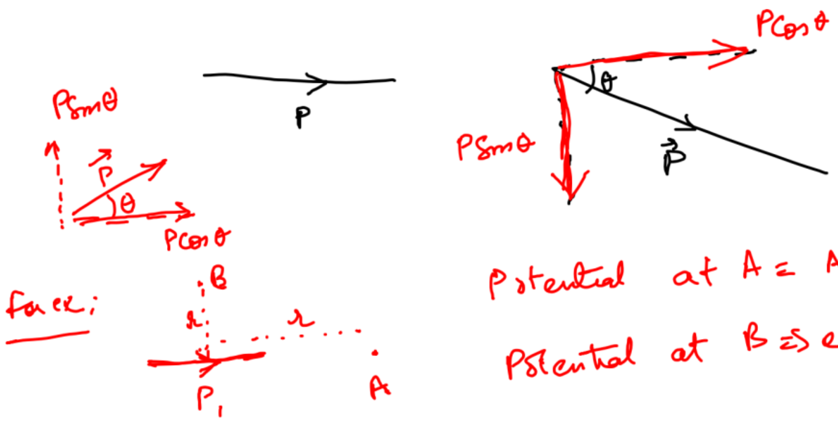
$V_{axis} = \frac{KP}{r^2 - \frac{d^2}{4}}$ if $r >> d$ $V_{axis} = \frac{KP}{r^2}$

$V_{equator} = 0$

Potential at any other Point :-

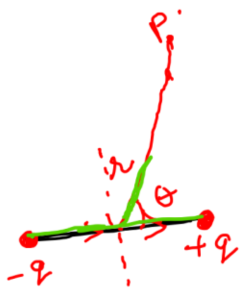
Dipole Resolution





Potential at A = Axis ⇒ $\frac{KP}{r^2}$

Potential at B ⇒ equator = 0



Let P is a point at distance r from the center of dipole, at angle θ

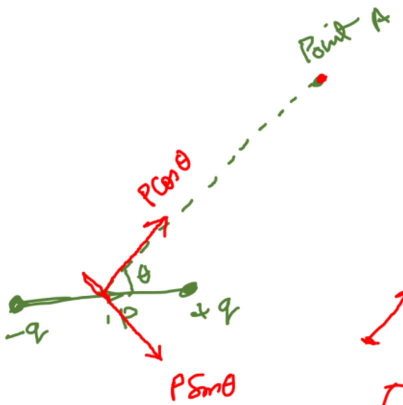
Now Dipole is resolved into two components.

P = Dipole Moment

$\vec{P} = qd$

We have two Dipole moments

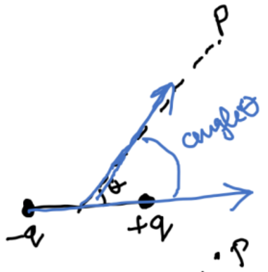
① PCosθ ② PSinθ



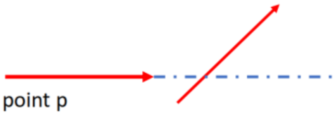
$V_1 = \frac{KP\cos\theta}{r^2}$

$V_2 = 0$

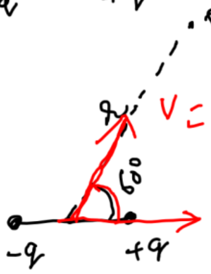
Net Potential $V = \frac{KP\cos\theta}{r^2}$



To determine angleθ
Join charge –q to +q
Join center of dipole to point p

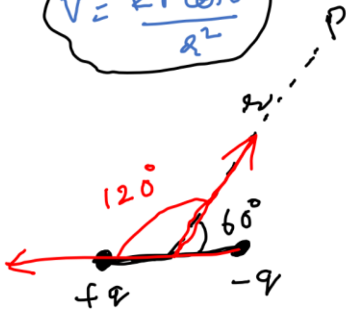


$$V = \frac{kP\cos\theta}{r^2}$$



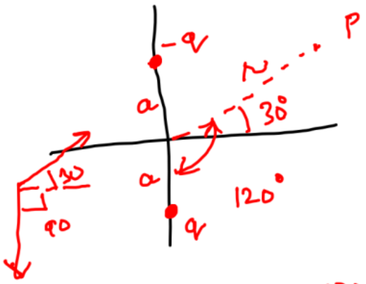
$$V = \frac{KP\cos 60}{r^2}$$

$$V = \frac{KP}{2r^2}$$



$$V = \frac{kP\cos 120}{r^2}$$

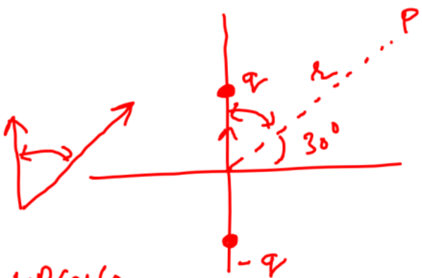
$$V = -\frac{kP}{2r^2}$$



$$V = \frac{kP\cos 120}{r^2}$$

$$\Rightarrow -\frac{kP}{2r^2}$$

Potential at P = ?



$$V = \frac{kP\cos 60}{r^2}$$

$$= \frac{kP}{2r^2}$$

Electric field due to Dipole :→

Potential $V = \frac{k p \cos\theta}{r^2}$ $V_{axis} = \frac{kp}{r^2}$ $V_{eq} = 0$

① At the axis

Two electric fields at point P

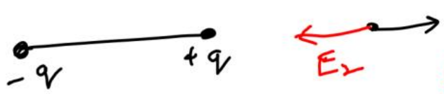
$E_1 = $ due to $+q \Rightarrow \frac{kq}{(r - \frac{d}{2})^2}$

$E_2 \Rightarrow$ due to $-q = \frac{kq}{(r + d/2)^2}$

(+q) ⟶ E ⇒ away from ⊕q

(-q) ⟵ E ⇒ towards the negative

$E_{net} = E_1 - E_2$



$E_{net} = kq\left[\frac{1}{(r - \frac{d}{2})^2} - \frac{1}{(r + \frac{d}{2})^2}\right]$

E

$$E = kq\left[\frac{\left(r+\frac{d}{2}\right)^2 - \left(r-\frac{d}{2}\right)^2}{\left(r-\frac{d}{2}\right)^2\left(r+\frac{d}{2}\right)^2}\right]$$

$$E = kq\left[\frac{2rd}{\left(r^2-\frac{d^2}{4}\right)^2}\right] = \frac{2k(qd)\,r}{\left(r^2-\frac{d^2}{4}\right)^2}$$

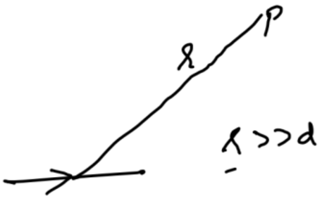
$$E = \frac{2kPr}{\left(r^2-\frac{d^2}{4}\right)^2}$$

If $r >>> d$

$$E_{axis} = \frac{2kP}{r^3}$$

special case ← $E_{axis} = \frac{2kP}{r^3}$ ⇒ For numericals Short Method

$$E_{axis} = \frac{2kPr}{\left(r^2-\frac{d^2}{4}\right)^2}$$



$r >> d$

$V_{axis} = \frac{kP}{r^2}$

Short Method

$$E = -\frac{dV}{dr}$$

$r^n = n\,r^{n-1}$

$r^{-2} = -2\,r^{-2-1} = \frac{-2}{r^3}$

$\frac{1}{r^2}$ diff $= \frac{-2}{r^3}$

$$E_{axis} = -\frac{dV}{dr}$$

$$V_{axis} = \frac{kp}{r^2}$$

$$= -kp\frac{d}{dr}\left(\frac{1}{r^2}\right)$$

$$\frac{1}{r^2} = r^{-2}$$

differentiate

$$= -kp\left[\frac{-2}{r^3}\right]$$

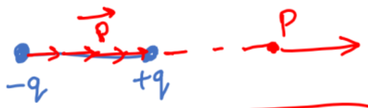
$$-2r^{-2-1} = -2r^{-3}$$

$$= -\frac{2}{r^3}$$

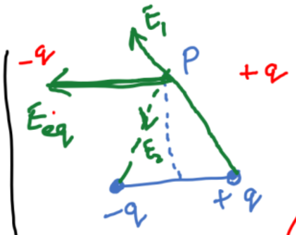
$$E_{axis} = \frac{2kp}{r^3}$$

Difference b/w Electric field at axis and eq.

$$E_{axis} = \frac{2kp}{r^3}$$



$E_{axis} \Rightarrow -q$ to $+q$



$$E_{eq} = \frac{kp}{r^3}$$

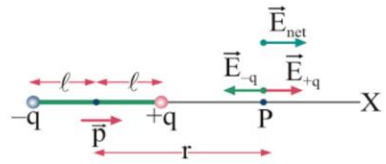
$$E_{eq} = \frac{E_{axis}}{2}$$

At equator E is opposite to E_{axis}, +q to -q

ELECTRIC DIPOLE

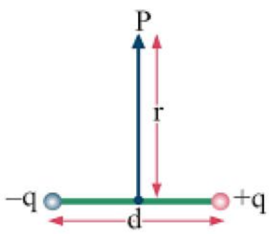
1. Electric field intensity due to an electric dipole at a point on the axial line

$$E_{axis} = \frac{2kp}{r^3} \quad (r \gg l)$$

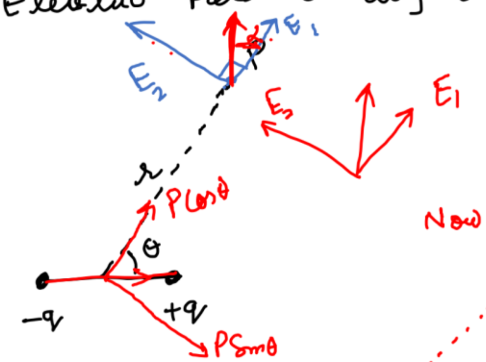


2. ·Electric field intensity due to an electric dipole at the equatorial point

$$E_{equatorial} = \frac{kp}{r^3} \quad (r \gg l)$$



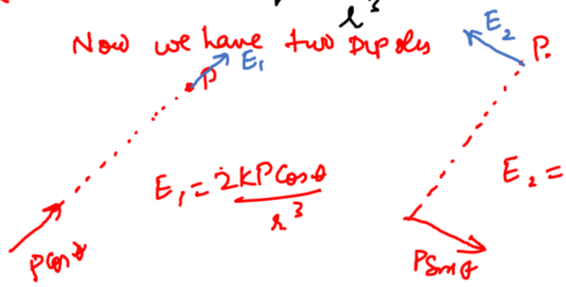
Electric field at any other Point



$E_{axis} = \frac{2kP}{r^3}$

$E_{eq} = \frac{kP}{r^3}$

Now we have two Dipoles



$E_1 = \frac{2kP\cos\theta}{r^3}$

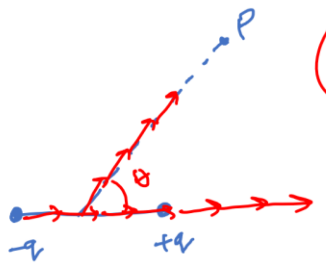
$E_2 = \frac{kP\sin\theta}{r^3}$

$E_{net} = \sqrt{E_1^2 + E_2^2}$

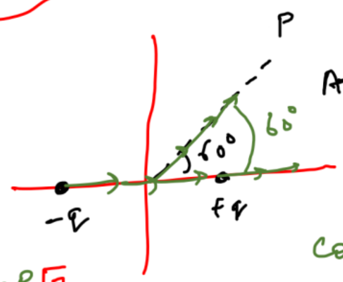
$$E = \frac{kP}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{kP}{r^3}\sqrt{3\cos^2\theta + (\cos^2\theta + \sin^2\theta)}$$

$$E = \frac{kP}{r^3}\sqrt{1 + 3\cos^2\theta}$$ → Important



$$E = \frac{kP}{r^3}\sqrt{1 + 3\cos^2\theta}$$



At P

E = ?

$$E = \frac{kP}{r^3}\sqrt{1 + 3\cos^2 60}$$

$$= \frac{kP}{r^3}\sqrt{1 + \frac{3}{4}} = \frac{kP\sqrt{7}}{2r^3}$$

$\cos 60 = \frac{1}{2}$

$\cos^2 60 = \frac{1}{4}$

$E_{axis} = \frac{2kp}{r^3}$ $E_{eq} = \frac{kp}{r^3}$

$E = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$

axis $\theta = 0$

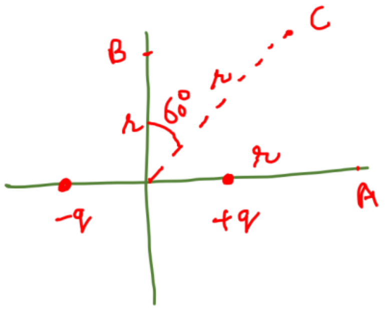
$E_{axis} = \frac{kp}{r^3}\sqrt{1+3}$

$= \frac{2kp}{r^3}$

equata $\theta = 90$

$\cos 90 = 0$

$E_{eq} = \frac{kp}{r^3}$



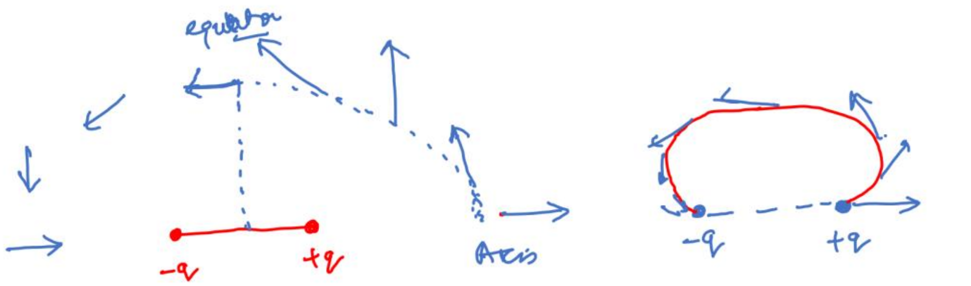
$E_A = \frac{2kp}{r^3}$

$E_B = \frac{kp}{r^3}$

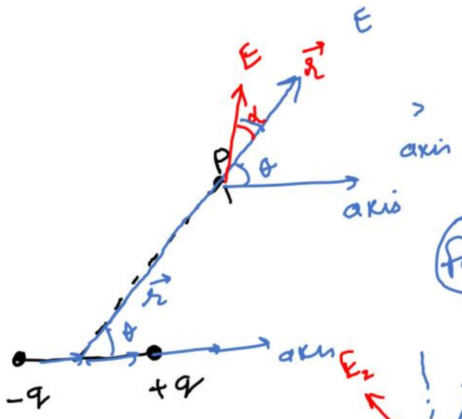
$E_C = \frac{kp}{r^3}\sqrt{1+3\cos^2 30}$

$\cos 30 = \frac{\sqrt{3}}{2}$

$\cos^2 30 = \frac{3}{4}$



From axis to equator Direction of E changes from 0° to 180°

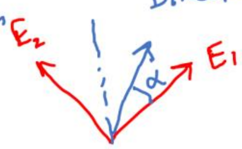


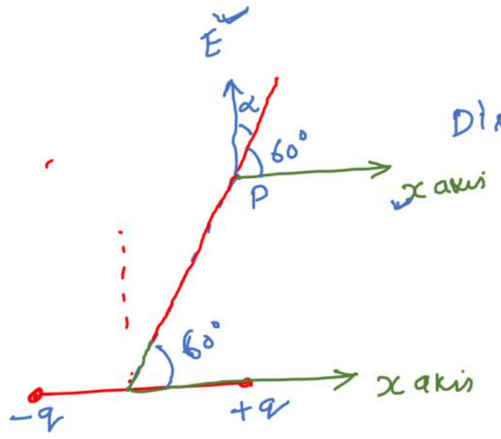
$$\tan\alpha = \frac{\tan\theta}{2}$$

$$\alpha = \tan^{-1}\left[\frac{\tan\theta}{2}\right]$$

from axis E = $\theta + \alpha$

Direction $\theta = \theta + \tan^{-1}\left[\frac{\tan\theta}{2}\right]$



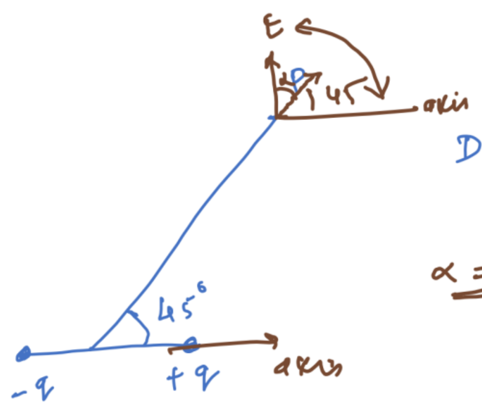


Direction of E at Point P with x axis?

$\tan\alpha = \frac{\tan\theta}{2} = \frac{\tan 60}{2}$

$\tan\alpha = \frac{\sqrt{3}}{2}$ $\quad \alpha = \tan^{-1}\left[\frac{\sqrt{3}}{2}\right]$

angle b/w E and x axis = $60^\circ + \alpha \Rightarrow 60^\circ + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$



Direction of E with axis?

$\alpha \Rightarrow \quad \tan\alpha = \frac{\tan 45}{2}$

$= \frac{1}{2}$

$\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

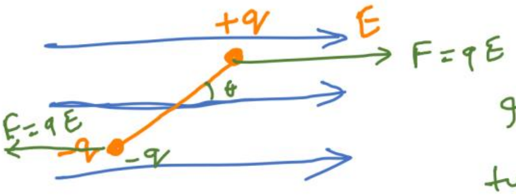
E and axis ⇒ angle = $45^\circ + \alpha \Rightarrow \frac{\pi}{4} + \tan^{-1}\left[\frac{1}{2}\right]$

Torque on Dipole

⇒ If a dipole is placed in uniform Electric field

Force on charge q

$F = qE$



If dipole is at angle θ two equal and opposite force act on dipole

Therefore a torque acts on dipole

Torque = Force × ⊥ distance

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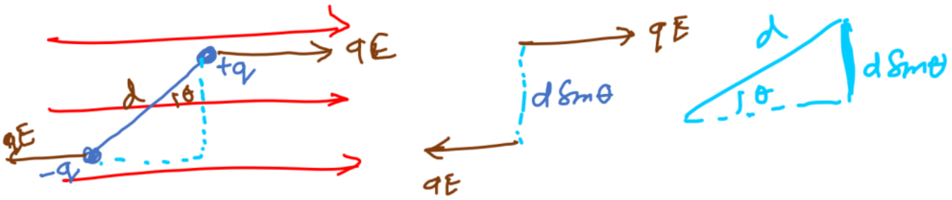
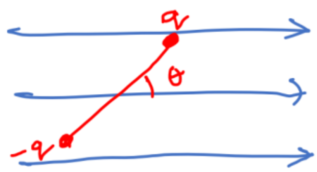


$\tau = (qE)(d\sin\theta) = E q d \sin\theta$

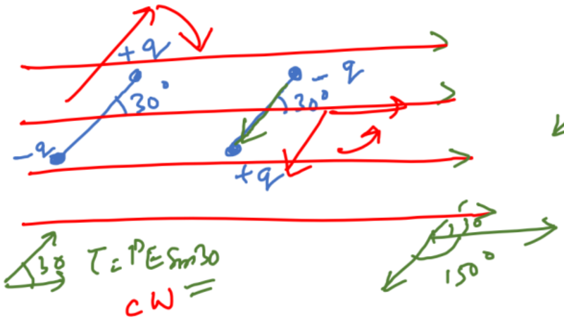
$\tau = EP\sin\theta$

$\tau = PE\sin\theta$



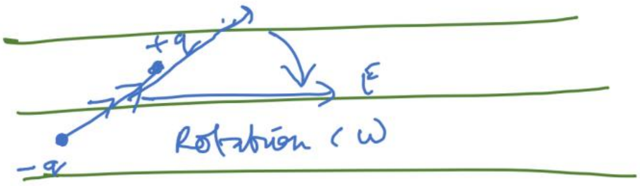
$\tau = PE \sin\theta$

For angle ⇒ Line −q to +q
Line of E



Sin30 = Sin150

$\tau = PE \sin 150$
ACW



Torque Direction
⇓
Right Hand $\vec{P}$ to $\vec{E}$
−q to +q

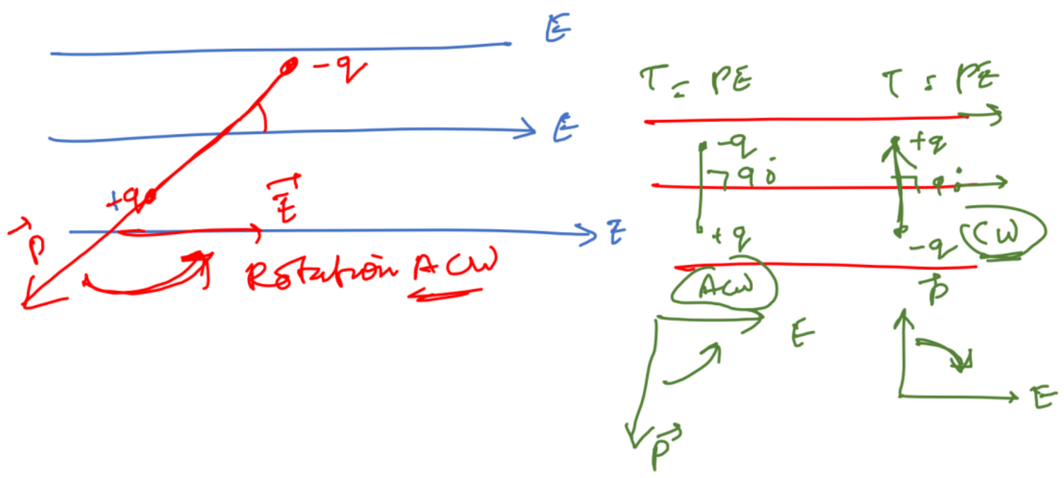
$\sin 30° = \frac{1}{2}$

$\sin 150$

$\sin(180-30)$

$\sin(90+60)$

$\cos 60 = \frac{1}{2}$



ELECTRIC DIPOLE

A small electric dipole is placed at origin with its dipole moment directed along positive x-axis. The direction of electric field at point $(2, 2\sqrt{2}, 0)$ is :

(A) along z-axis

(B) along y-axis

(C) along negative y-axis

(D) along negative z-axis

Determine the electric dipole moment of the system of the three charges, placed on the vertices of an equilateral triangle, as shown in the figure :

(A) $2q\ell\,\hat{j}$

(B) $(q\ell)\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(C) $\sqrt{3}\,q\ell\,\frac{\hat{j}-\hat{i}}{\sqrt{2}}$

(D) $-\sqrt{3}\,q\ell\,\hat{j}$