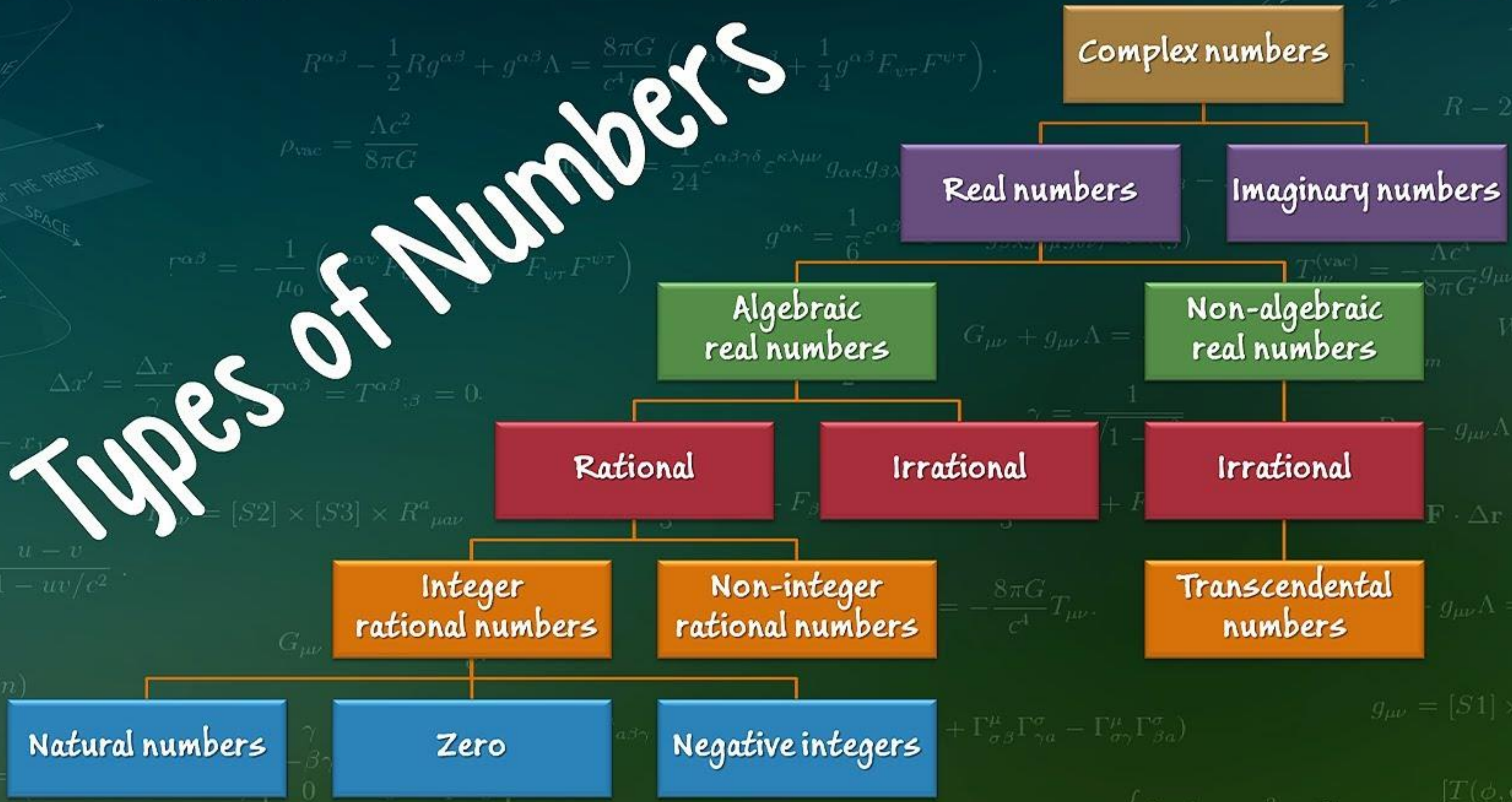
FUNCTION

Types of Numbers



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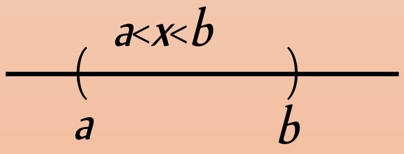
Arbitrary constant :

A constant which remains same in a particular operation, but changes with the change of reference, is called arbitrary constant

e.g. $y = mx + c$ represents a line. Here m and c are constants, but they are different for different lines. Therefore, m and c are arbitrary constants.

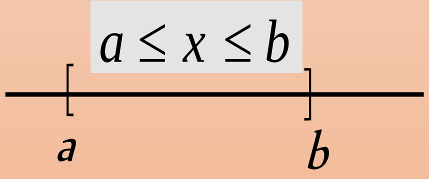
(1) **Open interval :** Let a and b be two real numbers such that $a<b$, then the set of all real numbers lying strictly between a and b is called an open interval and is denoted by $]a, b[$ or (a, b). Thus, $]a, b[$ or

$(a, b) = \{x \in R : a < x < b\}$



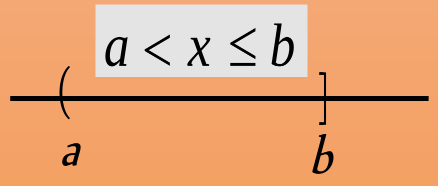
Open interval

(2) **Closed interval :** Let a and b be two real numbers such that $a<b$, then the set of all real numbers lying between a and b including a and b is called a closed interval and is denoted by $[a, b]$. Thus, $[a, b] = \{x \in R : a \leq x \leq b\}$



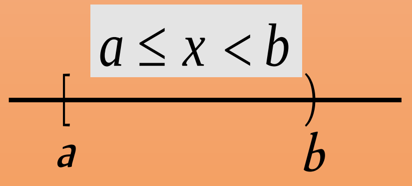
Closed interval

(3) **Open-Closed interval :** It is denoted by $]a, b]$ or $(a, b]$ and $]a, b]$ or $(a, b] = \{x \in R : a < x \leq b\}$

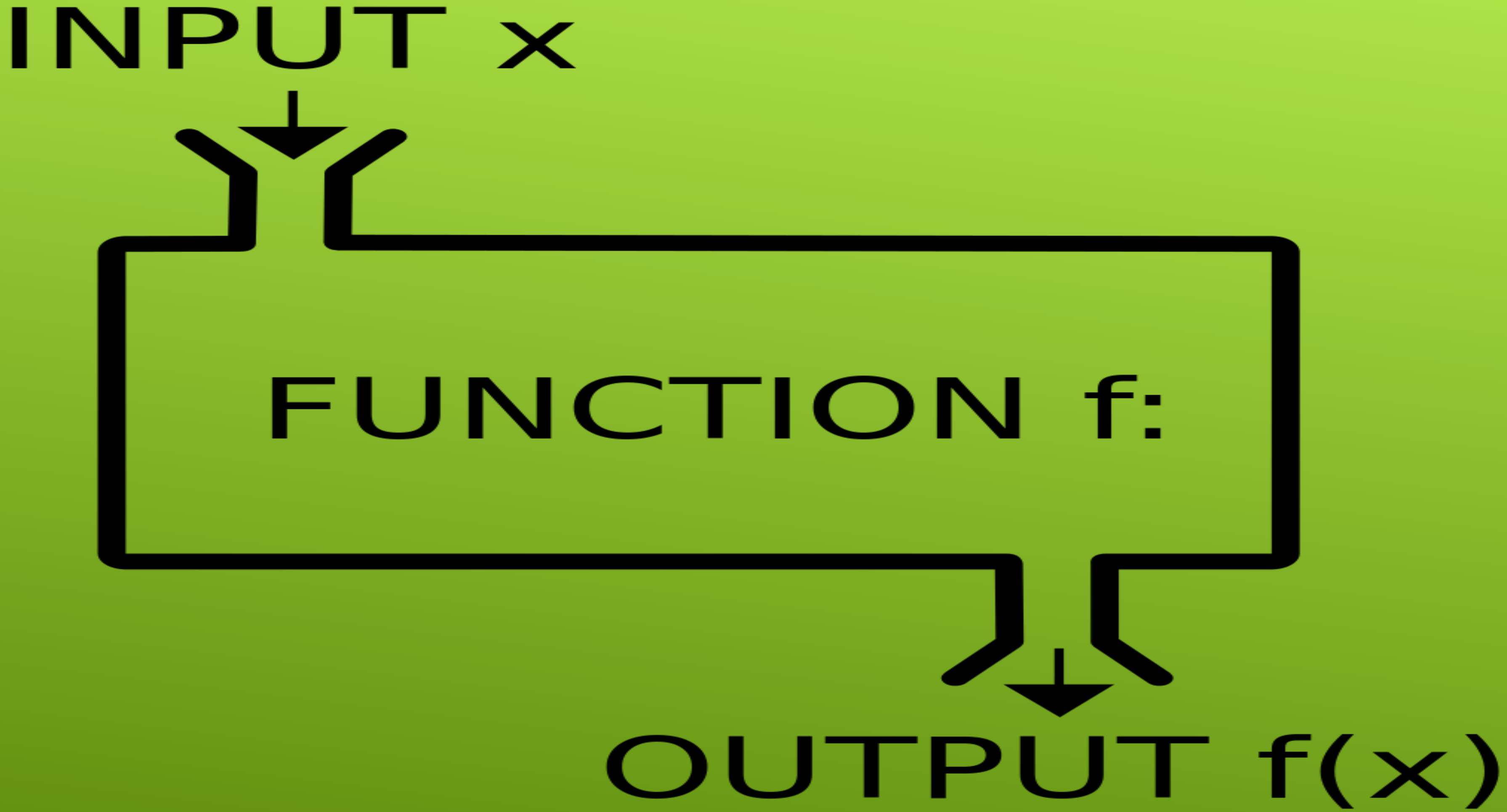


Open closed interval

(4) **Closed-Open interval :** It is denoted by $[a, b[$ or $[a, b)$ and $[a, b[$ or $[a, b) = \{x \in R : a \leq x < b\}$



Closed open interval


INPUT x
FUNCTION f:
OUTPUT f(x)

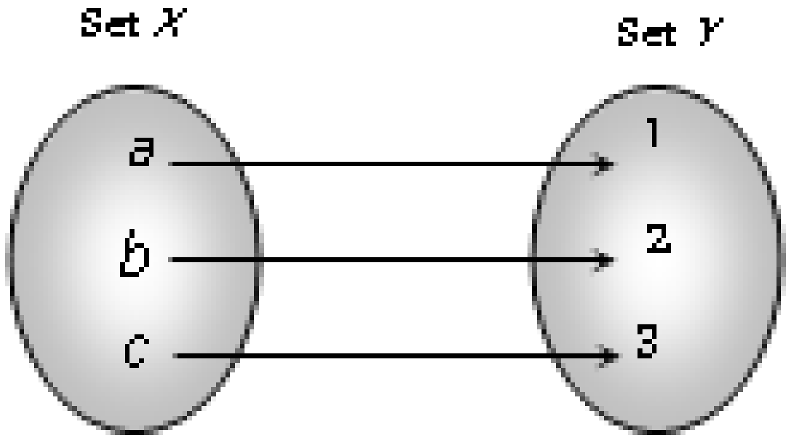
Function Notation

$$y = f(x)$$

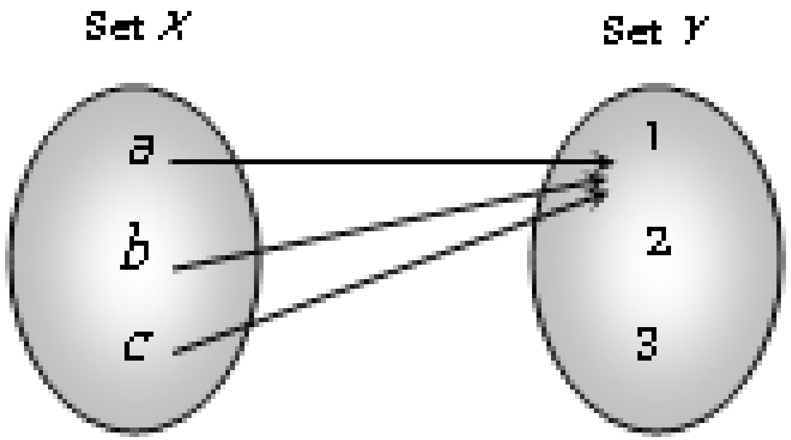
Output

Name of Function

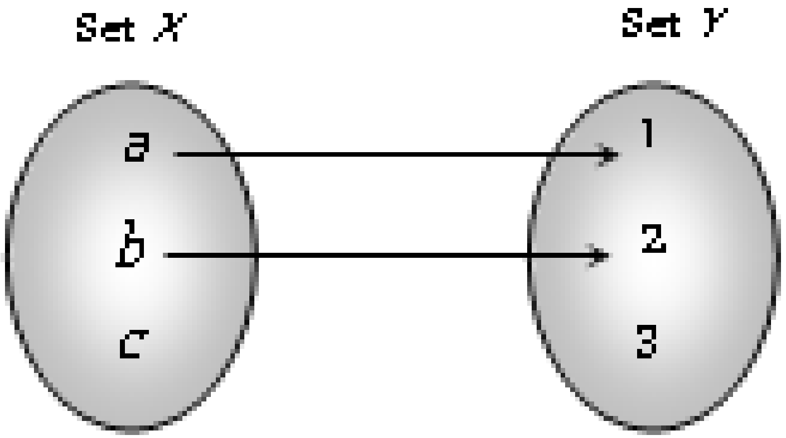
Input



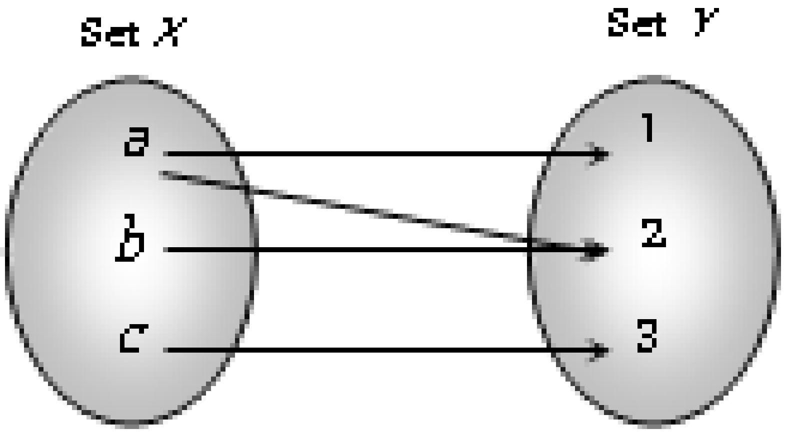
Function



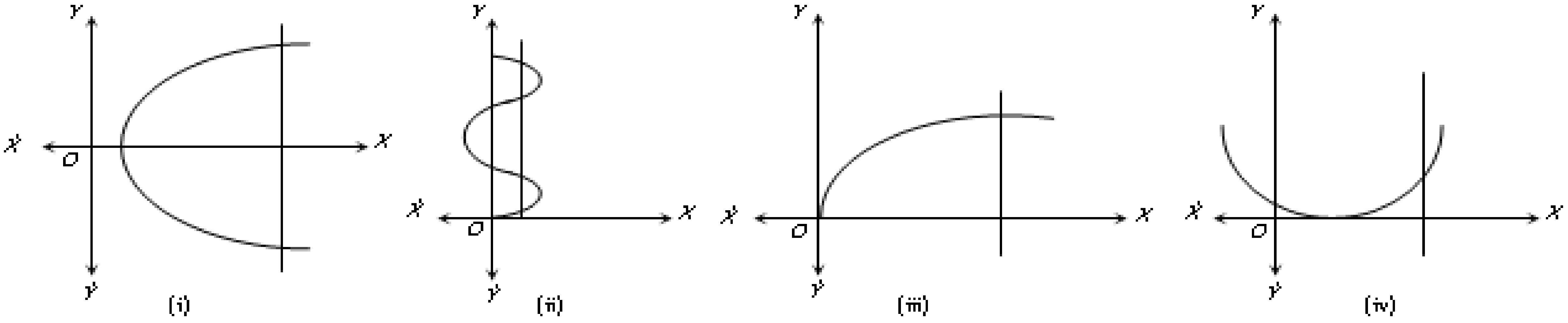
Function



Not function



Not function



Testing for a function by vertical line test :

A relation $f : A \to B$ is a function or not it can be checked by a graph of the relation.

If it is possible to draw a vertical line which cuts the given curve at more than one point then the given relation is not a function and when this vertical line means line parallel to Y-axis cuts the curve at only one point then it is a function.

Figure (iii) and (iv) represents a function.

Number of functions :

Let X and Y be two finite sets having m and n elements respectively.

Then each element of set X can be associated to any one of n elements of set Y.

So, total number of functions from set X to set Y is n^m .

If A contains 10 elements then total number of functions defined from A to A is [UPSEAT 1992]

(a) 10

(b) 2^{10}

(c) 10^{10}

(d) $2^{10}-1$

If $f(y) = \log y$, then $f(y) + f\left(\frac{1}{y}\right)$ is equal to [Rajasthan PET 1996]

(a) 2

(b) 1

(c) 0

(d) – 1

Given $f(y) = \log y$

$$\Rightarrow f(1/y) = \log(1/y),$$

then $f(y) + f\left(\dfrac{1}{y}\right) = \log y + \log(1/y) = \log 1 = 0.$

If $f(x) = \log\left[\frac{1+x}{1-x}\right]$, then $f\left[\frac{2x}{1+x^2}\right]$ is equal to

[MP PET 1999; Rajasthan PET 1999; UPSEAT 2003]

(a) $[f(x)]^2$ (b) $[f(x)]^3$

(c) $2f(x)$ (d) $3f(x)$

$$f(x) = \log\left(\frac{1+x}{1-x}\right)$$

$$\therefore\ f\left(\frac{2x}{1+x^2}\right) = \log\left[\frac{1+\dfrac{2x}{1+x^2}}{1-\dfrac{2x}{1+x^2}}\right] = \log\left[\frac{x^2+1+2x}{x^2+1-2x}\right]$$

$$= \log\left[\frac{1+x}{1-x}\right]^2 = 2\log\left[\frac{1+x}{1-x}\right] = 2f(x)$$

Domain, Co-domain and Range of Function

If a function f is defined from a set of A to set B then for $f: A \to B$

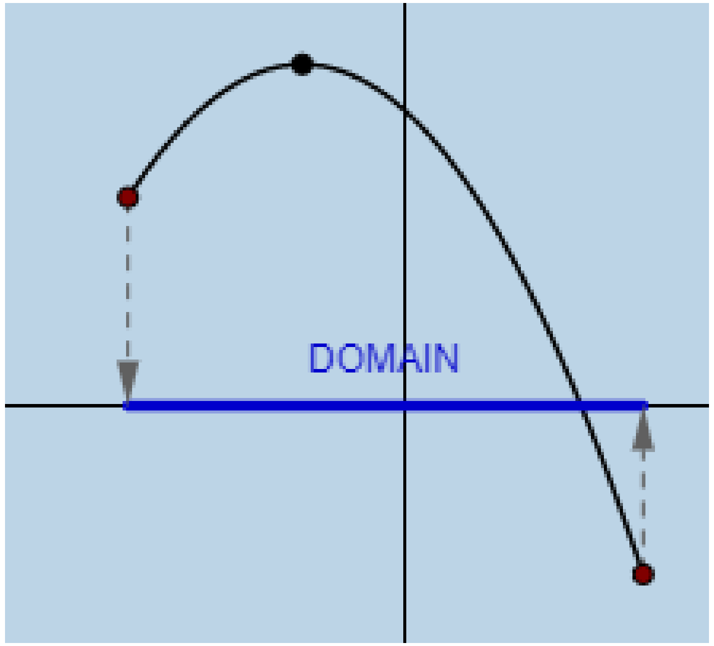
set A is called the domain of function f and set B is called the co-domain of function f.

The set of all f-images of the elements of A is called the range of function f.

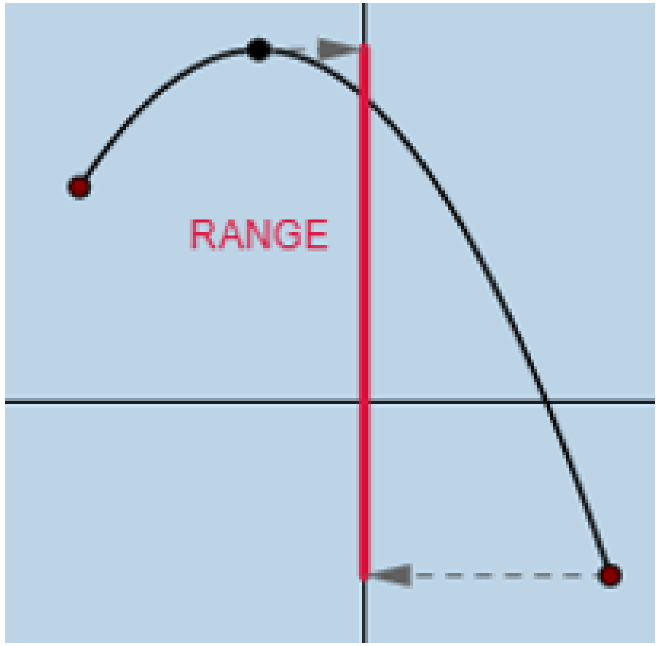
In other words, we can say Domain = All possible values of x for which $f(x)$ exists.

Range = For all values of x, all possible values of $f(x)$.

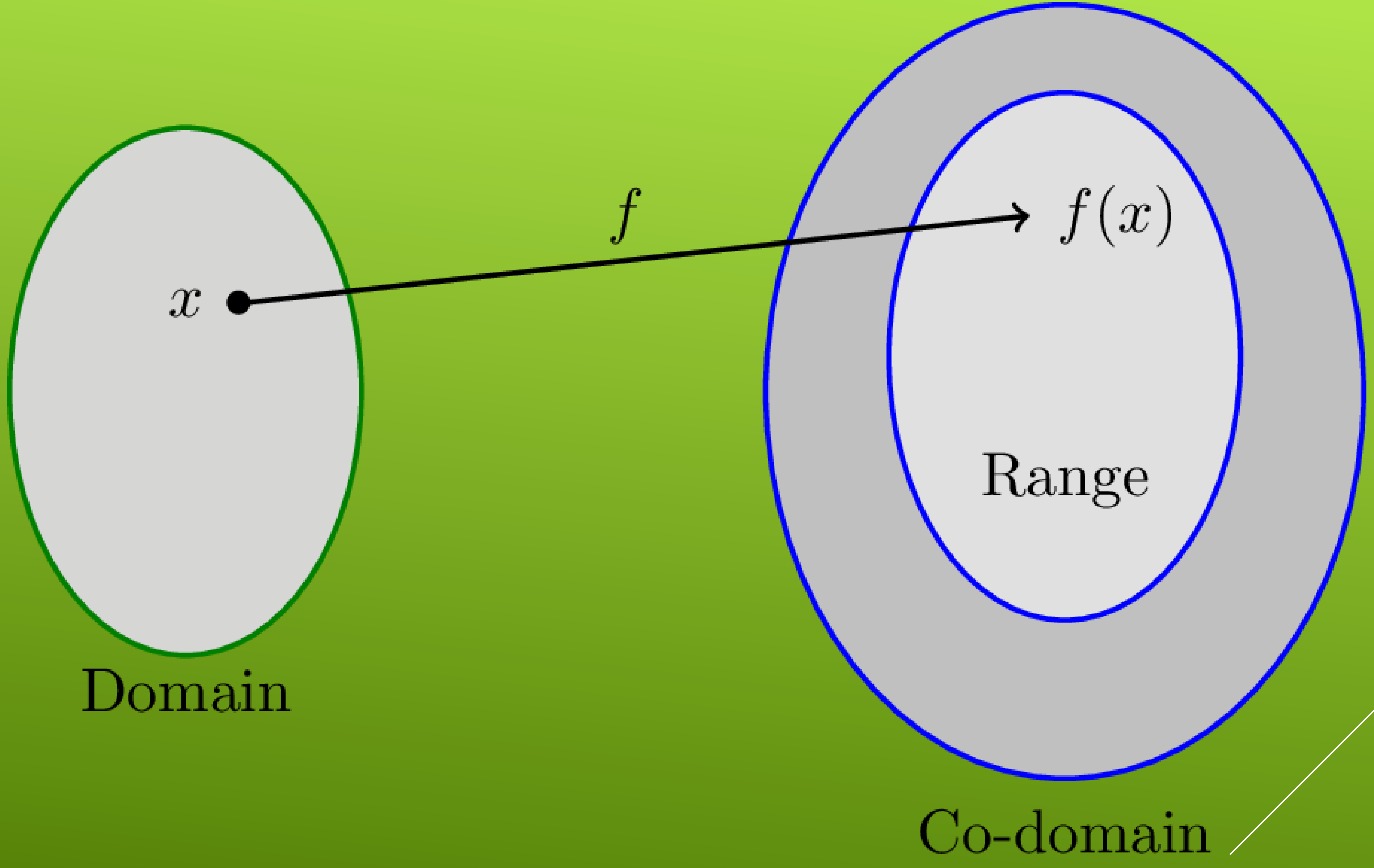
Domain and Range

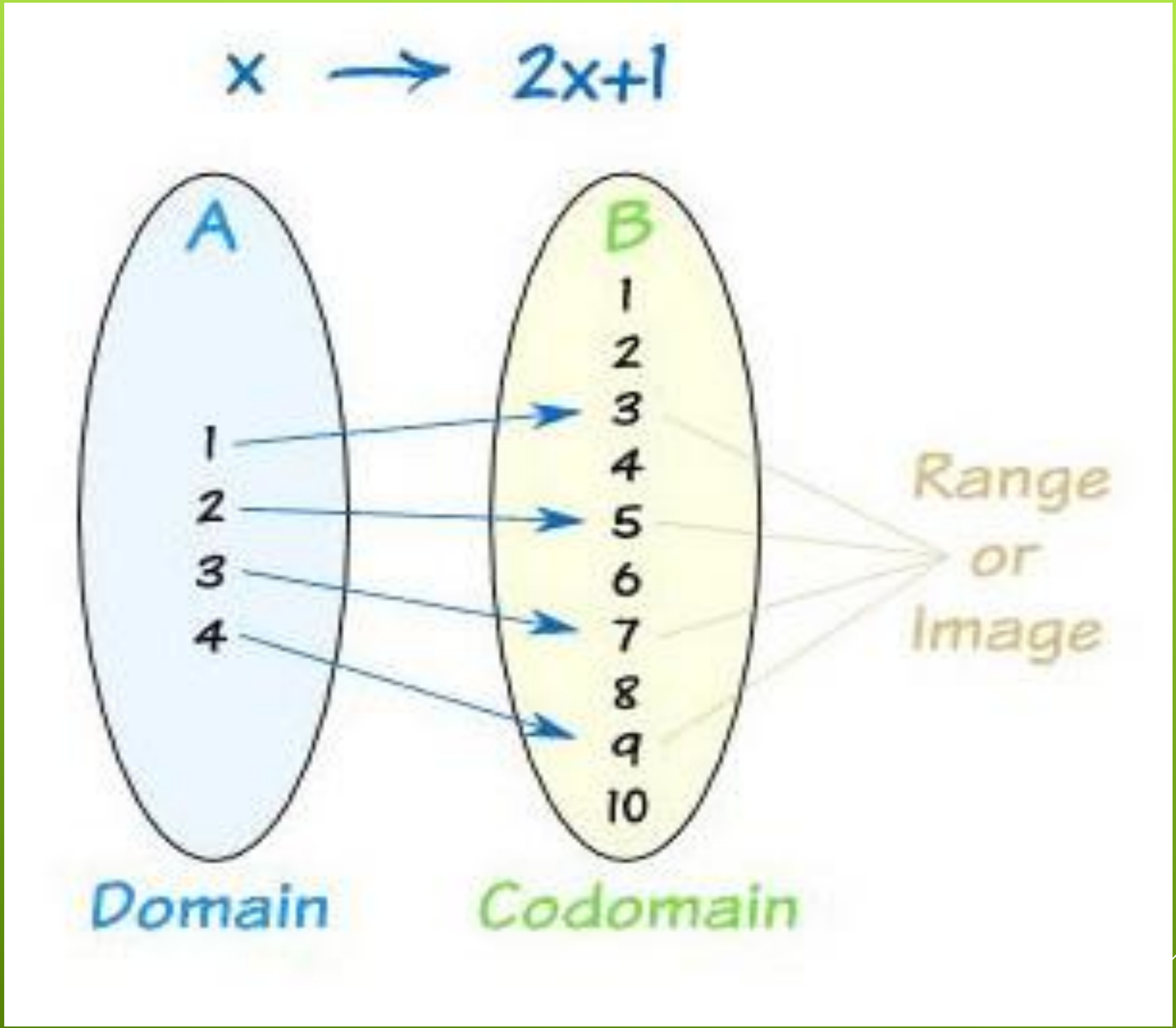


Domain is all the possible x values of a function.



Range is all the possible y values of a function.


f
x
$f(x)$
Range
Domain
Co-domain


x → 2x+1
A
1
2
3
4
B
1
2
3
4
5
6
7
8
9
10
Range
or
Image
Domain
Codomain

Domain

(a) Expression under even root (*i.e.*, square root, fourth root etc.) ≥ 0

(b) Denominator $\neq 0$.

(c) If domain of $y = f(x)$ and $y = g(x)$ are D_1 and D_2 respectively then the domain of $f(x) \pm g(x)$ or $f(x).g(x)$ is $D_1 \cap D_2$.

(d) While domain of $\dfrac{f(x)}{g(x)}$ is $D_1 \cap D_2 - \{g(x) = 0\}$.

Domain of $\left(\sqrt{f(x)}\right) = D_1 \cap \{x : f(x) \geq 0\}$

(i) **Range :** Range of $y = f(x)$ is collection of all outputs

$f(x)$ corresponding to each real number in the domain.

(a) If domain $\in$ finite number of points $\Rightarrow$ range $\in$ set of corresponding $f(x)$ values.

(b) If domain $\in R$ or $R - [\text{some finite points}]$. Then express x in terms of y.

From this find y for x to be defined (*i.e.*, find the values of y for which x exists).

If domain $\in$ a finite interval, find the least and greatest value for range using monotonicity

Domain of the function $\dfrac{1}{\sqrt{x^2-1}}$ is [Roorkee 1987; Rajasthan PET 2000]

(a) $(-\infty,-1)\cup(1,\infty)$

(b) $(-\infty,-1]\cup(1,\infty)$

(c) $(-\infty,-1)\cup[1,\infty)$

(d) None of these

For domain, $x^2-1>0 \Rightarrow (x-1)(x+1)>0$

$\Rightarrow x<-1$ or $x>1 \Rightarrow x\in(-\infty,-1)\cup(1,\infty)$.

The domain of the function $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is [Roorkee 1998]

(a) R^+ (b) R^-

(c) R_0 (d) R

For domain, $|x| - x > 0 \Rightarrow |x| > x$.

This is possible, only when $x \in R^-$.

Find the domain of definition of $f(x) = \dfrac{\log_2(x+3)}{x^2+3x+2}$ [IIT 2001; UPSEAT 2001]

(a) $(-3, \infty)$ (b) $\{-1, -2\}$

(c) $(-3, \infty) - \{-1, -2\}$ (d) $(-\infty, \infty)$

Here $f(x) = \dfrac{\log_2(x+3)}{x^2+3x+2} = \dfrac{\log_2(x+3)}{(x+1)(x+2)}$ exists if,

Numerator $x+3>0 \Rightarrow x>-3$ ...... (i)

and denominator $(x+1)(x+2) \neq 0 \Rightarrow x \neq -1, -2$ ...... (ii)

Thus, from (i) and (ii); we have domain of $f(x)$ is $(-3, \infty) - \{-1, -2\}$.

The domain of the function $f(x)=\sqrt{(2-2x-x^{2})}$ is [BIT Ranchi 1992]

(a) $-3 \le x \le \sqrt{3}$

(b) $-1-\sqrt{3} \le x \le -1+\sqrt{3}$

(c) $-2 \le x \le 2$

(d) None of these

The quantity square root is positive,

when $-1-\sqrt{3} \le x \le -1+\sqrt{3}$.

If the domain of function $f(x) = x^2 - 6x + 7$ is $(-\infty, \infty)$, then the range of function is [MP PET 1996]

(a) $(-\infty, \infty)$ (b) $[-2, \infty)$

(c) $(-2, 3)$ (d) $(-\infty, -2)$

$$x^2 - 6x + 7 = (x-3)^2 - 2$$

Obviously, minimum value is –2 and maximum ∞.

The domain of the function $f(x) = \sqrt{x - x^2} + \sqrt{4 + x} + \sqrt{4 - x}$ is

(a) $[-4, \infty)$

(b) $[-4, 4]$

(c) $[0, 4]$

(d) $[0, 1]$

$$f(x) = \sqrt{x - x^2} + \sqrt{4 + x} + \sqrt{4 - x}$$

clearly $f(x)$ is defined if

$$4 + x \geq 0 \Rightarrow x \geq -4$$

$$4 - x \geq 0 \Rightarrow x \leq 4$$

$$x(1 - x) \geq 0 \Rightarrow x \geq 0 \text{ and } x \leq 1$$

$$\therefore \text{ Domain of } f = (-\infty, 4] \cap [-4, \infty) \cap [0, 1] = [0, 1].$$

Domain of definition of the function $f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$, is [AIEEE 2003]

(a) $(1, 2)$

(b) $(-1, 0) \cup (1, 2)$

(c) $(1, 2) \cup (2, \infty)$

(d) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

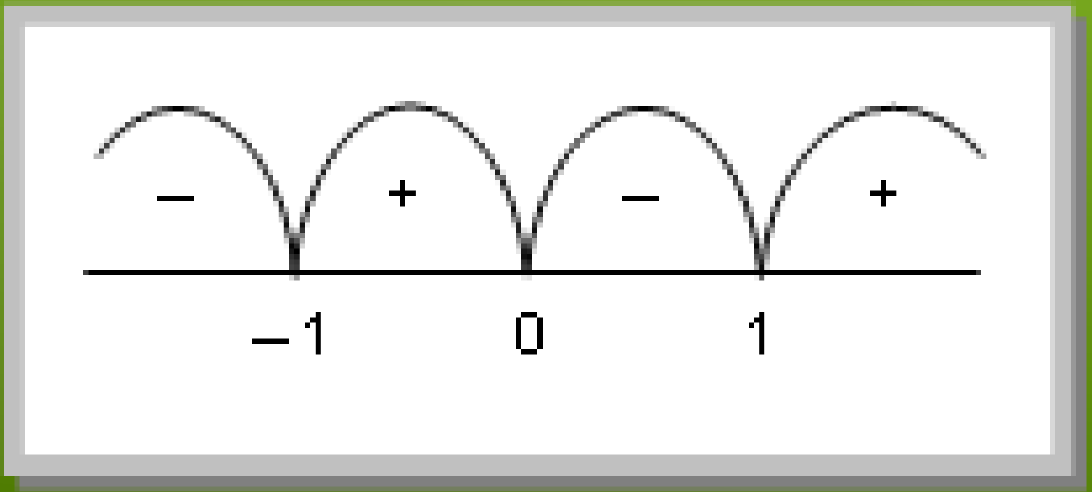
$$f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$$

So, $4 - x^2 \neq 0 \Rightarrow x \neq \pm\sqrt{4} \Rightarrow x \neq \pm 2$

and $x^3 - x > 0 \Rightarrow x(x^2 - 1) > 0 \Rightarrow x > 0, |x| > 1$

$\therefore \; D = (-1, 0) \cup (1, \infty) - \{2\}$

$$D = (-1, 0) \cup (1, 2) \cup (2, \infty).$$



The range of $\dfrac{1+x^2}{x^2}$ is [**Karnataka CET 1989**]

(a) $(0, 1)$

(b) $(1, \infty)$

(c) $[0, 1]$

(d) $[1, \infty)$

Let $y = \dfrac{1+x^2}{x^2}$ $\Rightarrow x^2 y = 1 + x^2 \Rightarrow x^2(y-1) = 1 \Rightarrow x^2 = \dfrac{1}{y-1}$

Now since, $x^2 > 0 \Rightarrow \dfrac{1}{y-1} > 0 \Rightarrow (y-1) > 0 \Rightarrow y > 1 \Rightarrow y \in (1, \infty)$

Trick : $y = \dfrac{1+x^2}{x^2} = 1 + \dfrac{1}{x^2}$. Now since, $\dfrac{1}{x^2}$ is always > 0 $\Rightarrow y > 1 \Rightarrow y \in (1, \infty)$.

Range of the function $f(x)=\dfrac{x^2+x+2}{x^2+x+1}; x \in R$ is [IIT Screening 2003]

(a) $(1, \infty)$

(b) $(1, 11/7)$

(c) $(1, 7/3]$

(d) $(1, 7/5]$

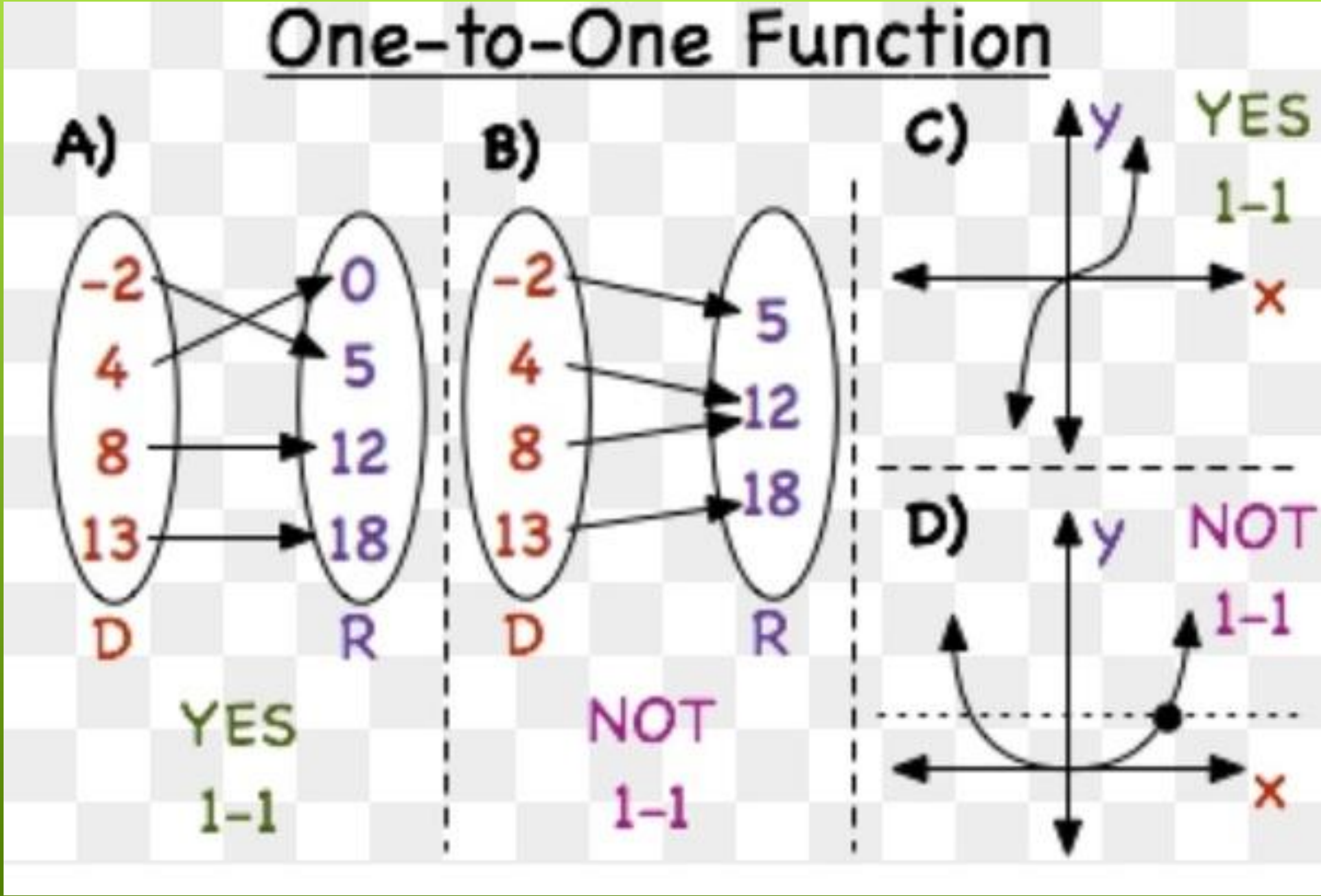
$$f(x)=1+\frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} \Rightarrow \text{Range } = (1, 7/3].$$

Kinds of Function

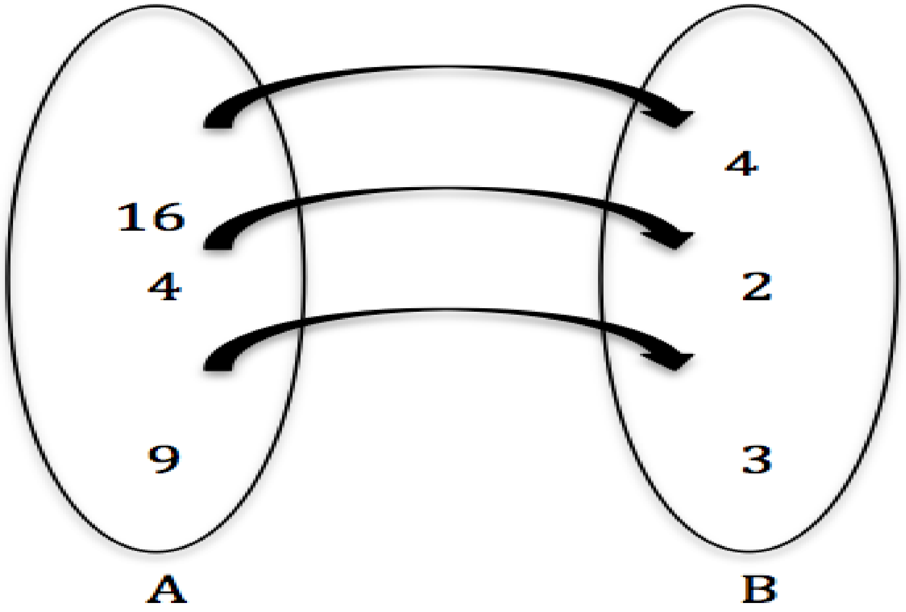
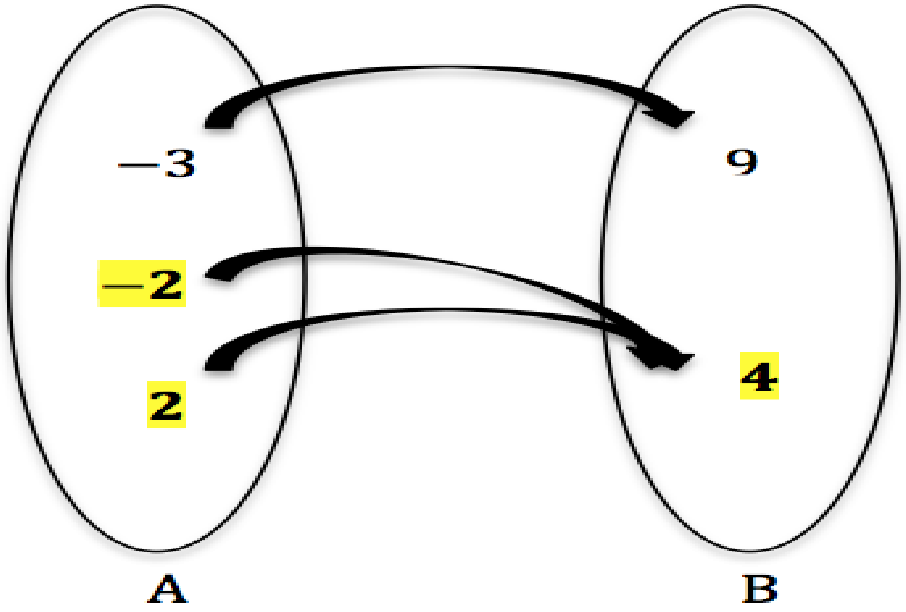
(1) **One-one function (injection) :** A function $f : A \to B$ is said to be a one-one function or an injection, if different elements of A have different images in B. Thus, $f : A \to B$ is one-one.

$$\Leftrightarrow a \neq b \Rightarrow f(a) \neq f(b)$$

for all $a, b \in A$ $\Leftrightarrow f(a) = f(b) \Rightarrow a = b$ for all $a, b \in A$.



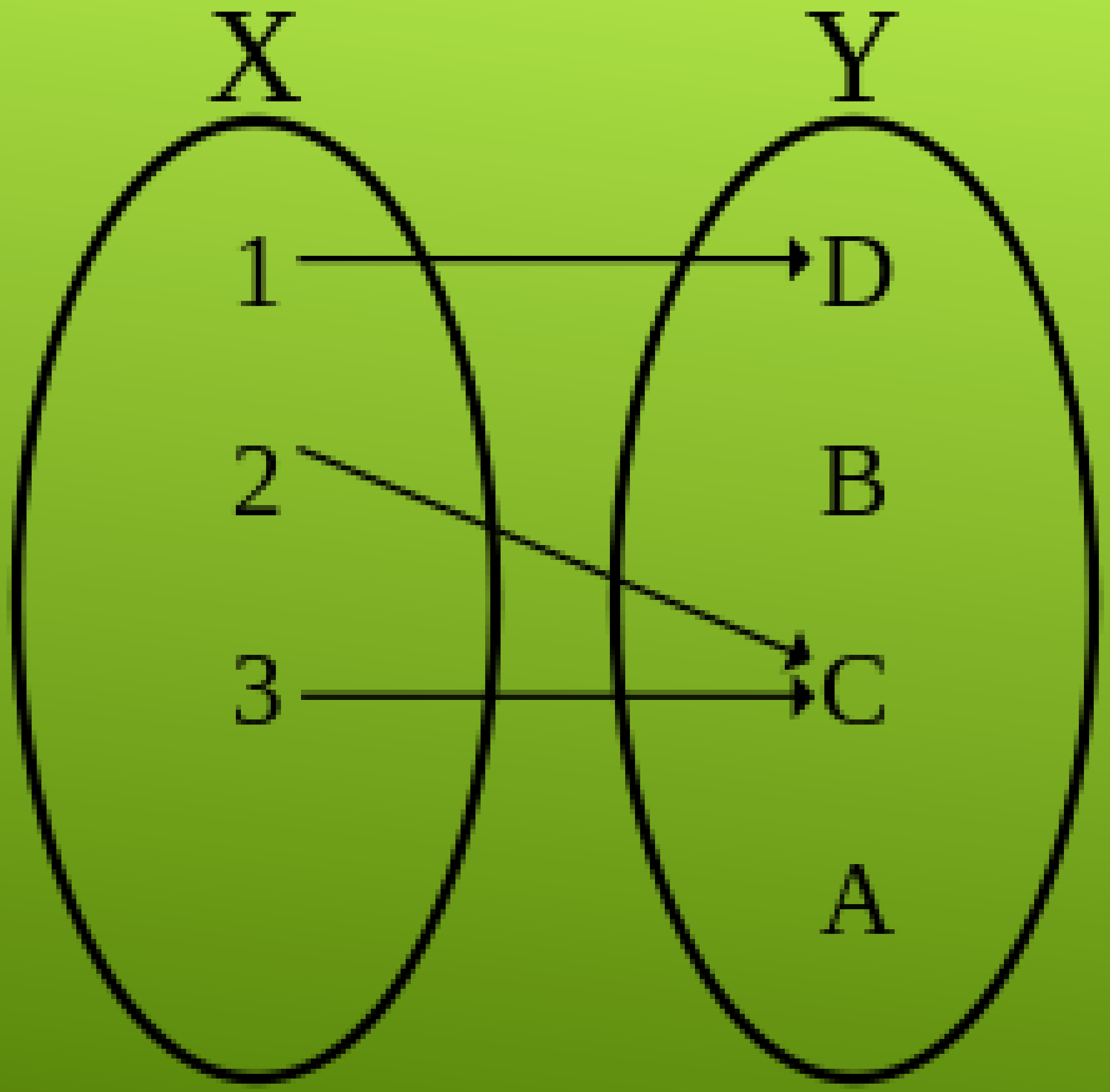
One-to-One Function
A)
-2
4
8
13
D
0
5
12
18
R
YES
1-1
B)
-2
4
8
13
D
5
12
18
R
NOT
1-1
C)
y
x
YES
1-1
D)
y
x
NOT
1-1

<table>
<tr><th>One-to-One (1-1)</th><th>Not One-to-One</th></tr>
<tr><td>$f(x) = \sqrt{x}$</td><td>$g(x) = x^2$</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$A = \{x \in \mathbb{R} \mid x \geq 0\}$</td><td>$A = \{x \in \mathbb{R}\}$</td></tr>
</table>

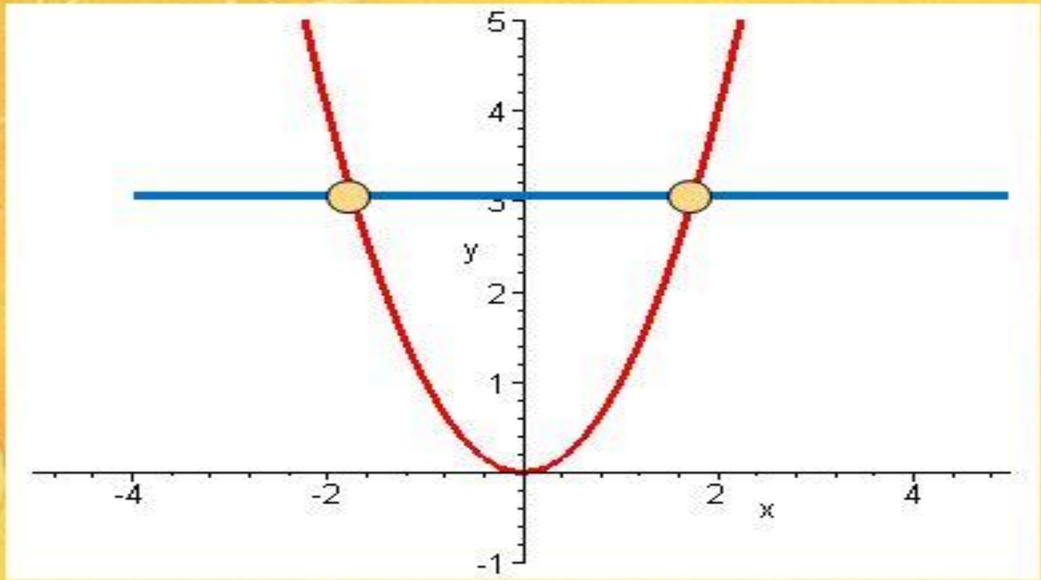
(1) **Many-one function :** A function $f : A \to B$ is said to be a many-one function if two or more elements of set A have the same image in B.

Thus, $f : A \to B$ is a many-one function if there exist $x, y \in A$ such that $x \neq y$ but $f(x) = f(y)$.

In other words, $f : A \to B$ is a many-one function if it is not a one-one function.

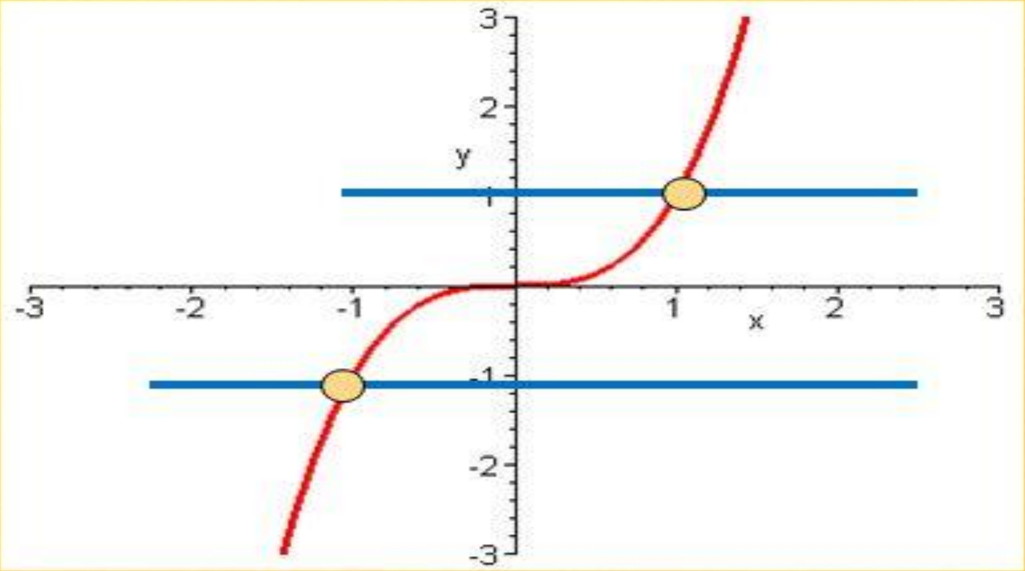

X
Y
1
2
3
D
B
C
A

To be a one-to-one function, each *y* value could only be paired with one *x*. Let's look at a couple of graphs.



Look at a ***y*** **value (for example** ***y*** **= 3)and see if there is only one** ***x*** **value on the graph for it.**

This is a many-to-one function



For any ***y*** **value, a horizontal line will only intersection the graph once so will only have one** ***x*** **value**

This then IS a one-to-one function

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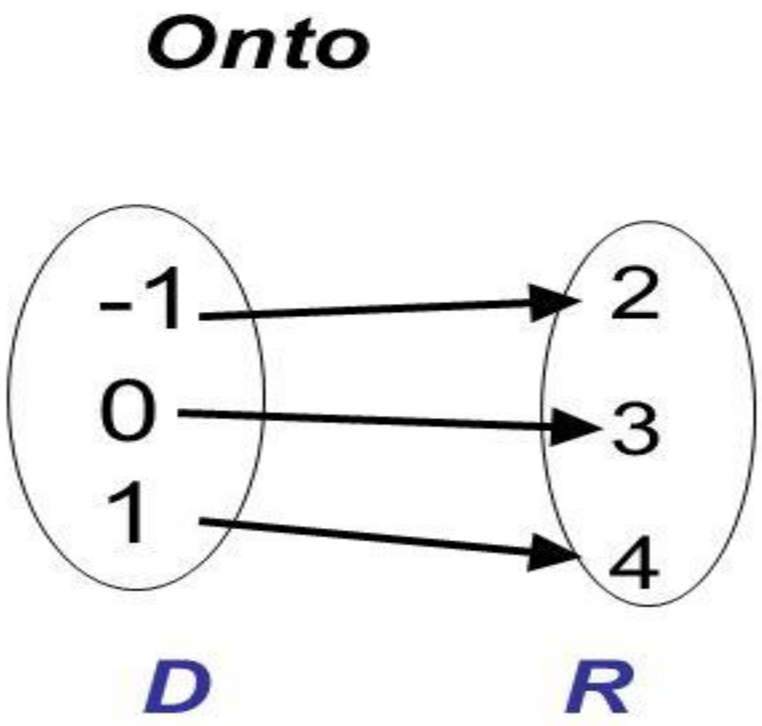
(1) Onto function (surjection) :

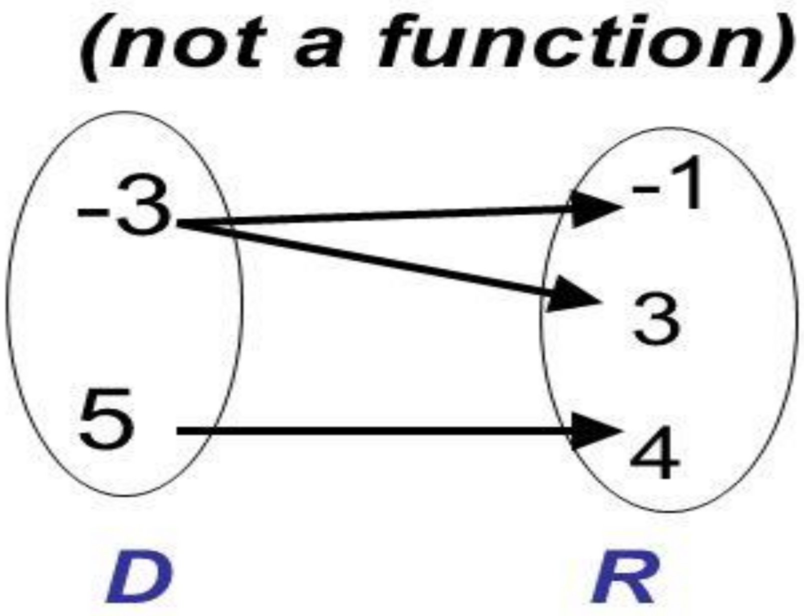
A function $f : A \to B$ is onto if each element of B has its pre-image in A.

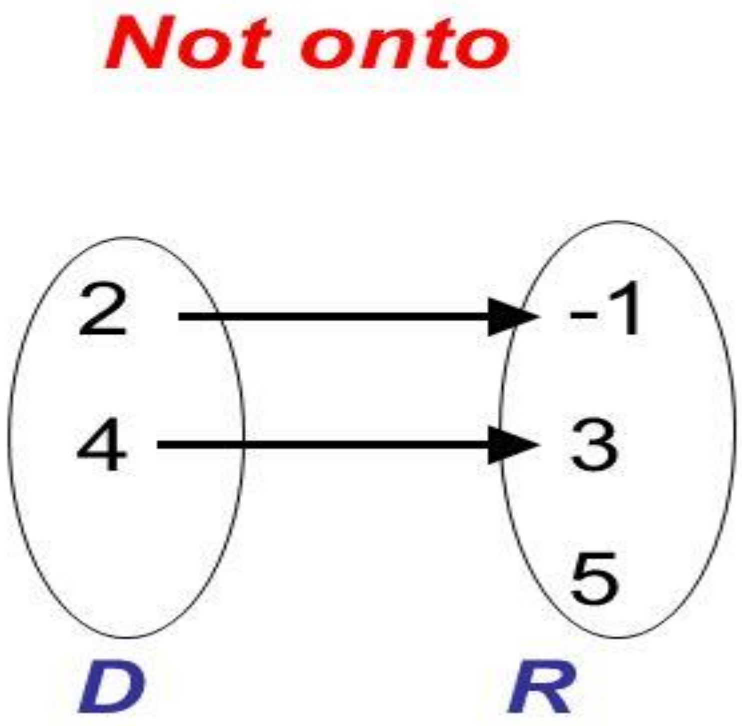
Therefore, if $f^{-1}(y) \in A, \ \forall y \in B$ then function is onto. In other words, Range of f = Co-domain of f.

*Onto**

Onto Functions: A function such that each element of the range is paired with exactly one unique element in the domain.







Onto Function

Surjection

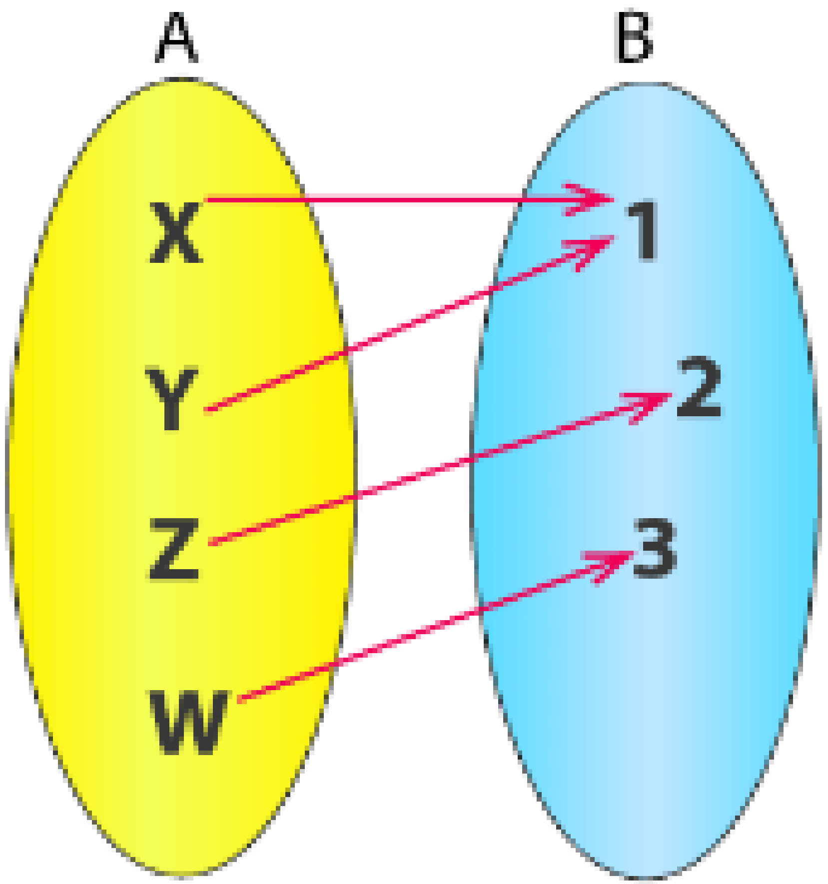


Fig. 1

Not a surjection

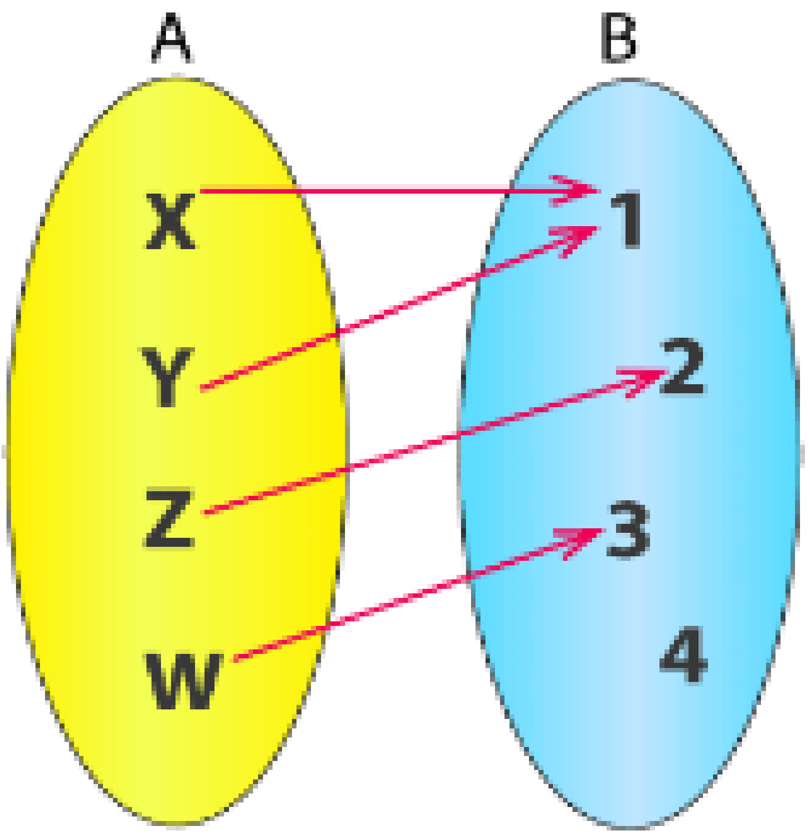
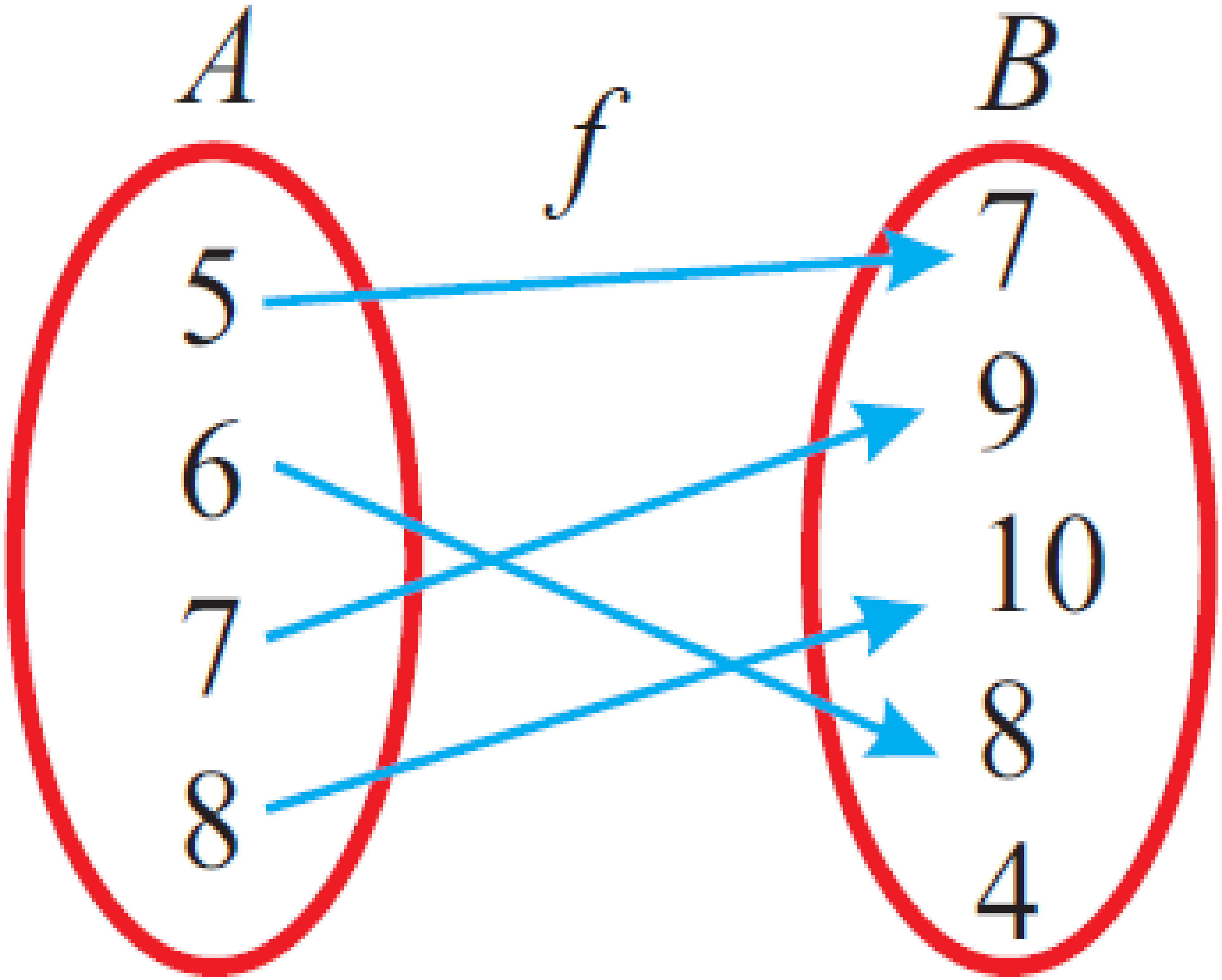


Fig. 2

Into function :

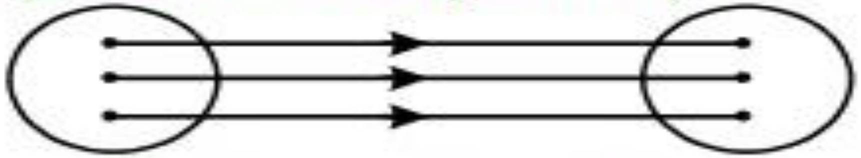
A function $f : A \to B$ is an into function if there exists an element in B having no pre-image in A.

In other words, $f : A \to B$ is an into function if it is not an onto function

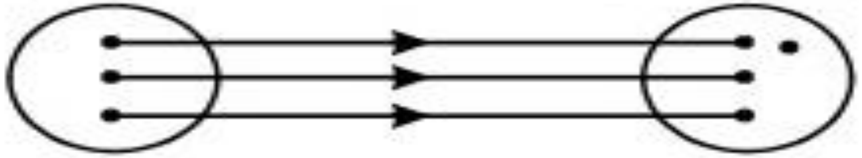

A
B
f
5
6
7
8
7
9
10
8
4

If a function is onto, it cannot be into and vice versa. Thus a function can be one of these four types :

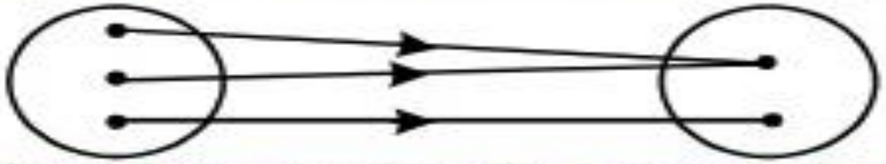
I. one-one onto (injective & surjective)



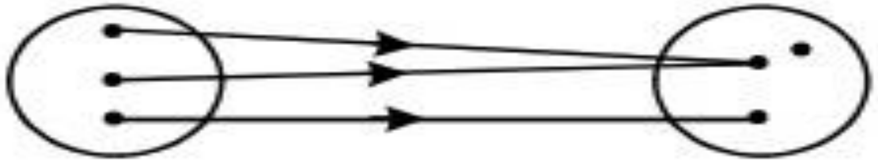
II. one-one into (injective but not surjective)



III. many-one onto (surjective but not injective)



IV. many-one into (neither surjective nor injective)



Q

- Any function, which is entirely increasing or decreasing in the whole of a domain, is one-one.
- Any continuous function $f(x)$, which has at least one local maximum or local minimum, is many-one.
- If any line parallel to the x-axis cuts the graph of the function at most at one point, then the function is one-one and if there exists a line which is parallel to the x-axis and cuts the graph of the function in at least two points, then the function is many-one.
- Any polynomial function $f : R \to R$ is onto if degree of f is odd and into if degree of f is even.
- An into function can be made onto by redefining the co-domain as the range of the original function.

Function $f : N \to N, f(x) = 2x + 3$ is [IIT 1973; UPSEAT 1983]

(a) One-one onto

(b) One-one into

(c) Many-one onto

(d) Many –one into

The function $f : R \to R$ defined by $f(x) = (x-1)(x-2)(x-3)$ is [Roorkee 1999]

(a) One-one but not onto
(b) Onto but not one-one
(c) Both one-one and onto
(d) Neither one-one nor onto

If $f : R \to R$, then $f(x) = |x|$ is [Rajasthan PET 2000]

(a) One-one but not onto

(b) Onto but not one-one

(c) One-one and onto

(d) None of these

Function	Domain	Range	Definition of the function
$\sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$	$y = \sin^{-1} x \Leftrightarrow x = \sin y$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$	$y = \cos^{-1} x \Leftrightarrow x = \cos y$
$\tan^{-1} x$	$(-\infty, \infty)$ or R	$(-\pi/2, \pi/2)$	$y = \tan^{-1} x \Leftrightarrow x = \tan y$
$\cot^{-1} x$	$(-\infty, \infty)$ or R	$(0, \pi)$	$y = \cot^{-1} x \Leftrightarrow x = \cot y$
$\operatorname{cosec}^{-1} x$	$R - (-1, 1)$	$[-\pi/2, \pi/2] - \{0\}$	$y = \operatorname{cosec}^{-1} x \Leftrightarrow x = \operatorname{cosec} y$
$\sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - [\pi/2]$	$y = \sec^{-1} x \Leftrightarrow x = \sec y$

Even function : If we put $(-x)$ in place of x in the given function and

if $f(-x) = f(x),\ \forall x \in$ domain then function $f(x)$ is called even function.

e.g. $f(x) = e^{x} + e^{-x}$, $f(x) = x^{2}$, $f(x) = x \sin x$, $f(x) = \cos x$, $f(x) = x^{2} \cos x$ all are even function.

Odd function :

If we put $(-x)$ in place of x in the given function and if

$f(-x) = -f(x),\ \forall x \in$ domain then $f(x)$ is called odd function.

e.g. $f(x) = e^{x} - e^{-x}$, $f(x) = \sin x$, $f(x) = x^{3}$,

$f(x) = x \cos x$, $f(x) = x^{2} \sin x$ all are odd function

Periodic Function

A function is said to be periodic function

if its each value is repeated after a definite interval.

So a function $f(x)$ will be periodic if a positive real number

T exist such that, $f(x+T)=f(x), \forall x \in$ domain.

Here the least positive value of T is called the period of the function.

Clearly $f(x)=f(x+T)=f(x+2T)=f(x+3T)=\ldots\ldots$ *e.g.*

$\sin x, \cos x, \tan x$ are periodic functions with period $2\pi, 2\pi$ and π respectively.

Some standard results on periodic functions

Functions	Periods
(1) $\sin^n x,\ \cos^n x,\ \sec^n x,\ \mathrm{cosec}^n x$	π; if n is even 2π; if n is odd or fraction
(2) $\tan^n x,\ \cot^n x$	$\pi; n$ is even or odd.
(3) $\lvert\sin x\rvert, \lvert\cos x\rvert, \lvert\tan x\rvert,$ $\lvert\cot x\rvert, \lvert\sec x\rvert, \lvert\mathrm{cosec}\, x\rvert$	π
(4) $x-[x]$	1
(5) Algebraic functions e.g., $\sqrt{x}, x^2, x^3+5, \ldots$etc	Period does not exist

☞ If $f(x)$ is periodic with period T, then $c.f(x)$ is periodic with period T, $f(x + c)$ is periodic with period T and $f(x) \pm c$ is periodic with period T. where c is any constant.

☞ If a function $f(x)$ has a period T, then the function $f(ax+b)$ will have a period $\dfrac{T}{|a|}$.

☞ If $f(x)$ is periodic with period T then $\dfrac{1}{f(x)}$ is also periodic with same period T.

☞ If $f(x)$ is periodic with period T, $\sqrt{f(x)}$ is also periodic with same period T.

☞ If $f(x)$ is periodic with period T, then $a f(x) + b$, where $a, b \in R\,(a \neq 0)$ is also a periodic function with period T.

☞ If $f_1(x)$, $f_2(x)$, $f_3(x)$ are periodic functions with periods T_1, T_2, T_3 respectively then; we have

$h(x) = af_1(x) \pm bf_2(x) \pm cf_3(x)$, *has period as,*

$$= \begin{cases} \text{L.C.M. of } \{T_1, T_2, T_3\}; & \text{if } h(x) \text{ is not an even function} \\ \dfrac{1}{2}\text{L.C.M. of } \{T_1, T_2, T_3\}; & \text{if } h(x) \text{ is an even function} \end{cases}$$

The period of the function $f(x) = 2\cos\frac{1}{3}(x - \pi)$ is [DCE 1998]

(a) 6π (b) 4π

(c) 2π (d) π

$$f(x) = 2\cos\frac{1}{3}(x-\pi) \quad = 2\cos\left(\frac{x}{3} - \frac{\pi}{3}\right)$$

Now, since $\cos x$ has period 2π

$$\Rightarrow \quad \cos\left(\frac{x}{3} - \frac{\pi}{3}\right) \text{ has period } \frac{2\pi}{\frac{1}{3}} = 6\pi$$

$$\Rightarrow \quad 2\cos\left(\frac{x}{3} - \frac{\pi}{3}\right) \text{ has period } = 6\pi .$$

The function $f(x) = \sin\frac{\pi x}{2} + 2\cos\frac{\pi x}{3} - \tan\frac{\pi x}{4}$ is periodic with period **[EAMCET 1992]**

(a) 6 (b) 3

(c) 4 (d) 12

$$\because \sin x \text{ has period } = 2\pi \Rightarrow \sin\frac{\pi x}{2} \text{ has period } = \frac{2\pi}{\frac{\pi}{2}} = 4$$

$$\because \cos x \text{ has period } = 2\pi \Rightarrow \cos\frac{\pi x}{3} \text{ has period } = \frac{2\pi}{\frac{\pi}{3}} = 6 \Rightarrow 2\cos\frac{\pi x}{3} \text{ has period} = 6$$

$$\because \tan x \text{ has period } = \pi \Rightarrow \tan\frac{\pi x}{4} \text{ has period } = \frac{\pi}{\frac{\pi}{4}} = 4 .$$

L.C.M. of 4, 6 and 4 =12, period of $f(x)$ = 12.

The period of $|\sin 2x|$ is

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) π

(d) 2π

Here $|\sin 2x| = \sqrt{\sin^2 2x} = \sqrt{\frac{(1-\cos 4x)}{2}}$

Period of $\cos 4x$ is $\frac{\pi}{2}$. Hence, period of $|\sin 2x|$ will be $\frac{\pi}{2}$

Trick : $\because$ $\sin x$ has period $= 2\pi \Rightarrow$ $\sin 2x$ has period $= \frac{2\pi}{2} = \pi$

Now, if $f(x)$ has period p then $|f(x)|$ has period $\frac{p}{2} \Rightarrow |\sin 2x|$ has period $= \frac{\pi}{2}$.

If $f(x)$ is an odd periodic function with period 2, then $f(4)$ equals [IIT 1991]

(a) 0 (b) 2

(c) 4 (d) – 4

Given, $f(x)$ is an odd periodic function. We can take $\sin x$, which is odd and periodic.

Now since, $\sin x$ has period = 2 and $f(x)$ has period = 2.

So, $f(x) = \sin(\pi x) \Rightarrow f(4) = \sin(4\pi) = 0$.

The period of the function $f(x) = \sin^2 x$ is [**UPSEAT 1991, 2002; AIEEE 2002**]

(a) $\frac{\pi}{2}$ (b) π

(c) 2π (d) None of these

$$\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow \text{Period} = \frac{2\pi}{2} = \pi.$$

Composite Function

If $f : A \to B$ and $g : B \to C$ are two function then the composite function of f and g, $gof\ A \to C$ will be defined as $gof(x) = g[f(x)], \forall x \in A$

(1) **Properties of composition of function :**

(i) f is even, g is even $\Rightarrow$ fog even function.

(ii) f is odd, g is odd $\Rightarrow$ fog is odd function.

(iii) f is even, g is odd $\Rightarrow$ fog is even function.

(iv) f is odd, g is even $\Rightarrow$ fog is even function

(v) Composite of functions is not commutative *i.e.,* $fog \neq gof$

(vi) Composite of functions is associative *i.e.,* $(fog)oh = fo(goh)$

(vii) If $f : A \rightarrow B$ is bijection and $g : B \rightarrow A$ is inverse of f.

Then $fog = I_B$ and $gof = I_A$.

where, I_A and I_B are identity functions on the sets A and B respectively.

(viii) If $f : A \rightarrow B$ and $g : B \rightarrow C$ are two bijections,

then $gof : A \rightarrow C$ is bijection and $(gof)^{-1} = (f^{-1}og^{-1})$.

(ix) $fog \neq gof$ but if , $fog = gof$ then either

$f^{-1} = g$ or $g^{-1} = f$ also, $(fog)(x) = (gof)(x) = (x)$.

If $f: R \to R, f(x) = 2x - 1$ and $g: R \to R$, $g(x) = x^2$ then $(gof)(x)$ equals [Rajasthan PET 1987]

(a) $2x^2 - 1$ (b) $(2x-1)^2$

(c) $4x^2 - 2x + 1$ (d) $x^2 + 2x - 1$

$$gof(x) = g\{f(x)\} = g(2x-1) = (2x-1)^2.$$

If $f : R \to R, f(x) = (x+1)^2$ and $g : R \to R, g(x) = x^2 + 1,$

then $(fog)(-3)$ is equal to [**Rajasthan PET 1999**]

(a) 121 (b) 144

(c) 112 (d) 11

$$fog(x) = f\{g(x)\} = f(x^2+1) = (x^2+1+1)^2 = (x^2+2)^2$$

$$\Rightarrow \quad fog(-3) = (9+2)^2 = 121 .$$

If $g(x) = x^2 + x - 2$ and $\frac{1}{2}(gof)(x) = 2x^2 - 5x + 2$, then $f(x)$ is equal to

(a) $2x - 3$

(b) $2x + 3$

(c) $2x^2 + 3x + 1$

(d) $2x^2 - 3x - 1$

$$g(x) = x^2 + x - 2 \Rightarrow (gof)(x) = g[f(x)] = [f(x)]^2 + f(x) - 2$$

$$\text{Given, } \frac{1}{2}(gof)(x) = 2x^2 - 5x + 2$$

$$\therefore \quad \frac{1}{2}[f(x)]^2 + \frac{1}{2}f(x) - 1 = 2x^2 - 5x + 2$$

$$\Rightarrow \quad [f(x)]^2 + f(x) = 4x^2 - 10x + 6$$

$$\Rightarrow \quad f(x)[f(x)+1] = (2x-3)[(2x-3)+1] \Rightarrow \quad f(x) = 2x - 3.$$

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If $f(x)=\dfrac{2x-3}{x-2}$, then $[f\{f(x)\}]$ equals**Rajasthan PET 1996]**

(a) x

(b) $-x$

(c) $\dfrac{x}{2}$

(d) $-\dfrac{1}{x}$

$$f[f(x)] = \frac{2\left(\dfrac{2x-3}{x-2}\right)-3}{\left(\dfrac{2x-3}{x-2}\right)-2} = x$$

If $f(x)=\dfrac{2x+1}{3x-2}$, then $(fof)(2)$ is equal to **[Kerala (Engg.) 2002]**

(a) 1 (b) 3

(c) 4 (d) 2

Here $f(2)=\dfrac{5}{4}$

Hence $(fof)(2)=f(f(2))=f\left(\dfrac{5}{4}\right)=\dfrac{2\times\dfrac{5}{4}+1}{3\times\dfrac{5}{4}-2}=2$.

Inverse Function

If $f : A \to B$ be a one-one onto (bijection) function,

then the mapping $f^{-1} : B \to A$ which associates

each element $b \in B$ with element $a \in A$, such that $f(a) = b$,

is called the inverse function of the function $f : A \to B$

$$f^{-1} : B \to A,\ f^{-1}(b) = a \Rightarrow f(a) = b$$

In terms of ordered pairs inverse function is defined as $f^{-1} = (b, a)$ if $(a, b) \in f$.

Note : ❑ For the existence of inverse function, it should be one-one and onto.

- *Inverse of a bijection is also a bijection function.*
- *Inverse of a bijection is unique.*
- $(f^{-1})^{-1} = f$
- *If f and g are two bijections such that (gof) exists then* $(gof)^{-1} = f^{-1}og^{-1}$.
- *If* $f : A \rightarrow B$ *is a bijection then* $f^{-1} : B \rightarrow A$ *is an inverse function of f.* $f^{-1}of = I_A$ *and* $fof^{-1} = I_B$. *Here* I_A *is an identity function on set A, and* I_B *is an identity function on set B.*

If $f: R \to R$ is given by $f(x) = 3x - 5$, then $f^{-1}(x)$ [IIT Screening 1998]

(a) Is given by $\frac{1}{3x-5}$

(b) Is given by $\frac{x+5}{3}$

(c) Does not exist because f is not one-one

(d) Does not exist because f is not onto

Clearly, $f: R \to R$ is a one-one onto function. So, it is invertible.

Let $f(x) = y$. then, $3x - 5 = y \Rightarrow x = \frac{y+5}{3} \Rightarrow f^{-1}(y) = \frac{y+5}{3}$.

Hence, $f^{-1}(x) = \frac{x+5}{3}$.

Let $f : R \to R$ be defined by $f(x) = 3x - 4$, then $f^{-1}(x)$ is [UPSEAT 1993]

(a) $3x + 4$

(b) $\frac{1}{3}x - 4$

(c) $\frac{1}{3}(x + 4)$

(d) $\frac{1}{3}(x - 4)$

$$f(x) = 3x - 4 = y \Rightarrow y = 3x - 4$$

$$\Rightarrow x = \frac{y+4}{3} \Rightarrow f^{-1}(y) = \frac{y+4}{3} \Rightarrow f^{-1}(x) = \frac{x+4}{3}$$

If $f:[1,\infty)\to[1,\infty)$ is defined as $f(x)=2^{x(x-1)}$ then $f^{-1}(x)$ is equal to [**IIT Screening 1999**]

(a) $\left(\frac{1}{2}\right)^{x(x-1)}$

(b) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(c) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$

(d) Not defined

Given $f(x) = 2^{x(x-1)} \Rightarrow x(x-1) = \log_2 f(x)$

$$\Rightarrow x^2 - x - \log_2 f(x) = 0 \Rightarrow x = \frac{1 \pm \sqrt{1 + 4\log_2 f(x)}}{2}$$

Only $x = \dfrac{1 + \sqrt{1 + 4\log_2 f(x)}}{2}$ lies in the domain

$$\therefore \quad f^{-1}(x) = \frac{1}{2}[1 + \sqrt{1 + 4\log_2 x}]$$