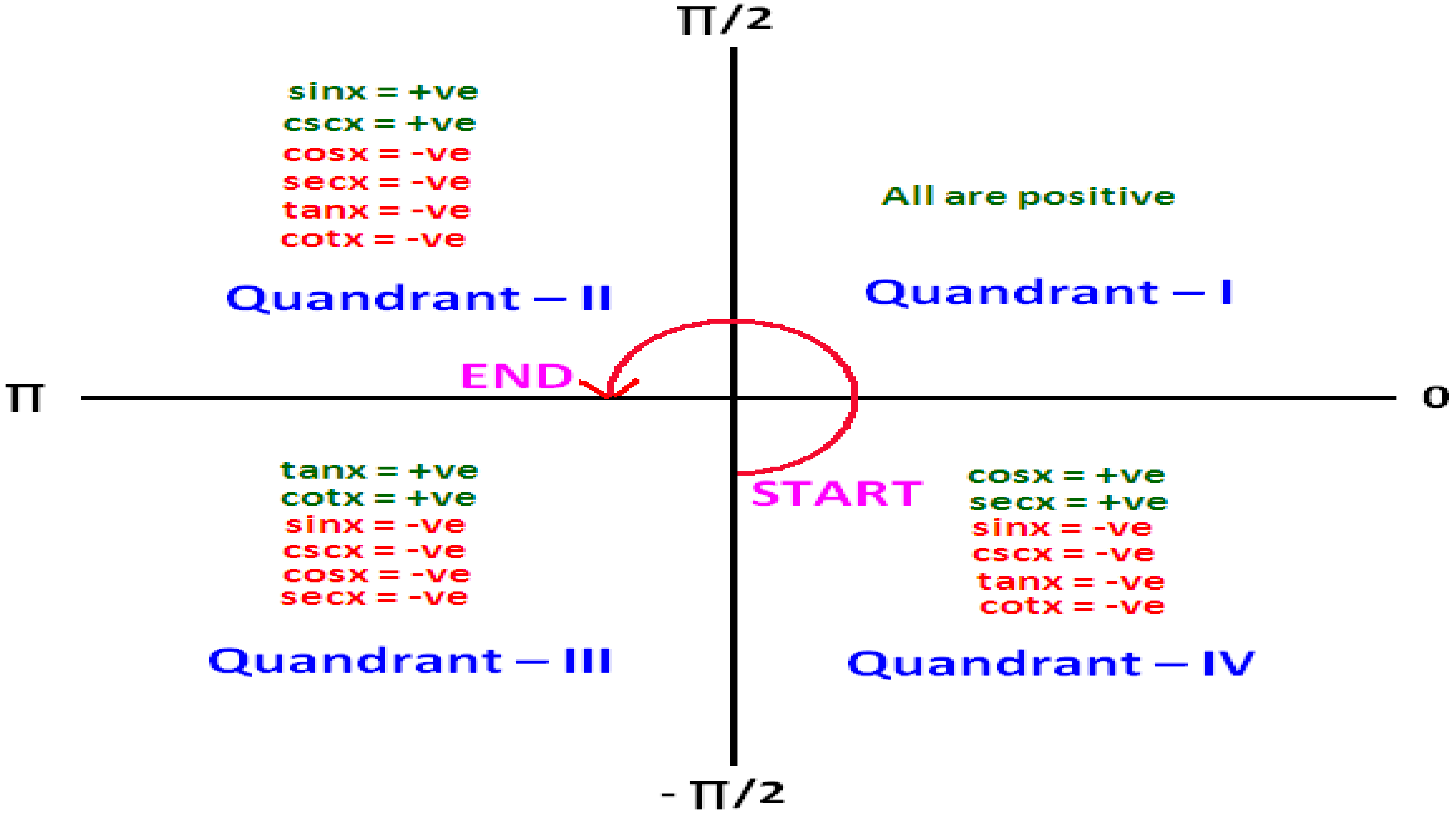
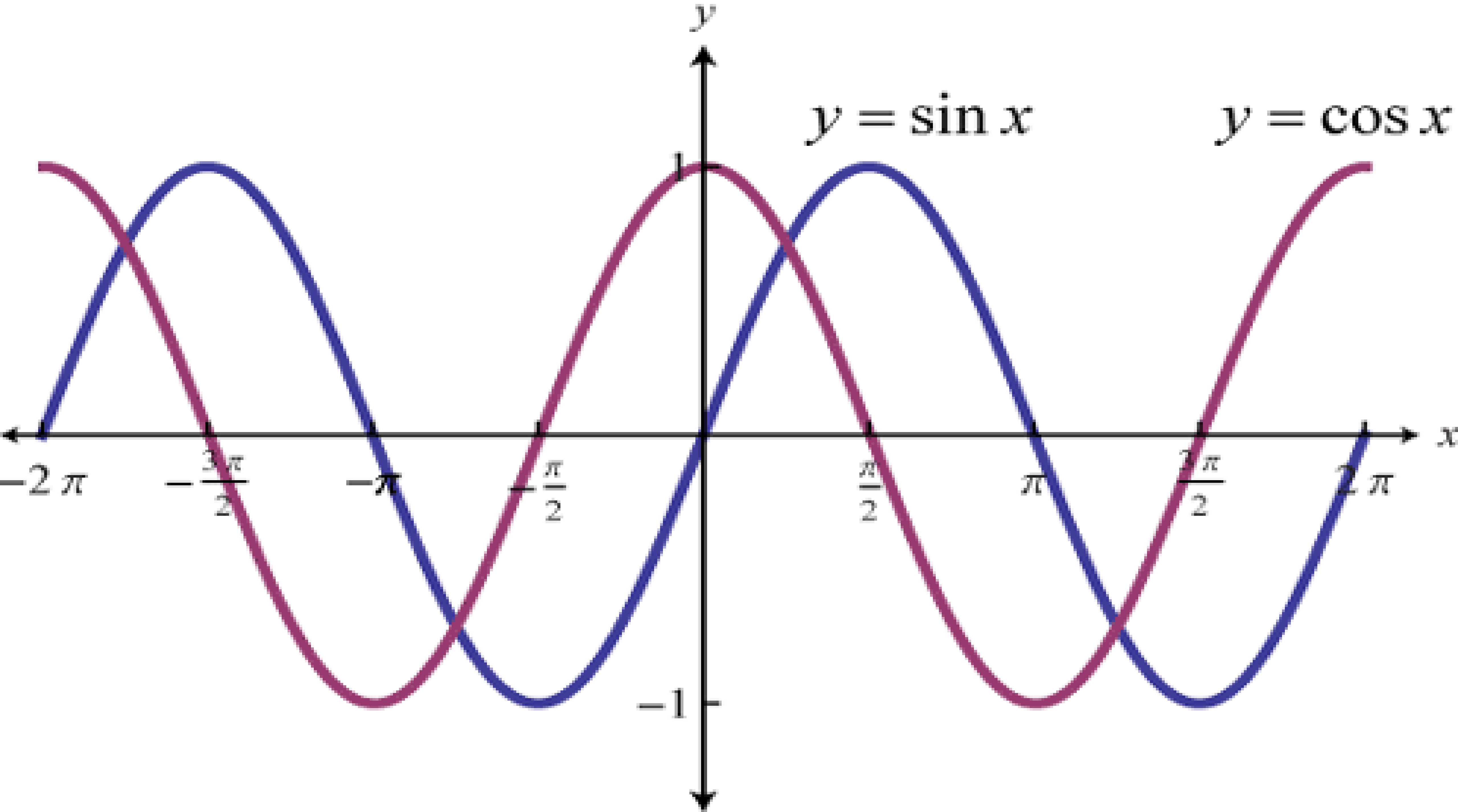
HARIOM CHAUDHARY



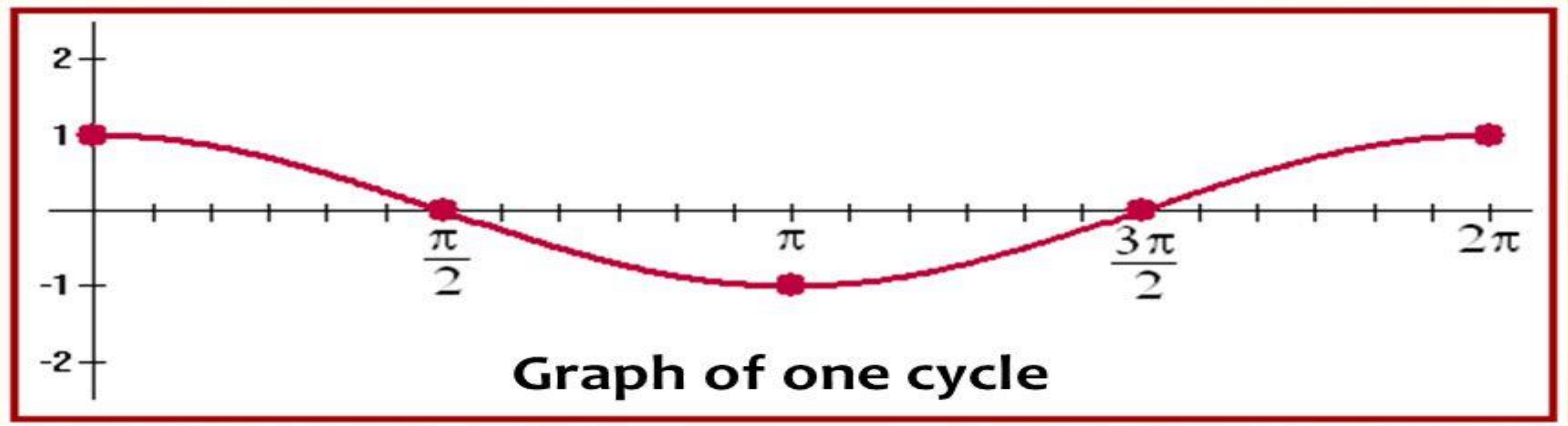
π/2
sinx = +ve
cscx = +ve
cosx = -ve
secx = -ve
tanx = -ve
cotx = -ve
Quandrant – II
All are positive
Quandrant – I
END
π
0
START
tanx = +ve
cotx = +ve
sinx = -ve
cscx = -ve
cosx = -ve
secx = -ve
Quandrant – III
cosx = +ve
secx = +ve
sinx = -ve
cscx = -ve
tanx = -ve
cotx = -ve
Quandrant – IV
- π/2

Inverse Trigonometric Functions

Function	Domain	Range
$\arcsin x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$\arccos x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$\arctan x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$\text{arccot}\ x = \arctan \frac{1}{x}$	$-\infty < x < \infty$	$0 < y < \pi$
$\text{arcsec}\ x = \arccos \frac{1}{x}$	$\lvert x \rvert \geq 1$	$0 \leq y \leq \pi, \quad y \neq \frac{\pi}{2}$
$\text{arccsc}\ x = \arcsin \frac{1}{x}$	$\lvert x \rvert \geq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, \quad y \neq 0$


y
y = sin x
y = cos x
1
−1
x
−2 π
−3π/2
−π
−π/2
π/2
π
3π/2
2 π

Cosine Graph: $y = \cos\theta$



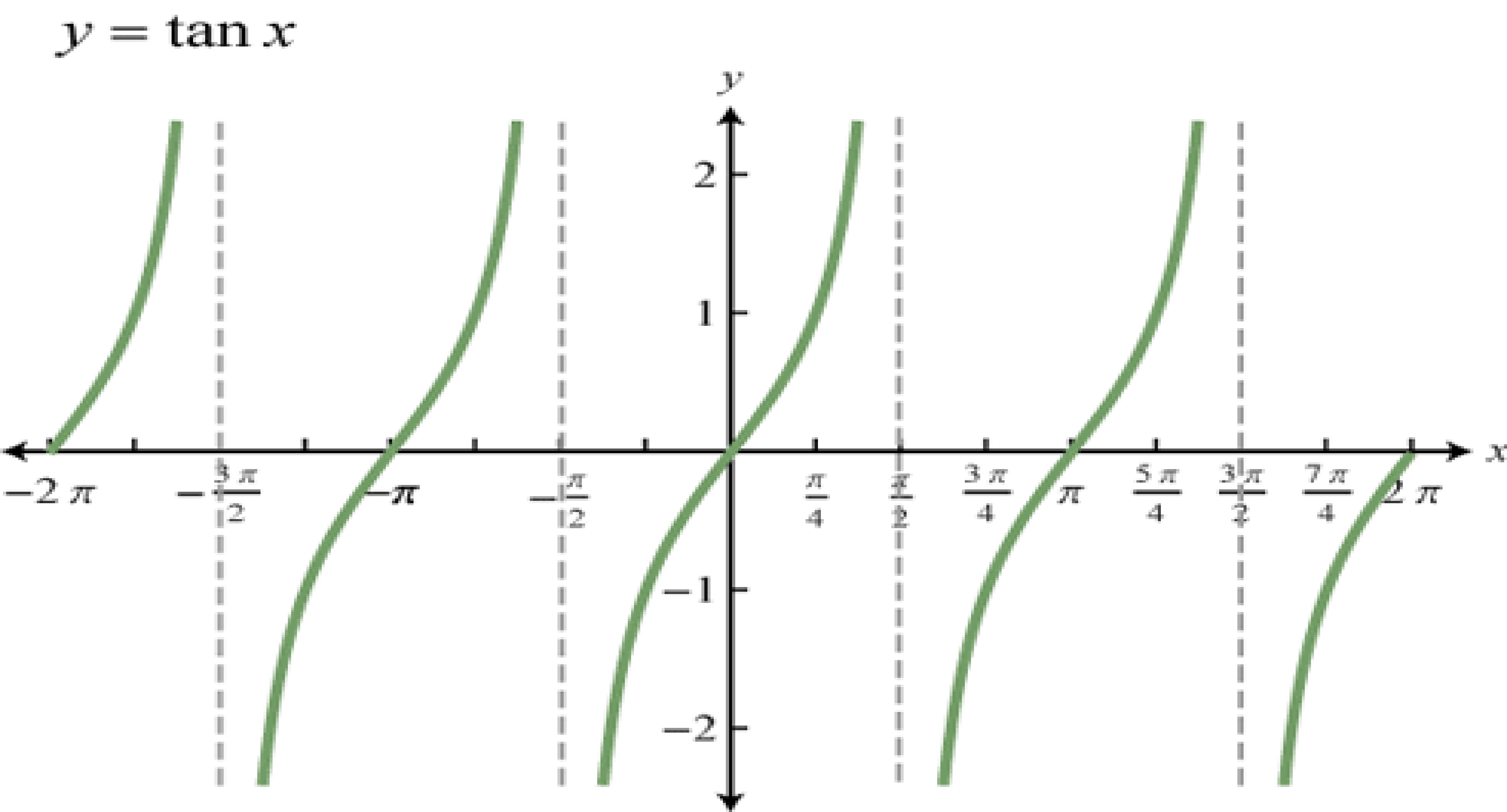
Domain: all □

Range: $-1 \le y \le 1$

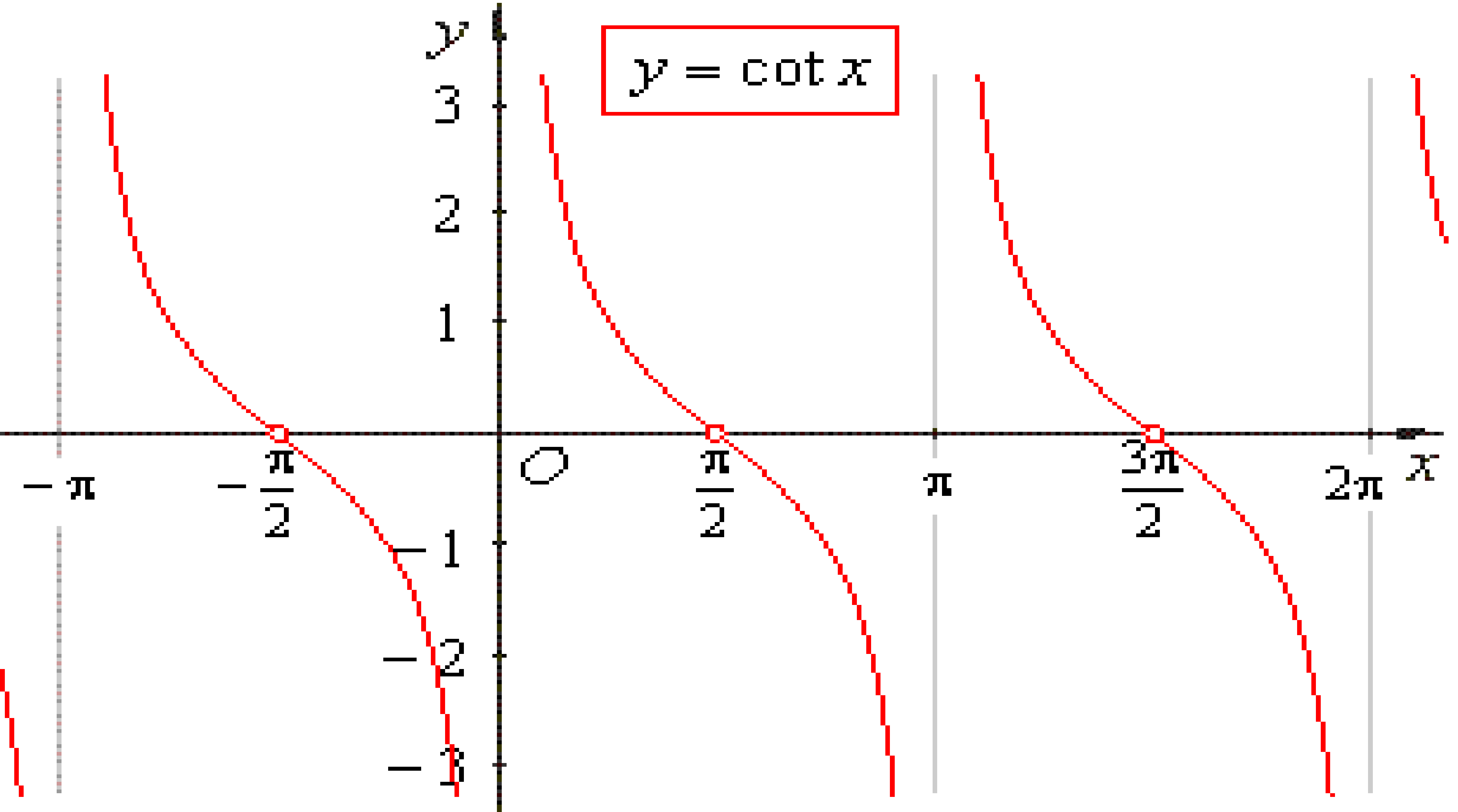
***y*-int:** $(0, 1)$

Amplitude: 1

Period: 2π

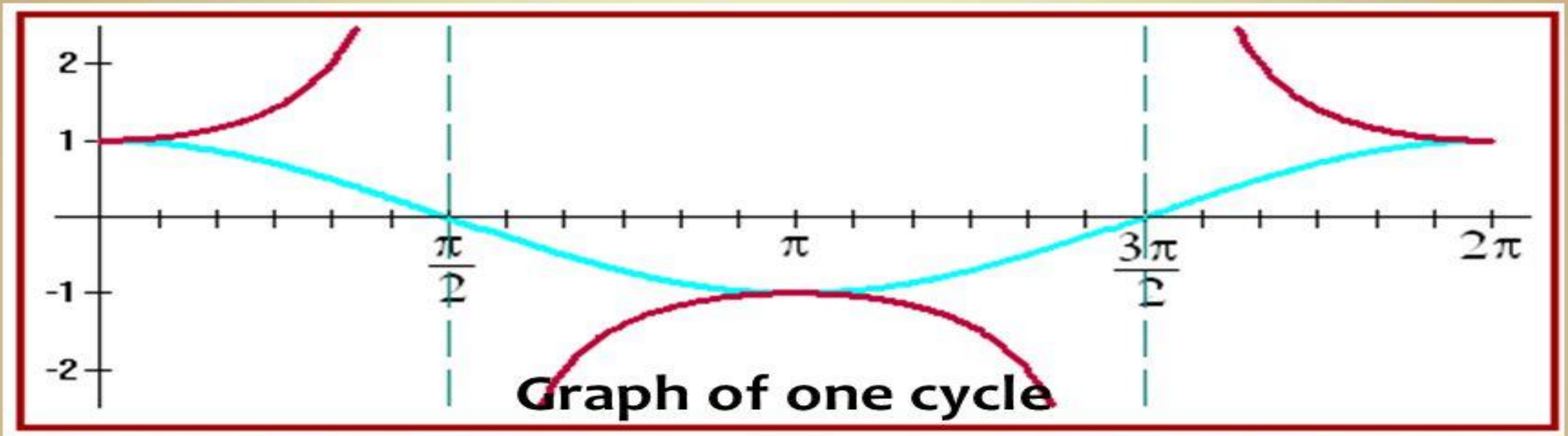


y = tan x
y
x
2
1
−1
−2
−2π
−3π/2
−π
−π/2
π/4
π/2
3π/4
π
5π/4
3π/2
7π/4
2π



y = cot x
y
x
O
3
2
1
−1
−2
−3
−π
−π/2
π/2
π
3π/2
2π

Secant Graph: $y = \sec \theta$



Domain: all □ $, x \neq \frac{\pi}{2} + \pi k$

Range: $y \leq -1$ or $y \geq 1$

***y*-int:** $(0,1)$

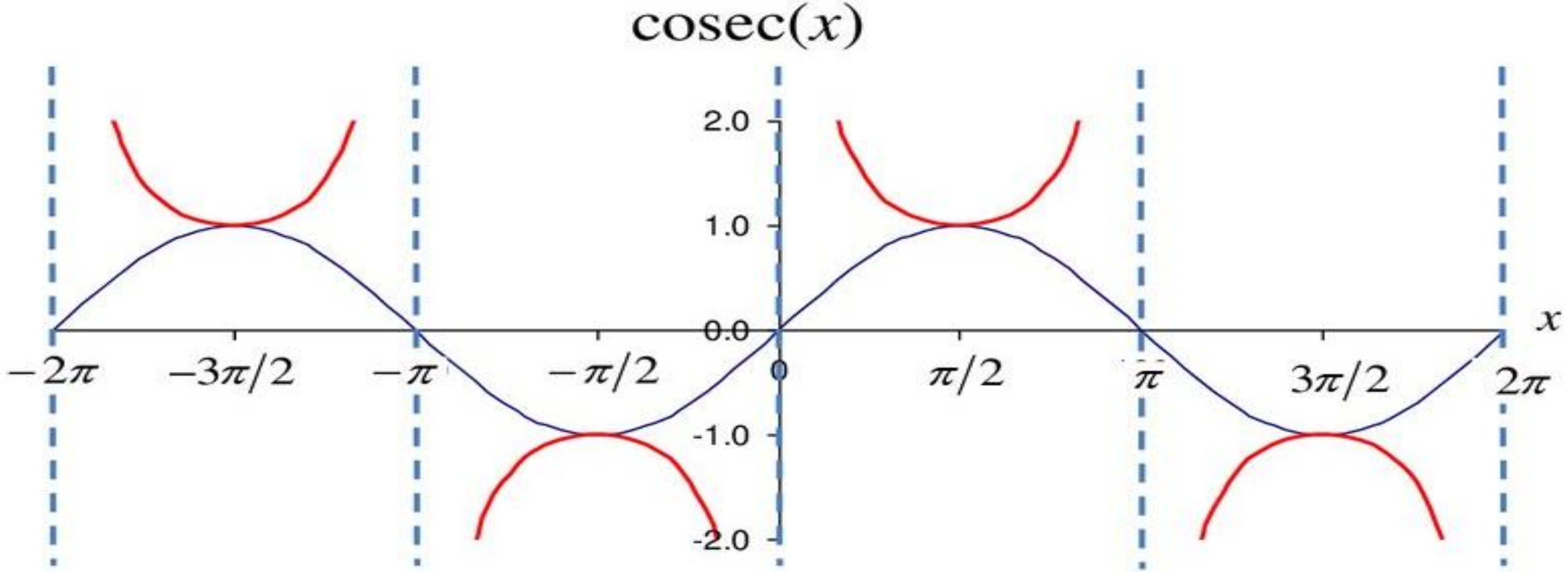
Amplitude: undefined

Period: 2π

Asymptotes: $x = \frac{\pi}{2} + \pi k$

Graph of $\operatorname{cosec}(x)$

The graph of $y = \operatorname{cosec}(x),\ \ x \in \mathbb{R}$ is 2π periodic. It has vertical asymptotes for all x for which $\sin(x) = 0$, i.e.
$x = n\pi, \quad n \in \mathbb{Z}.$



Fundamental Trigonometrical Identities :

(a) $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$

(b) $\cos\theta = \dfrac{1}{\sec\theta}$

(c) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

(d) $1 + \tan^2\theta = \sec^2\theta$

or, $\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta - \tan\theta) = \dfrac{1}{(\sec\theta + \tan\theta)}$

(e) $\sin^2\theta + \cos^2\theta = 1$

(f) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

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5. Trigonometrical ratio of allied angles

Two angles are said to be allied when their sum or difference is either zero or a multiple of 90º.

similarly,

Allied angles	(– θ)	(90° – θ)	(90° + θ)	(180° – θ)	(180° + θ)	(270° – θ)	(270° + θ)	(360° – θ)
Trigo. ratio		or $\left(\frac{\pi}{2}-\theta\right)$	or $\left(\frac{\pi}{2}+\theta\right)$	or $(\pi - \theta)$	or $(\pi + \theta)$	or $\left(\frac{3\pi}{2}-\theta\right)$	or $\left(\frac{3\pi}{2}+\theta\right)$	or $(2\pi - \theta)$
sin	–sin θ	cos θ	cos θ	sin θ	– sin θ	– cos θ	– cos θ	– sin θ
cos	cos θ	sin θ	– sin θ	– cos θ	– cos θ	– sin θ	sin θ	cos θ
tan	– tan θ	cot θ	– cot θ	– tan θ	tan θ	cot θ	– cot θ	– tan θ

6. Sum & Difference formulae

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

(b) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(c) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(d) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(e) $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(f) $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(g) $\cot(A + B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

(h) $\cot(A - B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

*(a) $\sin(A+B).\sin(A-B) = \sin^2 A - \sin^2 B$

$= \cos^2 B - \cos^2 A$

*(b) $\cos(A+B).\cos(A-B) = \cos^2 A - \sin^2 B$

$= \cos^2 B - \sin^2 A$

(c) $\sin(A+B+C) = \sin A \cos B \cos C$

$+ \cos A \sin B \sin C + \cos A \cos B \sin C$

$- \sin A \sin B \sin C$

(d) $\cos(A+B+C) = \cos A \cos B \cos C$

$- \cos A \sin B \sin C - \sin A \cos B \sin C$

$- \sin A \sin B \cos C$

(e) $\tan(A+B+C)$

$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

(Note : * Important)

7. Formulae for product into sum or difference conversion

Formulae :

(a) $2 \sin A \cos B = \sin (A + B) + \sin (A - B)$

(b) $2 \cos A \sin B = \sin (A + B) - \sin (A - B)$

(c) $2 \cos A \cos B = \cos (A + B) + \cos (A - B)$

(d) $2 \sin A \sin B = \cos (A - B) - \cos (A + B)$

(i) $\sin 2\theta = 2 \sin \theta \cos \theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(iii) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(iv) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(v) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(vi) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

(vii) $\sin \theta/2 = \sqrt{\dfrac{1-\cos\theta}{2}}$

(viii) $\cos \theta/2 = \sqrt{\dfrac{1+\cos\theta}{2}}$

(ix) $\tan \theta/2 = \sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

The greatest and least values of a sinθ + b cosθ are respectively $\sqrt{a^2 + b^2}$ and $-\sqrt{a^2 + b^2}$

$$\text{(a)}\quad \tan(A+B+C) = \frac{\sum \tan A - \tan A \tan B \tan C}{1 - \sum \tan A.\tan B}$$

$$\text{(b)}\quad \tan\theta = \cot\theta - 2\cot 2\theta$$

$$\text{(c)}\quad \tan 3\theta = \tan\theta.\tan(60^\circ - \theta).\tan(60^\circ + \theta)$$

$$\text{(d)}\quad \tan(A+B) - \tan A - \tan B = \tan A.\tan B.\tan(A+B)$$

$$\text{(e)}\quad \sin\theta \sin(60^\circ - \theta)\sin(60^\circ + \theta) = \frac{1}{4}\sin 3\theta$$

$$\text{(f)}\quad \cos\theta \cos(60^\circ - \theta)\cos(60^\circ + \theta) = \frac{1}{4}\cos 3\theta$$

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$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots\ldots\ldots + \text{to n terms}$$

$$= \frac{\sin\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right]\left[\sin\left(\frac{n\beta}{2}\right)\right]}{\sin\left(\frac{\beta}{2}\right)} ; \beta \neq 2n\pi$$

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots\ldots + \text{ to n terms}$$

$$= \frac{\cos\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right]\left[\sin\left(\frac{n\beta}{2}\right)\right]}{\sin\left(\frac{\beta}{2}\right)};\ \beta \neq 2n\pi$$

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	15°	18°	22½°	36°
sin	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$
cos	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$
tan	$2-\sqrt{3}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{2}-1$	$\sqrt{5-2\sqrt{5}}$

An Increasing Product series :

$$p = \cos\alpha . \cos 2\alpha . \cos 2^2\alpha \ldots\ldots . \cos (2^{n-1}\alpha) = \begin{cases} \dfrac{\sin 2^n \alpha}{2^n \sin\alpha}, \text{if } \alpha \neq n\pi \\ 1, \text{ if } \alpha = 2k\pi \\ -1, \text{if } \alpha = (2k+1)\pi \end{cases}$$

$\sqrt{3}$ cosec20° - sec20° =

(A) 1 (B) 2

(C) 3 (D) 4

$$\text{Given} = \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ}$$

$$= \frac{4.\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{2\sin 20^\circ \cos 20^\circ} = 4.\frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\sin 40^\circ}$$

$$= 4\frac{\sin 40}{\sin 40} = 4$$

Hence (D) is the correct answer.

tan A + 2 tan 2A + 4 tan 4A + 8 cot 8A =

(A) Cot A

(B) tan 6A

(C) cot 4A

(D) None of these

$\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A$

$$= \tan A + 2\tan 2A + 4\tan 4A + 8\frac{\left(1-\tan^2 4A\right)}{2\tan 4A}$$

$$= \tan A + 2\tan 2A + 4\cot 4A = \tan A + 2\tan 2A + 4\frac{\left(1-\tan^2 2A\right)}{2\tan 2A}$$

$$= \tan A + 2\cot 2A = \tan A + \frac{1-\tan^2 A}{\tan A} = \cot A$$

Hence (A) is the correct answer

The value of sin 12°. sin48°.sin54° =

(A) 1/8 (B) 1/6

(C) 1/4 (D) 1/2

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sin 12°. sin48°.sin54° =

$$\frac{1}{2}\left[\cos 36^\circ - \cos 60^\circ\right]\sin 54^\circ = \frac{1}{2}\left[\cos 36^\circ \sin 54^\circ - \frac{1}{2}\sin 54^\circ\right]$$

$$= \frac{1}{4}\left[2\cos 36^\circ \sin 54^\circ - \sin 54^\circ\right] = \frac{1}{4}\left[\sin 90^\circ + \sin 18^\circ - \sin 54^\circ\right]$$

$$= \frac{1}{4}\left[1 - \left(\sin 54^\circ - \sin 18^\circ\right)\right] = \frac{1}{4}\left[1 - 2\sin 18^\circ \cos 36^\circ\right]$$

$$= \frac{1}{4}\left[1 - \frac{2\sin 18^\circ}{\cos 18^\circ}\cos 18^\circ \cos 36^\circ\right] = \frac{1}{4}\left[1 - \frac{\sin 36^\circ \cos 36^\circ}{\cos 18^\circ}\right]$$

$$\frac{1}{4}\ 1 - \frac{2\sin 36^\circ \cos 36^\circ}{2\cos 18^\circ} = \frac{1}{4}\ 1 - \frac{\sin 72^\circ}{2\sin 72^\circ} = \frac{1}{4}\ 1 - \frac{1}{2} = \frac{1}{8}$$

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Alternative Method

Let $\theta = 12°$

$$\sin 12°.\sin 48°.\sin 54° = \frac{1}{\sin 72°}\sin 12°\sin 48°\sin 72°\sin 54$$

$$= \frac{1}{4}\frac{\sin 3(12°)\sin 54°}{\sin 72°} = \frac{\sin 36°\sin 54°}{8\sin 36°\cos 36°} = \frac{\cos 36°}{8\cos 36°} = \frac{1}{8}$$

Hence (A) is the correct answer.

The smallest positive value of x (in degrees) for which

$\tan(x + 100°) = \tan(x + 50°) \tan x \tan(x - 50°)$ is :

(A) 30° (B) 45°

(C) 60° (D) 90°

The relation may be written as $\frac{\tan(x+100^\circ)}{\tan(x-50^\circ)} = \tan(x+50^\circ)\tan x$

$$\Rightarrow \frac{\sin(x+100°)\cos(x-50°)}{\sin(x-50°)\cos(x+100°)} = \frac{\sin(x+50°)\sin x}{\cos(x+50°)\cos x}$$

$$\Rightarrow \frac{\sin(2x+50°)+\sin(150°)}{\sin(2x+50°)-\sin(150°)} = \frac{\cos(50°)-\cos(2x+50°)}{\cos(50°)+\cos(2x+50°)}$$

$$\Rightarrow \frac{\sin(2x+50°)}{\sin 150°} = \frac{-\cos 50°}{\cos(2x+50°)} \Rightarrow \cos 50° + 2\sin(2x + 50°) \cos(2x + 50°) = 0$$

$\Rightarrow \cos 50° + \sin(4x + 100°) = 0 \Rightarrow \cos 50° + \cos(4x + 10°) = 0$

$\Rightarrow \cos(2x + 30°) \cos(2x - 20°) = 0 \Rightarrow x = 30°, 55°$

$\Rightarrow$ The smallest value of $x = 30°$

Hence (A) is the correct answer.

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If $\sin\theta = 3\sin(\theta + 2\alpha)$, then the value of $\tan(\theta + \alpha) + 2\tan\alpha$ is:

(A) 0 (B) 2

(C) 4 (D) 1

If $\sin\theta = 3\sin(\theta + 2\alpha)$, then the value of $\tan(\theta + \alpha) + 2\tan\alpha$ is:

(A) 0 (B) 2

(C) 4 (D) 1

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Given $\sin\theta = 3\sin(\theta + 2\alpha)$

$$\Rightarrow \sin(\theta + \alpha - \alpha) = 3\sin(\theta + \alpha + \alpha)$$

$$\Rightarrow \sin(\theta + \alpha)\cos\alpha - \cos(\theta + \alpha)\sin\alpha$$

$$= 3\sin(\theta + \alpha)\cos\alpha + 3\cos(\theta + \alpha)\sin\alpha$$

$$\Rightarrow -2\sin(\theta + \alpha)\cos\alpha = 4\cos(\theta + \alpha)\sin\alpha$$

$$\Rightarrow \frac{-\sin(\theta + \alpha)}{\cos(\theta + \alpha)} = \frac{2\sin\alpha}{\cos\alpha}$$

$$\Rightarrow \tan(\theta+\alpha) + 2\tan\alpha = 0$$

Hence (A) is the correct answer

The minimum value of $3\tan^2\theta + 12\cot^2\theta$ is:

(A) 6 (B) 8

(C) 10 (D) None of these

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$$\text{A.M.} \geq \text{G.M} \Rightarrow \frac{1}{2}\left(3\tan^2\theta + 12\cot^2\theta\right) \geq 6$$

$\Rightarrow$ $3\tan^2\theta + 12\cot^2\theta$ has minimum value 12.

Hence (D) is the correct answer.

If $A + B + C = 180^o$ then the value of tanA + tanB + tanC is :

(A) $\geq 3\sqrt{3}$

(B) $\geq 2\sqrt{3}$

(C) $> 3\sqrt{3}$

(D) $> 2\sqrt{3}$

$$\tan(A + B) = \tan(180^\circ - C)$$

$$\text{or, } \frac{\tan A + \tan B}{1 - \tan A \tan B} = \tan C$$

or, tanA + tanB + tanC = tana tanB tanC

$$\frac{\tan A + \tan B + \tan C}{3} \geq \sqrt[3]{\tan A \tan B \tan C} \text{ [since A.M. } \geq \text{ G.M.]}$$

$$\text{or, } \tan A \tan B \tan C \geq \sqrt[3]{\tan A \tan B \tan C}$$

$$\text{or, } \tan^2 A \tan^2 B \tan^2 C \geq 27 \text{ [cubing both sides]}$$

$$\text{or } \tan A \tan B \tan C \geq 3\sqrt{3}$$

$$\Rightarrow \tan A + \tan B + \tan C \geq 3\sqrt{3}.$$

Hence (A) is the correct answer

Let $0 < A, B < \frac{\pi}{2}$ satisfying the equalities $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$.

Then A + 2B = :

(A) $\frac{p}{4}$

(B) $\frac{p}{3}$

(C) $\frac{\pi}{2}$

(D) None of these.

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Let $0 < A, B < \frac{\pi}{2}$ satisfying the equalities $3\ sin^2 A + 2\ sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$.

Then A + 2B = :

(A) $\frac{p}{4}$

(B) $\frac{p}{3}$

(C) $\frac{\pi}{2}$

(D) None of these.

From the second equation, we have

$$\sin 2B = \frac{3}{2}\sin 2A \qquad \ldots(1)$$

and from the first equality

$$3\sin^2 A = 1 - 2\sin^2 B = \cos 2B \qquad \ldots(2)$$

Now $\cos(A + 2B) = \cos A.\cos 2B - \sin A.\sin 2B$

$$= 3\cos A.\sin^2 A - \frac{3}{2}.\sin A.\sin 2A$$

$$= 3\cos A.\sin^2 A - 3\sin^2 A.\cos A = 0$$

$$\Rightarrow A + 2B = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

Given that $0 < A < \frac{\pi}{2}$ and $0 < B < \frac{\pi}{2} \Rightarrow 0 < A + 2B < \pi + \frac{\pi}{2}$

Hence $A + 2B = \frac{\pi}{2}$.

Hence (C) is the correct answer.

If $a\cos^3\theta + 3a\cos\theta\sin^2\theta = x$ and $a\sin^3\theta + 3a\cos^2\theta\sin\theta = y$, then $(x+y)^{2/3} + (x-y)^{2/3} =$

(A) $2a^{2/3}$ (B) $a^{2/3}$

(C) $3a^{2/3}$ (D) $2a^{1/3}$

$a\cos^3\theta + 3a\cos\theta\sin^2\theta = x$

$a\sin^3\theta + 3a\cos^2\theta\sin\theta = y$

$x + y = a[\sin^3\theta + \cos^3\theta + 3\sin\theta\cos\theta(\sin\theta + \cos\theta)] = a(\sin\theta + \cos\theta)^3$

$$\left(\frac{x+y}{a}\right)^{1/3} = \sin\theta + \cos\theta \quad \ldots\ldots(1)$$

$x - y = a[\cos^3\theta - \sin^3\theta + 3\cos\theta\sin^2\theta - 3\cos^2\theta\sin\theta] = a[\cos\theta - \sin\theta]^3$

$$\left(\frac{x-y}{a}\right)^{1/3} = \cos\theta - \sin\theta \quad \ldots\ldots(2)$$

$$(\sin\theta + \cos\theta)^2 + (\cos\theta - \sin\theta)^2 = \frac{(x+y)^{2/3} + (x-y)^{2/3}}{a^{2/3}}$$

$$2(\sin^2\theta + \cos^2\theta) = \frac{(x+y)^{2/3} + (x-y)^{2/3}}{a^{2/3}}$$

$(x + y)^{2/3} + (x - y)^{2/3} = 2a^{2/3}$.

Hence (A) is the correct answer.

If $\left(1+\sqrt{1+a}\right)\tan\alpha = 1+\sqrt{1-a}$, then $\sin 4\alpha =$

(A) a/2 (B) a

(C) $a^{2/3}$ (D) 2a

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Let $a = \sin 4\theta \Rightarrow \sqrt{1+a} = \cos 2\theta + \sin 2\theta$ and $\sqrt{1-a} = \cos 2\theta - \sin 2\theta$

$(1 + \sqrt{1+a})\tan\alpha = (1 + \sqrt{1-a})$

$\Rightarrow (1 + \cos 2\theta + \sin 2\theta)\tan\alpha = 1 + \cos 2\theta - \sin 2\theta$

$$\Rightarrow \frac{2\cos\theta(\cos\theta + \sin\theta)}{2\cos\theta(\cos\theta - \sin\theta)} = \cot\alpha$$

$$\Rightarrow \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \cot\alpha \qquad \Rightarrow \frac{1+\tan\theta}{1-\tan\theta} = -\cot\alpha$$

$$\Rightarrow \tan\left(\frac{\pi}{4} + \theta\right) = \tan\left(\frac{\pi}{2} + \alpha\right) \qquad \Rightarrow \theta = \left(\frac{\pi}{4} - \alpha\right)$$

$\Rightarrow a = \sin 4\theta = \sin(\pi - 4\alpha) = \sin 4\alpha$

Hence (B) is the correct answer.

The number of solutions of the equation $x^3 + 2x^2 + 5x + 2\cos x = 0$ in $[0, 2\pi]$ is:

(A) 0 (B) 1

(C) 2 (D) 3

Let $f(x) = x^3 + 2x^2 + 5x + 2\cos x$

$\Rightarrow f'(x) = 3x^2 + 4x + 5 - 2\sin x$

$$= 3\left(x + \frac{2}{3}\right)^2 + \frac{11}{3} - 2\sin x$$

Now $\frac{11}{3} - 2\sin x > 0 \ \forall x$ (as $-1 \le \sin x \le 1$)

$\Rightarrow f'(x) > 0 \ \forall x$

$\Rightarrow$ f(x) is an increasing function.

Now $f(0) = 2$

$\Rightarrow f(x) = 0$ has no solution in $[0, 2\pi]$.

Hence (A) is the correct answer.

If $\tan x = n \cdot \tan y$, $n \in \mathbf{R}^{+}$, then maximum value of $\sec^2(x - y)$ is equal to:

(A) $\dfrac{(n+1)^2}{2n}$ (B) $\dfrac{(n+1)^2}{n}$

(C) $\dfrac{(n+1)^2}{2}$ (D) $\dfrac{(n+1)^2}{4n}$

tanx = n tany, cos(x – y)

= cosx. cosy + sinx.siny.

$\Rightarrow$ cos(x – y) = cosx.cosy(1 + tanx.tany)

= cosx. cosy (1 + n tan^{2}y)

$$\Rightarrow \sec^2(x-y) = \frac{\sec^2 x \sec^2 y}{(1+n\tan^2 y^2)}$$

$$= \frac{(1+\tan^2 x)(1+\tan^2 y)}{(1+n\tan^2 y)^2}$$

$$= \frac{(1+n^2\tan^2 y)(1+\tan^2 y)}{(1+n\tan^2 y)^2}$$

$$= 1+\frac{(n-1)^2\tan^2 y}{(1+n\tan^2 y)^2}$$

Now, $\left(\frac{1+n\tan^2 y}{2}\right)^2 \geq n\tan^2 y.$

$$\Rightarrow \frac{\tan^2 y}{(1+n\tan^2 y)^2} \leq \frac{1}{4n}$$

$$\Rightarrow \sec^2(x-y) \leq 1+\frac{(n-1)^2}{4n} = \frac{(n+1)^2}{4n}$$

Hence (D) is the correct answer.

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If $3\sin\theta + 5\cos\theta = 5$, then the value of $5\sin\theta - 3\cos\theta$ is equal to

(A) 5

(B) 3

(C) 4

(D) none of these

$3\sin\theta = 5(1 - \cos\theta) = 5 \times 2\sin^2\theta/2 \Rightarrow \tan\theta/2 = 3/5$

$$5\sin\theta - 3\cos\theta = 5\times\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} - 3\frac{\left(1-\tan^2\frac{\theta}{2}\right)}{1+\tan^2\frac{\theta}{2}} = 5\times\frac{2\times\frac{3}{5}}{1+\frac{9}{25}} - \frac{3\times\left(1-\frac{9}{25}\right)}{1+\frac{9}{25}} = 3$$

Hence (B) is the correct answer.

If $\pi < 2\theta < \frac{3\pi}{2}$, then $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ equals to

(A) $-2\cos\theta$ (B) $-2\sin\theta$

(C) $2\cos\theta$ (D) $2\sin\theta$

HARIOM CHAUDHARY

$$\sqrt{2+\sqrt{2(1+\cos 4\theta)}} = \sqrt{2+2|\cos 2\theta|} = \sqrt{2(1-\cos 2\theta)}$$

$$= 2|\sin\theta| = 2\sin\theta \text{ as } \frac{\pi}{2} < \theta < \frac{3\pi}{4}$$

Hence (D) is the correct answer.

If sin x cos y = 1/4 and 3 tan x = 4 tan y, then find the value of sin (x + y).

(A) 1/16

(B) 7/16

(C) 5/16

(D) none of these

$3 \tan x = 4 \tan y \Rightarrow 3 \sin x \cos y = 4 \cos x \sin y$

$\Rightarrow 3/4 = 4 \cos x \sin y \Rightarrow \cos x \sin y = 3/16$

$\therefore \sin(x + y) = \sin x \cos y + \cos x \sin y = \dfrac{1}{4} + \dfrac{3}{16} = \dfrac{7}{16}.$

Hence (B) is the correct answer.

The maximum value of $4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$ is

(A) $4+\sqrt{2}$ (B) $3+\sqrt{2}$

(C) 9 (D) 4

Maximum value of $4\sin^2 x + 3\cos^2 x$ i.e. $\sin^2 x + 3$ is 4 and that of $\sin\frac{x}{2} + \cos\frac{x}{2}$ is $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \sqrt{2}$, both attained at $x = \pi/2$. Hence the given function has maximum value $4+\sqrt{2}$

Hence (A) is the correct answer

If α and β are solutions of $\sin^2 x + a \sin x + b = 0$ as well as that of $\cos^2 x + c \cos x + d = 0$, then $\sin(\alpha + \beta)$ is equal to

(A) $\dfrac{2bd}{b^2 + d^2}$

(B) $\dfrac{a^2 + c^2}{2ac}$

(C) $\dfrac{b^2 + d^2}{2bd}$

(D) $\dfrac{2ac}{a^2 + c^2}$

According to the given condition, sinα+sinβ = –a and cosα +cosβ= -c.

$$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -a \ \ \& \ \ 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -c$$

$$\Rightarrow \tan\frac{\alpha+\beta}{2} = \frac{a}{c} \Rightarrow \sin(\alpha+\beta) = \frac{2\tan\frac{\alpha+\beta}{2}}{1+\tan^2\frac{\alpha+\beta}{2}} = \frac{2ac}{a^2+c^2}$$

Hence (D) is the correct answer.

If $S = \cos^2\frac{\pi}{n} + \cos^2\frac{2\pi}{n} + \ldots\ldots\ldots\ldots\ldots + \cos^2\frac{(n-1)\pi}{n}$, then S equals

(A) $\frac{n}{2}(n+1)$

(B) $\frac{1}{2}(n-1)$

(C) $\frac{1}{2}(n-2)$

(D) $\frac{n}{2}$

$$S = \cos^2\frac{\pi}{n} + \cos^2\frac{2\pi}{n} + \ldots\ldots\ldots\ldots\ldots + \cos^2(n-1)\frac{\pi}{n}$$

$$= \frac{1}{2}\left[1+\cos\frac{2\pi}{n} + 1+\cos\frac{4\pi}{n} + 1+\cos\frac{6\pi}{n} + \ldots + 1+\cos 2(n-1)\frac{\pi}{n}\right]$$

$$= \frac{1}{2}\left[n-1+\sum_{k=1}^{n-1}\cos\frac{2k\pi}{n}\right] = \frac{1}{2}[n-1-1] = \frac{1}{2}(n-2)$$

Hence (C) is the correct answer

If in a ΔABC, $\angle C = 90°$, then the maximum value of sin A sin B is

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) None

$$\sin A \sin B = \frac{1}{2} \times 2 \sin A \sin B$$

$$= \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right] = \frac{1}{2}\left[\cos(A-B) - \cos 90°\right] = \frac{1}{2}\cos(A-B) \leq \frac{1}{2}$$

$$\Rightarrow \text{Maximum value of } \sin A \sin B = \frac{1}{2}$$

Hence (A) is the correct answer.

If in a ΔABC, $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is always

(A) isosceles triangle (B) right angled

(C) acute angled (D) obtuse angled

$\sin^2 A + \sin^2 B + \sin^2 C = 2 \Rightarrow 2\cos A \cos B \cos C = 0$

$\Rightarrow$ either $A = 90^o$ or $B = 90^o$ or $C = 90^o$

Hence (B) is the correct answer.

Maximum value of the expression 2sinx + 4cosx + 3 is

(A) $2\sqrt{5} + 3$ (B) $2\sqrt{5} - 3$

(C) $\sqrt{5} + 3$ (D) none of these

Maximum value of 2sinx + 4cosx = $2\sqrt{5}$.

Hence the maximum value of 2sinx + 4cosx +3 is $2\sqrt{5} + 3$

Hence (A) is the correct answer.

If $\tan\alpha$ and $\tan\beta$ are the roots of $x^2 - 3x - 1 = 0$, then value of $\tan(\alpha + \beta)$ is

(A) $\frac{1}{2}$ (B) 1

(C) $\frac{3}{2}$ (D) None of these.

$\because$ $\tan\alpha, \tan\beta$ are the roots of $x^2 - 3x - 1 = 0$

$\therefore$ $\tan\alpha + \tan\beta = 3$ and $\therefore$ $\tan\alpha + \tan\beta = -1.$

$$\therefore \quad \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} = \frac{3}{2}.$$

Hence (C) is the correct answer.

Trigonometric identities

HARIOM CHAUDHARY

General solution of the equation sin θ = sinα: If $\sin\theta = \sin\alpha$ or $\sin\theta - \sin\alpha = 0$

or, $2\sin\left(\frac{\theta-\alpha}{2}\right)\cos\left(\frac{\theta+\alpha}{2}\right) = 0 \quad \Rightarrow \quad \sin\left(\frac{\theta-\alpha}{2}\right) = 0 \text{ or } \cos\left(\frac{\theta+\alpha}{2}\right) = 0$

or, $\frac{\theta-\alpha}{2} = m\pi; m \in I$ or $\frac{\theta+\alpha}{2} = (2m+1)\frac{\pi}{2}; m \in I$

$\Rightarrow \theta = 2m\pi + \alpha; m \in I$ or $\theta = (2m+1)\pi - \alpha; m \in I$

$\Rightarrow \theta$ = (any even multiple of π) + α or θ = (any odd multiple of π) – α

$$\boxed{\theta = n\pi + (-1)^n \alpha \; ; \; n \in I}$$

(2) **General solution of the equation cosθ = cosα :** If $\cos\theta = \cos\alpha \Rightarrow \cos\theta - \cos\alpha = 0 \Rightarrow$ $-2\sin\left(\frac{\theta+\alpha}{2}\right).\sin\left(\frac{\theta-\alpha}{2}\right) = 0 \Rightarrow \sin\left(\frac{\theta+\alpha}{2}\right) = 0$ or $\sin\left(\frac{\theta-\alpha}{2}\right) = 0$, $\Rightarrow \frac{\theta+\alpha}{2} = n\pi; n \in I$ or $\frac{\theta-\alpha}{2} = n\pi; n \in I$

$\Rightarrow \theta = 2n\pi - \alpha; n \in I$ or $\theta = 2n\pi + \alpha; n \in I$. for the general solution of $\cos\theta = \cos\alpha$, combine these two result which gives $\boxed{\theta = 2n\pi \pm \alpha;\ n \in I}$

General solution of the equation tan θ = tan α : If $\tan\theta = \tan\alpha \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha}{\cos\alpha}$

$\Rightarrow \sin\theta\cos\alpha - \cos\theta\sin\alpha = 0 \Rightarrow \sin(\theta-\alpha) = 0 \Rightarrow \theta - \alpha = n\pi; n \in I$ $\boxed{\theta = n\pi + \alpha; n \in I}$

General Solution of Some Particular Equations

(1) $\sin\theta = 0 \Rightarrow \theta = n\pi$, $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}$ or $n\pi + \frac{\pi}{2}$, $\tan\theta = 0 \Rightarrow \theta = n\pi$

(2) $\sin\theta = 1 \Rightarrow \theta = (4n+1)\frac{\pi}{2}$ or $2n\pi + \frac{\pi}{2}$, $\cos\theta = 1 \Rightarrow \theta = 2n\pi$, $\tan\theta = 1 \Rightarrow \theta = (4n+1)\frac{\pi}{4}$ or $n\pi + \frac{\pi}{4}$

(3) $\sin\theta = -1 \Rightarrow \theta = (4n+3)\frac{\pi}{2}$ or $2n\pi + \frac{3\pi}{2}$, $\cos\theta = -1 \Rightarrow \theta = (2n+1)\pi$, $\tan\theta = -1 \Rightarrow \theta = (4n-1)\frac{\pi}{4}$ or $n\pi - \frac{\pi}{4}$

(4) $\tan\theta$ = not defined $\Rightarrow \theta = (2n+1)\frac{\pi}{2}$, $\cot\theta$ = not defined $\Rightarrow \theta = n\pi$

$\operatorname{cosec}\theta$ = not defined $\Rightarrow \theta = n\pi$, $\sec\theta$ = not defined $\Rightarrow \theta = (2n+1)\frac{\pi}{2}$.

If $\sin\theta = \frac{\sqrt{3}}{2}$, then the general value of θ is [MP PET 1988]

(a) $2n\pi \pm \frac{\pi}{6}$

(b) $2n\pi \pm \frac{\pi}{3}$

(c) $n\pi + (-1)^n \frac{\pi}{3}$

(d) $n\pi + (-1)^n \frac{\pi}{6}$

$$\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \sin\theta = \sin\frac{\pi}{3} \Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{3}$$

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The general solution of $\tan 3x = 1$ is [**Karnataka CET 1991**]

(a) $n\pi + \frac{\pi}{4}$

(b) $\frac{n\pi}{3} + \frac{\pi}{12}$

(c) $n\pi$

(d) $n\pi \pm \frac{\pi}{4}$

$$\tan 3x = \tan \frac{\pi}{4} \Rightarrow 3x = n\pi + \frac{\pi}{4} \Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{12}.$$

If $\sin 3\theta = \sin \theta$, then the general value of θ is

(a) $2n\pi, (2n+1)\frac{\pi}{3}$

(b) $n\pi, (2n+1)\frac{\pi}{4}$

(c) $n\pi, (2n+1)\frac{\pi}{3}$

(d) None of these

$$\sin 3\theta = \sin\theta \text{ or } 3\theta = m\pi + (-1)^m \theta$$

For (m) even *i.e.*, $m = 2n$ then $\theta = \dfrac{2n\pi}{2} = n\pi$

And for (m) odd, *i.e.*, $m = (2n+1)$ then $\theta = (2n+1)\dfrac{\pi}{4}$.

The general solution of $2\sin^2\theta - 3\sin\theta - 2 = 0$ is **[Roorkee 1993]**

(a) $n\pi + (-1)^n \dfrac{\pi}{2}$

(b) $n\pi + (-1)^n \dfrac{\pi}{6}$

(c) $n\pi + (-1)^n \dfrac{7\pi}{6}$

(d) $n\pi - (-1)^n \dfrac{\pi}{6}$

$$2\sin^2\theta - 3\sin\theta - 2 = 0$$

$$\Rightarrow 2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0 \Rightarrow 2\sin\theta(\sin\theta - 2) + (\sin\theta - 2) = 0$$

$$(2\sin\theta + 1)(\sin\theta - 2) = 0$$

$\sin\theta = +2$ (which is impossible)

$$\Rightarrow \therefore \sin\theta = -\frac{1}{2} \Rightarrow \sin\theta = \sin(-\pi/6) \Rightarrow \theta = n\pi - (-1)^n \pi/6$$

If $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$, then **[UPSEAT 2001]**

(a) $\theta = \frac{(6n+1)\pi}{18}, \forall n \in I$ (b) $\theta = \frac{(6n+1)\pi}{9}, \forall n \in I$

(c) $\theta = \frac{(3n+1)\pi}{9}, \forall n \in I$ (d) None of these

$$\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$$

$$\Rightarrow \tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$$

$$\Rightarrow \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3} \Rightarrow \tan 3\theta = \tan\left(\frac{\pi}{3}\right)$$

$$\Rightarrow 3\theta = n\pi + \frac{\pi}{3} \Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{9} = (3n+1)\frac{\pi}{9}$$

General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is [IIT 1996]

(a) $m\pi, n\pi + \frac{\pi}{3}$

(b) $m\pi, n\pi \pm \frac{\pi}{3}$

(c) $m\pi, n\pi \pm \frac{\pi}{6}$

(d) None of these

$$\tan^2\theta + \sec 2\theta = 1 \quad \Rightarrow \tan^2\theta + \frac{1+\tan^2\theta}{1-\tan^2\theta} = 1$$

$$\Rightarrow \tan^2\theta - \tan^4\theta + 1 + \tan^2\theta = 1 - \tan^2\theta$$

$$\tan^4\theta - 3\tan^2\theta = 0 \Rightarrow \tan^2\theta(\tan^2\theta - 3) = 0 \Rightarrow \tan^2\theta = 0 \text{ and } \tan^2\theta = 3$$

$$\tan^2\theta = \tan^2 0 \text{ and } \tan^2\theta = \tan^2\frac{\pi}{3} \Rightarrow \theta = m\pi \text{ and } \theta = n\pi \pm \frac{\pi}{3}.$$

If $\sec^2\theta = \frac{4}{3}$, then the general value of θ is **[MP PET 1988]**

(a) $2n\pi \pm \frac{\pi}{6}$ (b) $n\pi \pm \frac{\pi}{6}$

(c) $2n\pi \pm \frac{\pi}{3}$ (d) $n\pi \pm \frac{\pi}{3}$

$$\sec^2\theta = \frac{4}{3} \Rightarrow \cos^2\theta = \frac{3}{4} = \left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow \cos^2\theta = \cos^2\frac{\pi}{6}$$

$$\therefore \theta = n\pi \pm \frac{\pi}{6}.$$

The most general value of θ satisfying the equation

$$\tan\theta = -1 \text{ and } \cos\theta = \frac{1}{\sqrt{2}} \text{ is}$$

[MP PET 2003; UPSEAT 2002, 1982; Roorkee 1990]

(a) $n\pi + \frac{7\pi}{4}$

(b) $n\pi + (-1)^n \frac{7\pi}{4}$

(c) $2n\pi + \frac{7\pi}{4}$

(d) None of these

$$\tan\theta = -1 = \tan\left(2\pi - \frac{\pi}{4}\right) \text{ and } \cos\theta = \frac{1}{\sqrt{2}} = \cos\left(2\pi - \frac{\pi}{4}\right)$$

Hence, general value is $2n\pi + \left(2\pi - \frac{\pi}{4}\right) = 2n\pi + \frac{7\pi}{4}$.

If $\left(\frac{\sin\theta}{\sin\phi}\right)^2 = \frac{\tan\theta}{\tan\phi} = 3$, then the value of θ and ϕ are

(a) $\theta = n\pi \pm \frac{\pi}{3}, \phi = n\pi \pm \frac{\pi}{6}$

(b) $\theta = n\pi - \frac{\pi}{3}, \phi = n\pi - \frac{\pi}{6}$

(c) $\theta = n\pi \pm \frac{\pi}{2}, \phi = n\pi + \frac{\pi}{3}$

(d) None of these

The number of integral values of *k*, for which the equation

$7\cos x + 5\sin x = 2k + 1$ has a solution is **[IIT Screening 2002]**

(a) 4 (b) 8

(c) 10 (d) 12

HARIOM CHAUDHARY

$$\left(\frac{\sin\theta}{\sin\phi}\right)^2 = \frac{\tan\theta}{\tan\phi} \quad \Rightarrow \sin\theta\cos\theta = \sin\phi\cos\phi \quad \Rightarrow \sin 2\theta = \sin 2\phi$$

$$2\theta = \pi - 2\phi \quad \Rightarrow \theta = \frac{\pi}{2} - \phi$$

But, $\dfrac{\tan\theta}{\tan\phi} = 3 \Rightarrow \dfrac{\tan\theta}{\cot\theta} = 3 \Rightarrow \tan^2\theta = 3 \Rightarrow \theta = n\pi \pm \dfrac{\pi}{3}$, so that $\phi = n\pi \pm \dfrac{\pi}{6}$

Trick: Check with the options for $n = 0, n = 1$.

$$-\sqrt{7^2+5^2} \le (7\cos x + 5\sin x) \le \sqrt{7^2+5^2}$$

So, for solution $-\sqrt{74} \le (2k+1) \le \sqrt{74}$

or $-8.6 \le 2k+1 \le 8.6$ or $-9.6 \le 2k \le 7.6$ or $-4.8 \le k \le 3.8$.

So, integral values of k are $-4, -3, -2, -1, 0, 1, 2, 3$ (eight values)

If $\sqrt{3}\cos\theta+\sin\theta=\sqrt{2}$, then general value of θ is [MP PET 2002, 1991; UPSEAT 1999]

(a) $n\pi+(-1)^n\frac{\pi}{4}$

(b) $(-1)^n\frac{\pi}{4}-\frac{\pi}{3}$

(c) $n\pi+\frac{\pi}{4}-\frac{\pi}{3}$

(d) $n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{3}$

$$\sqrt{3}\cos\theta + \sin\theta = \sqrt{2} \quad \Rightarrow \frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\frac{\pi}{3}\cos\theta + \cos\frac{\pi}{3}\sin\theta = \frac{1}{\sqrt{2}} \quad \Rightarrow \sin\left(\theta + \frac{\pi}{3}\right) = \sin\frac{\pi}{4} \quad \Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{3}.$$

If $(\cos x - \sin x)\left(2\tan x + \frac{1}{\cos x}\right) + 2 = 0$ then $x =$

(a) $2n\pi \pm \frac{\pi}{3}$

(b) $n\pi \pm \frac{\pi}{3}$

(c) $2n\pi \pm \frac{\pi}{6}$

(d) None of these

Let $t = \tan\frac{x}{2}$, and using the formula. We get,

$$\left\{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} - \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right\}\left\{\frac{4\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} + \frac{1+\tan^2\frac{x}{2}}{1-\tan^2\frac{x}{2}}\right\} + 2 = 0$$

$$\left(\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2}\right)\left(\frac{4t}{1-t^2} + \frac{1+t^2}{1-t^2}\right) + 2 = 0 \quad \Rightarrow \quad \frac{3t^4 + 6t^3 + 8t^2 - 2t - 3}{(t^2+1)(1-t^2)} = 0$$

Its roots are; $t_1 = \frac{1}{\sqrt{3}}$ and $t_2 = -\frac{1}{\sqrt{3}}$.

Thus the solution of the equation reduces to that of two elementary equations,

$$\tan\frac{x}{2} = \frac{1}{\sqrt{3}}, \tan\frac{x}{2} = -\frac{1}{\sqrt{3}} \quad \Rightarrow \quad \frac{x}{2} = n\pi \pm \frac{\pi}{6} \quad \Rightarrow \quad x = 2n\pi \pm \frac{\pi}{3}, \text{ is required solution.}$$

PROPERTIES OF ANGLES

HARIOM CHAUDHARY

The law of sines or sine rule :

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

HARIOM CHAUDHARY

If the angles of a triangle are in the ratio 4 : 1 : 1,

then the ratio of the longest side to the perimeter is **[IIT Screening 2003]**

(a) $\sqrt{3} : (2 + \sqrt{3})$ (b) 1 : 6

(c) $1 : (2 + \sqrt{3})$ (d) 2 : 3

$$4x + x + x = 180 \Rightarrow 6x = 180 \Rightarrow x = 30^o$$

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$$\frac{\sin 120^o}{a} = \frac{\sin 30^o}{b} = \frac{\sin 30^o}{c}$$

$$\therefore \quad a : (a + b + c) = (\sin 120^o) : (\sin 120^o + \sin 30^o + \sin 30^o)$$

$$= \frac{\sqrt{3}}{2} : \frac{\sqrt{3} + 2}{2} = \sqrt{3} : \sqrt{3} + 2 .$$

HARIOM CHAUDHARY

In a triangle ABC, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$ and D divides BC internally in the ratio 1 : 3. Then $\frac{\sin \angle BAD}{\sin \angle CAD}$ is equal to

[UPSEAT 2003, 2001; IIT 1995]

(a) $\frac{1}{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{6}}$

(d) $\sqrt{\frac{2}{3}}$

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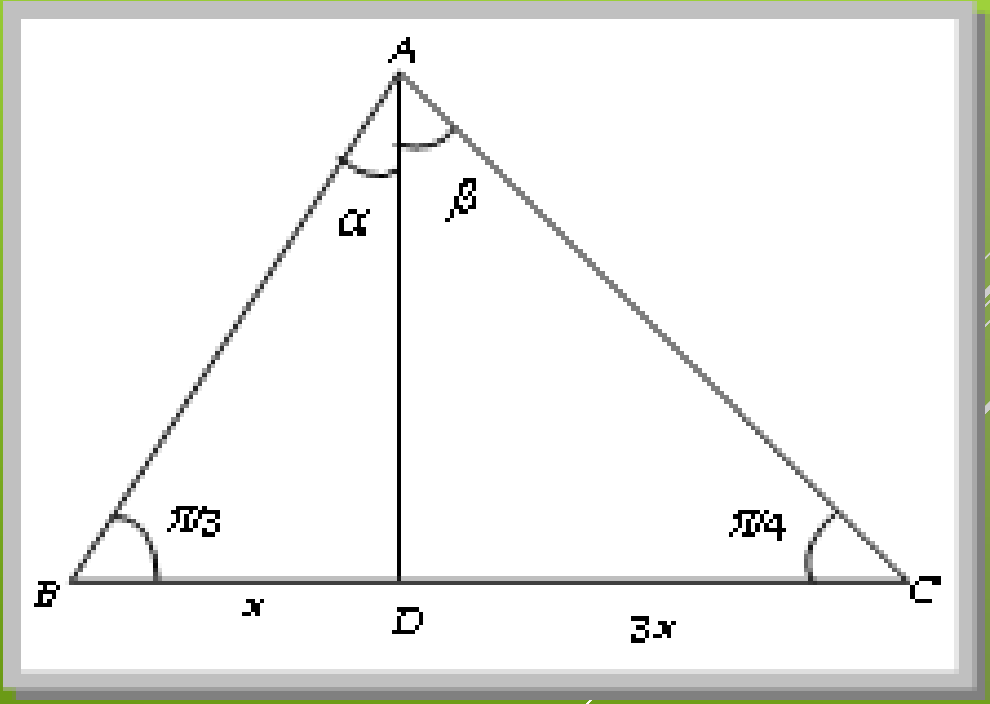
Let $\angle BAD = \alpha, \angle CAD = \beta$

In ΔADB , applying sine formulae, we get $\dfrac{x}{\sin\alpha} = \dfrac{AD}{\sin\left(\dfrac{\pi}{3}\right)}$ ....(i)

In ΔADC, applying sine formulae, we get, $\dfrac{3x}{\sin\beta} = \dfrac{AD}{\sin\left(\dfrac{\pi}{4}\right)}$ ....(ii)



Dividing (i) by (ii), we get,

$$\Rightarrow \quad \frac{x}{\sin\alpha} \times \frac{\sin\beta}{3x} = \frac{AD}{\sin\left(\frac{\pi}{3}\right)} \times \frac{\sin\left(\frac{\pi}{4}\right)}{AD}$$

$$\Rightarrow \frac{\sin\beta}{3\sin\alpha} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}} = \sqrt{\frac{2}{3}} \Rightarrow \frac{\sin\beta}{\sin\alpha} = 3\sqrt{\frac{2}{3}} = \sqrt{6}$$

$$\therefore \quad \frac{\sin\angle BAD}{\sin\angle CAD} = \frac{\sin\alpha}{\sin\beta} = \frac{1}{\sqrt{6}}.$$

HARIOM CHAUDHARY

In a $\Delta ABC, A:B:C = 3:5:4$. Then $[a + b + c\sqrt{2}]$ is equal to [DCE 2001]

(a) $2b$ (b) $2c$

(c) $3b$ (d) $3a$

$$A:B:C=3:5:4 \quad \Rightarrow \quad A+B+C=12x=180^{o} \quad \Rightarrow \quad x=15^{o}$$

$$\therefore \quad A=45^{o},\ B=75^{o},\ C=60^{o}$$

$$\frac{a}{\sin 45^{o}}=\frac{b}{\sin 75^{o}}=\frac{c}{\sin 60^{o}}=k \quad (\text{say})$$

$$i.e.,\ a=\frac{1}{\sqrt{2}}K,\quad b=\frac{\sqrt{3}+1}{2\sqrt{2}}K,\quad c=\frac{\sqrt{3}}{2}K.\ \text{Hence } [a+b+c\sqrt{2}]=3b$$

The Law of Cosines or Cosine Rule.

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(1) $a^2 = b^2 + c^2 - 2bc\cos A \Rightarrow \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

(2) $b^2 = c^2 + a^2 - 2ca\cos B \Rightarrow \cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$

(3) $c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow \cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

HARIOM CHAUDHARY

The smallest angle of the ΔABC,

when $a = 7, b = 4\sqrt{3}$ and $c = \sqrt{13}$, is [MP PET 2003]

(a) 30^o (b) 15^o

(c) 45^o (d) None of these

Smallest angle is opposite to smaller side

$$\therefore \cos C = \frac{b^2 + a^2 - c^2}{2ab} = \frac{49 + 48 - 13}{2 \times 7 \times 4\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow \angle C = 30^o .$$

In a ΔABC, if $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then $\cos C =$ [Karnataka CET 2003]

(a) $\frac{7}{5}$

(b) $\frac{5}{7}$

(c) $\frac{17}{36}$

(d) $\frac{16}{17}$

$$\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = \lambda \text{ (Let)}$$

$\therefore\ b+c = 11\lambda$ ......(i)

$c+a = 12\lambda$ ....(ii) and $a+b = 13\lambda$ ....(iii)

From (i) + (ii) + (iii), $2(a+b+c) = 36\lambda$ $\therefore\ a+b+c = 18\lambda$ ....(iv)

Now subtract (i), (ii) and (iii) from (iv), $a = 7\lambda, b = 6\lambda, c = 5\lambda$.

$$\text{Now } \cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{(7\lambda)^2+(6\lambda)^2-(5\lambda)^2}{2.7\lambda.6\lambda} = \frac{49\lambda^2+36\lambda^2-25\lambda^2}{84\lambda^2} = \frac{60\lambda^2}{84\lambda^2} = \frac{5}{7}.$$

In a ΔABC, $2ac\sin\left(\frac{A-B+C}{2}\right)$ is equal to **[IIT Screening 2000]**

(a) $a^2+b^2-c^2$ (b) $c^2+a^2-b^2$

(c) $b^2-c^2-a^2$ (d) $c^2-a^2-b^2$

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$$2ac\sin\frac{A-B+C}{2}=2ac\sin\frac{\pi-2B}{2}=2ac\cos B$$

$$=2ac\frac{c^2+a^2-b^2}{2ca}=c^2+a^2-b^2$$

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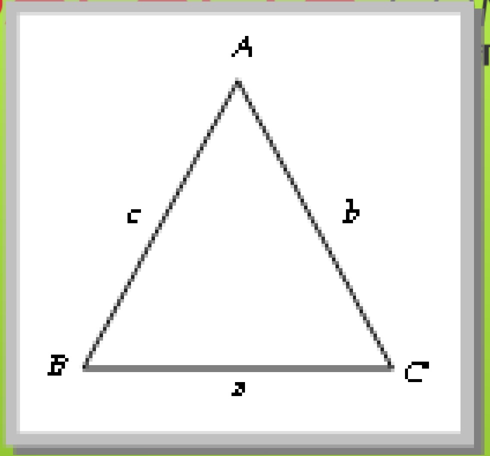
Projection Formulae



(i) $a = b\cos c + c\cos B$

(ii) $b = c\cos A + a\cos C$

(iii) $c = a\cos B + b\cos A$

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In a $\Delta ABC, \dfrac{\cos C + \cos A}{c + a} + \dfrac{\cos B}{b}$ is equal to **[EAMCET 2001]**

(a) $\dfrac{1}{a}$

(b) $\dfrac{1}{b}$

(c) $\dfrac{1}{c}$

(d) $\dfrac{c + a}{b}$

$$\frac{\cos C + \cos A}{c + a} + \frac{\cos B}{b} = \frac{(b\cos C + b\cos A) + (c\cos B + a\cos B)}{b(c+a)}$$

$$= \frac{(b\cos C + c\cos B) + (b\cos A + a\cos B)}{b(c+a)} = \frac{a+c}{b(c+a)} \text{ (Using projection formulae)} = \frac{1}{b}.$$

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$$b\cos^2\frac{C}{2}+c\cos^2\frac{B}{2}=\frac{b}{2}(1+\cos C)+\frac{c}{2}(1+\cos B)$$

$$=\frac{b}{2}+\frac{c}{2}+\frac{1}{2}(b\cos C+c\cos B)=\frac{a+b+c}{2}=\frac{k}{2}$$

Napier's Analogy (Law of Tangents)

For any triangle ABC,

(1) $$\tan\left(\frac{A-B}{2}\right)=\left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$$

(2) $$\tan\left(\frac{B-C}{2}\right)=\left(\frac{b-c}{b+c}\right)\cot\frac{A}{2}$$

(3) $$\tan\left(\frac{C-A}{2}\right)=\left(\frac{c-a}{c+a}\right)\cot\frac{B}{2}$$

- **Mollweide's formula:** For any triangle,

$$\frac{a+b}{c}=\frac{\cos\frac{1}{2}(A-B)}{\sin\frac{1}{2}C},\quad \frac{a-b}{c}=\frac{\sin\frac{1}{2}(A-B)}{\cos\frac{1}{2}C}.$$

If $\tan\frac{B-C}{2} = x\cot\frac{A}{2}$, then x equal to **[MP PET 1992, 2002]**

(a) $\frac{c-a}{c+a}$

(b) $\frac{a-b}{a+b}$

(c) $\frac{b-c}{b+c}$

(d) None of these

We know, $\tan\dfrac{B-C}{2}=\dfrac{b-c}{b+c}\cot\dfrac{A}{2} \Rightarrow x=\dfrac{b-c}{b+c}.$

If in a ΔABC $a = 6, b = 3$ and $\cos(A - B) = \frac{4}{5}$, then [Roorkee 1997]

(a) $C = \frac{\pi}{4}$

(b) $A = \sin^{-1}\frac{2}{\sqrt{5}}$

(c) $ar(\Delta ABC) = 9$

(d) None of these

$$\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}=\frac{1}{3}\cot\frac{C}{2}$$

$$\therefore\ \frac{4}{5}=\cos(A-B)=\frac{1-\tan^2\frac{A-B}{2}}{1+\tan^2\frac{A-B}{2}}=\frac{1-\frac{1}{9}\cot^2\frac{C}{2}}{1+\frac{1}{9}\cot^2\frac{C}{2}}$$

$$\therefore\ \tan^2\frac{C}{2}=1\ \Rightarrow\ C=\frac{\pi}{2}\quad\therefore\ ar(\Delta ABC)=\frac{1}{2}ab=\frac{1}{2}.6.3=9$$

$$\text{Also, }\sin A=\frac{6}{\sqrt{3^2+6^2}}=\frac{2}{\sqrt{5}}.$$

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Area of Triangle

(1) **When two sides and the included angle be given:**

The area of triangle ABC is given by, $\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \frac{1}{2} ab \sin C$

$i.e., \; \Delta = \frac{1}{2}$ (Product of two sides) × sine of included angle

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(2) When three sides are given:

$$\text{Area of } \Delta ABC = \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

where semiperimeter of triangle $s = \dfrac{a+b+c}{2}$

(2) **When three sides and the circum-radius be given:** Area of triangle $\Delta = \dfrac{abc}{4R}$,

where R be the circum-radius of the triangle.

(4) **When two angles and included side be given :**

$$\Delta = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin(B+C)} = \frac{1}{2}b^2 \frac{\sin A \sin C}{\sin(A+C)} = \frac{1}{2}c^2 \frac{\sin A \sin B}{\sin(A+B)}$$

In a ΔABC if $a = 2x, b = 2y$ and $\angle C = 120^o$, then the area of the triangle is **[MP PET 1986, 2002]**

(a) xy (b) $xy\sqrt{3}$

(c) $3xy$ (d) $2xy$

$$\Delta = \frac{1}{2} ab \sin C = \frac{1}{2} .2x.2y \sin 120^o = \sqrt{3}xy .$$

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In a ΔABC, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and $a = 2$, then the area of a triangle is [MP PET 2000; IIT 1993]

(a) 1 (b) 2

(c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{3}$

By sine rule, $\tan A = \tan B = \tan C$; $\therefore$ Triangle is equilateral .

Hence, $\Delta = \frac{1}{2}.a.a.\sin 60^o = \frac{1}{2}.2.2.\frac{\sqrt{3}}{2} = \sqrt{3}$.

Half Angle Formulae

(1) **Formulae for** $\sin\frac{A}{2}, \sin\frac{B}{2}, \sin\frac{C}{2}$:

(i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ (ii) $\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ca}}$ (iii) $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

(2) **Formulae for** $\cos\frac{A}{2}, \cos\frac{B}{2}, \cos\frac{C}{2}$:

(i) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ (ii) $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$ (iii) $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

(3) **Formulae for** $\tan\frac{A}{2}, \tan\frac{B}{2}, \tan\frac{C}{2}$:

(i) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ (ii) $\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$ (iii) $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

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If in any ΔABC; $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in A.P., then **[MP PET 2003]**

(a) $\cot\frac{A}{2}\cot\frac{B}{2} = 4$

(b) $\cot\frac{A}{2}\cot\frac{C}{2} = 3$

(c) $\cot\frac{B}{2}\cot\frac{C}{2} = 1$

(d) $\cot\frac{B}{2}\tan\frac{C}{2} = 0$

Trick: Take $A = B = C = 60^o$,

then $\cot\frac{A}{2}, \cot\frac{B}{2}$ and $\cot\frac{C}{2}$

are in A.P. with common difference zero. Now option (b) satisfies.

In a ΔABC, if $3a = b + c$, then the value of $\cot\frac{B}{2}\cot\frac{C}{2}$ is [Roorkee 1986; MP PET 1990, 97, 98]

(a) 1 (b) 2

(c) $\sqrt{3}$ (d) $\sqrt{2}$

$$\cot\frac{B}{2}.\cot\frac{C}{2}=\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}.\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}=\frac{s}{s-a}$$

$$\text{Given } 3a=b+c\Rightarrow a+b+c=4a \Rightarrow \quad \therefore \quad \cot\frac{B}{2}.\cot\frac{C}{2}=\frac{2a}{a}=2$$

In ΔABC , $\left(\cot\frac{A}{2}+\cot\frac{B}{2}\right)\left(a\sin^2\frac{B}{2}+b\sin^2\frac{A}{2}\right)$ equal to [Roorkee 1988]

(a) $\cot C$ (b) $c\cot C$

(c) $\cot\frac{C}{2}$ (d) $c\cot\frac{C}{2}$

$$\left\{\cot\frac{A}{2}+\cot\frac{B}{2}\right\}\left\{a\sin^2\frac{B}{2}+b\sin^2\frac{A}{2}\right\}=\left\{\frac{\cos\frac{C}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}}\right\}\left\{a\sin^2\frac{B}{2}+b\sin^2\frac{A}{2}\right\}=\left\{\cos\frac{C}{2}\right\}\left\{a\frac{\sin\frac{B}{2}}{\sin\frac{A}{2}}+b\frac{\sin\frac{A}{2}}{\sin\frac{B}{2}}\right\}$$

$$=\sqrt{\frac{s(s-c)}{ab}}\left\{a\frac{\sqrt{\frac{(s-a)(s-c)}{ac}}}{\sqrt{\frac{(s-b)(s-c)}{bc}}}+b\frac{\sqrt{\frac{(s-b)(s-c)}{bc}}}{\sqrt{\frac{(s-a)(s-c)}{ac}}}\right\}=\sqrt{\frac{s(s-c)}{ab}}\left\{\sqrt{\left(\frac{s-a}{s-b}\right)ab}+\sqrt{\left(\frac{s-b}{s-a}\right)ab}\right\}$$

$$=\sqrt{s(s-c)}\left\{\frac{s-a+s-b}{\sqrt{(s-a)(s-b)}}\right\}=\sqrt{s(s-c)}\left\{\frac{2s-a-b}{\sqrt{(s-a)(s-b)}}\right\}=c\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}=c\cot\frac{C}{2}.$$

Trick : Such type of unconditional problems can be checked by putting the particular values for

$a=1$, $b=\sqrt{3}$, $c=2$ and $A=30^0$, $B=60^0$, $C=90^0$, Here expression is equal to 2 which is given by (d).

Height and distance

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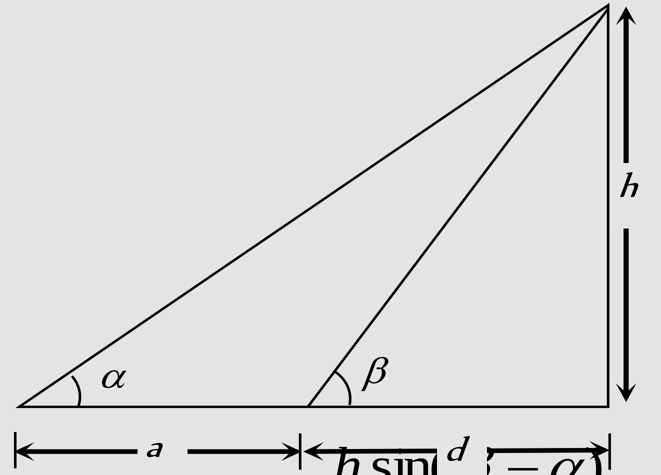
Some Important Results

(1)

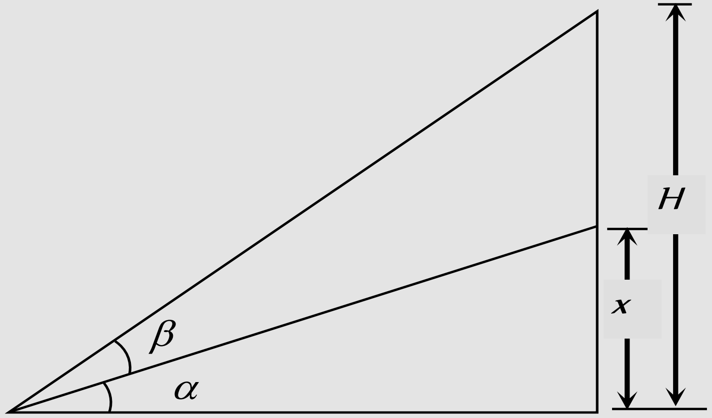


$$a = h(\cot\alpha - \cot\beta) = \frac{h\sin(\beta-\alpha)}{\sin\alpha.\sin\beta}$$

$$\therefore h = a\sin\alpha\sin\beta\,\text{cosec}\,(\beta-\alpha) \text{ and}$$

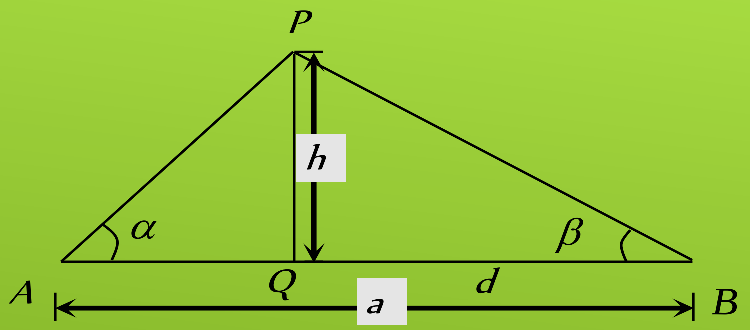
$$d = h\cot\beta = a\sin\alpha.\cos\beta.\text{cosec}(\beta-\alpha)$$

(2)



$$H = x\cot\alpha\tan(\alpha+\beta)$$

(3)

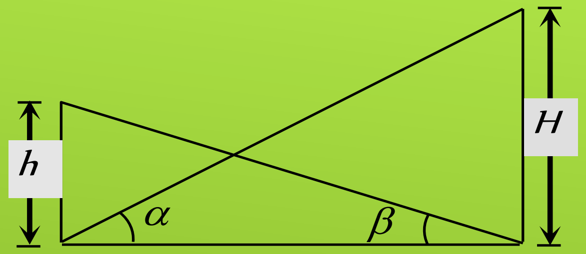


$a = h(\cot\alpha + \cot\beta)$, where by

$h = a\sin\alpha.\sin\beta.\operatorname{cosec}(\alpha+\beta)$ and

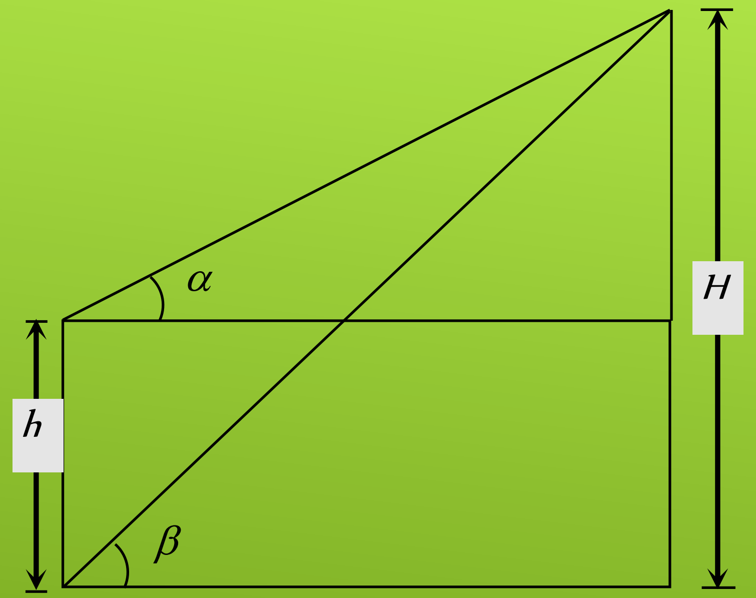
$d = h\cot\beta = a\sin\alpha.\cos\beta.\operatorname{cosec}(\alpha+\beta)$

(4)



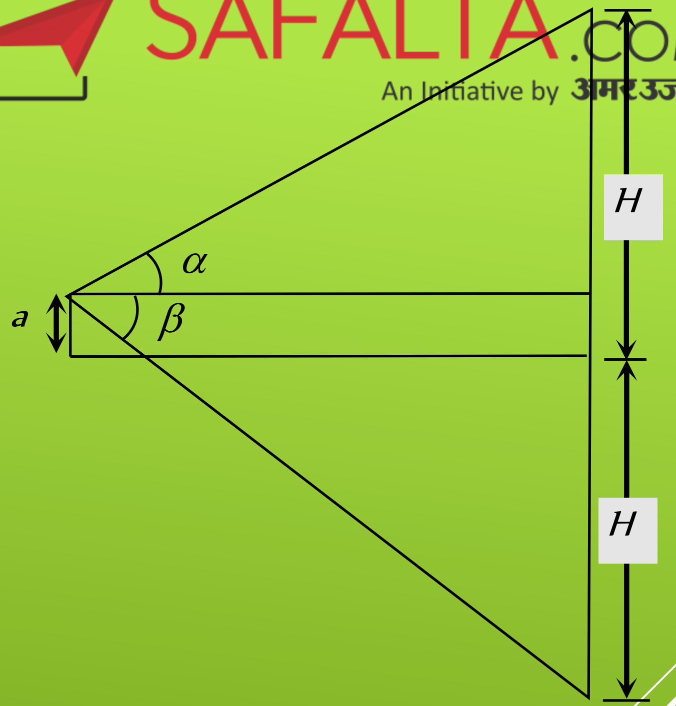
$$H = \frac{h\cot\beta}{\cot\alpha}$$

(5)



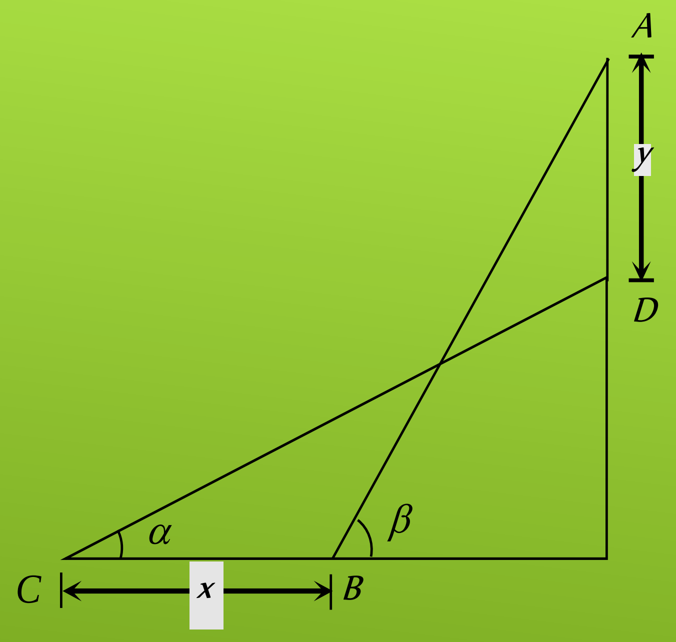
$$h = \frac{H\sin(\beta - \alpha)}{\cos\alpha \sin\beta}$$ or $$H = \frac{h\cot\alpha}{\cot\alpha - \cot\beta}$$

(6)



$$H = \frac{a\sin(\alpha + \beta)}{\sin(\beta - \alpha)}$$

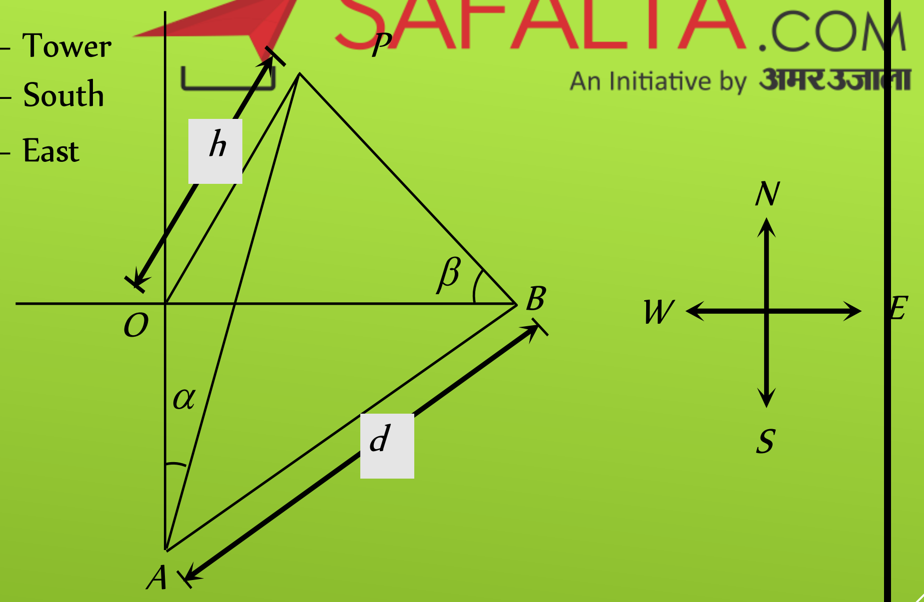
(7)



$AB = CD$. Then, $x = y\tan\left(\frac{\alpha+\beta}{2}\right)$

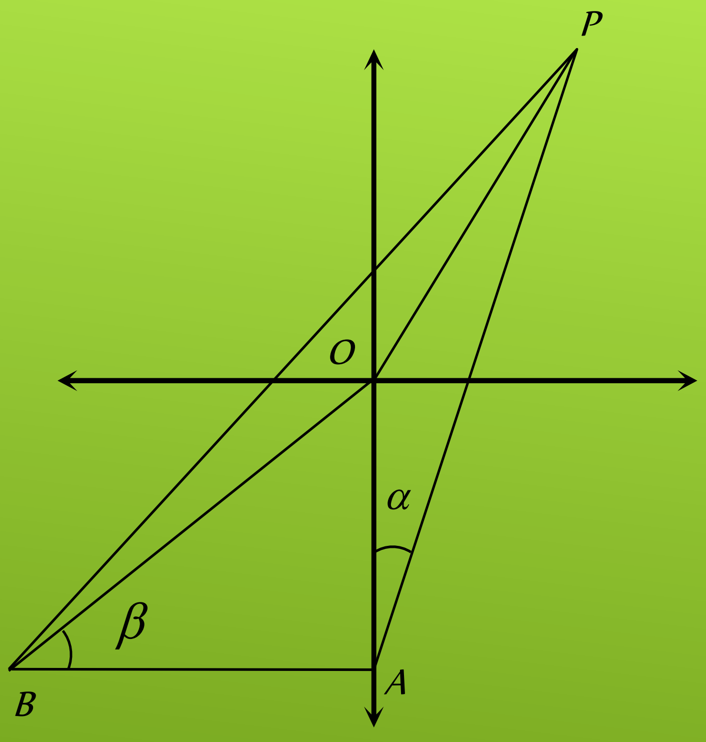
(8)

OP – Tower
A – South
B – East



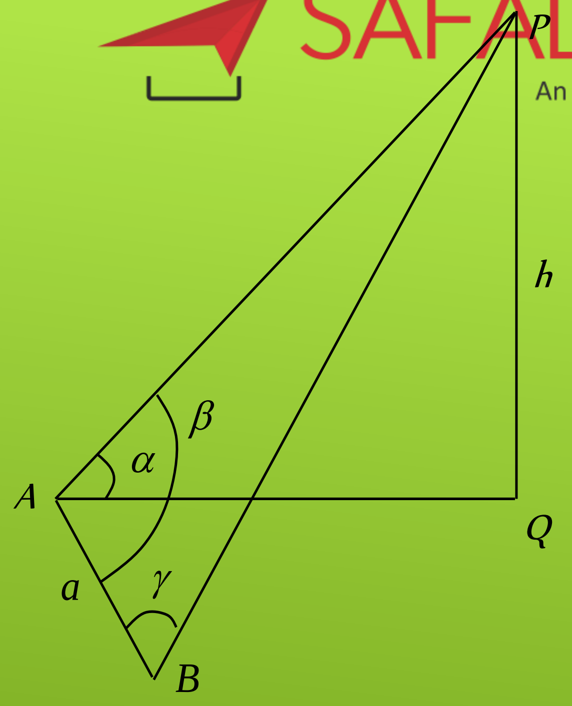
$$h = \frac{d}{\sqrt{\cot^2\beta + \cot^2\alpha}}$$

(9)



$$h = \frac{AB}{\sqrt{\cot^2 \beta - \cot^2 \alpha}}$$

(10)



$h = AP \sin \alpha = a \sin \alpha . \sin \gamma . \text{cosec} \, (\beta - \gamma)$ and

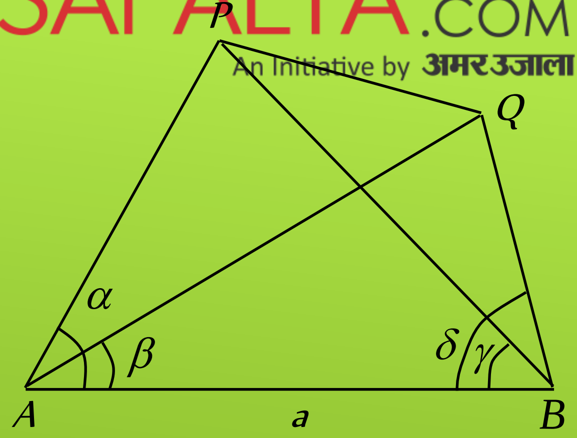
if $AQ = d$, then $d = AP \cos \alpha = a \cos \alpha . \sin \gamma . \text{cosec} \, (\beta - \gamma)$

(ii) $AP = a\sin\gamma.\text{cosec}(\alpha-\gamma)$

$AQ = a\sin\delta.\text{cosec}(\beta-\delta)$

and apply,

$$PQ^2 = AP^2 + AQ^2 - 2AP.AQ\cos\theta$$



The angle of elevation of the top of a tower from a point 20 *meters* away from its base is 45^o. The height of the tower is

[MP PET 1984, 1989]

(a) 10 m (b) 20 m

(c) 40 m (d) $20\sqrt{3}\ m$

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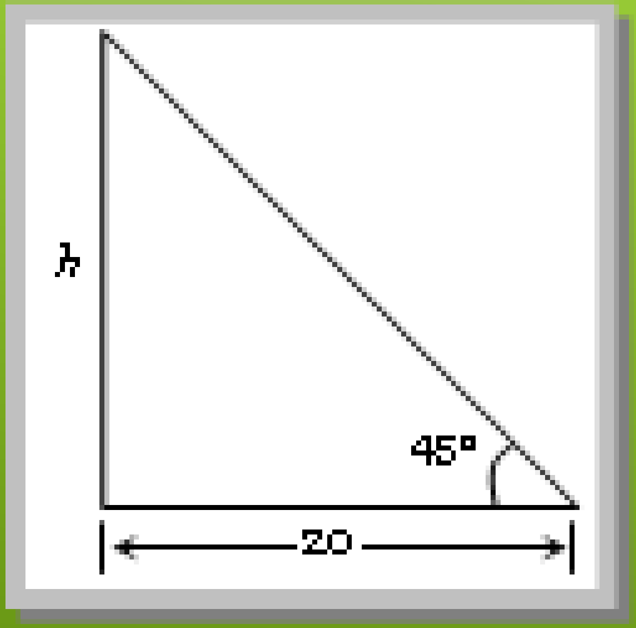
Let height of the tower be h.



$$\frac{h}{20} = \tan 45^{o}$$

$$h = 20\,m\ .$$

If the angle of elevation of the top of a tower at a distance 500 m from its foot is 30^o, then height of the tower is

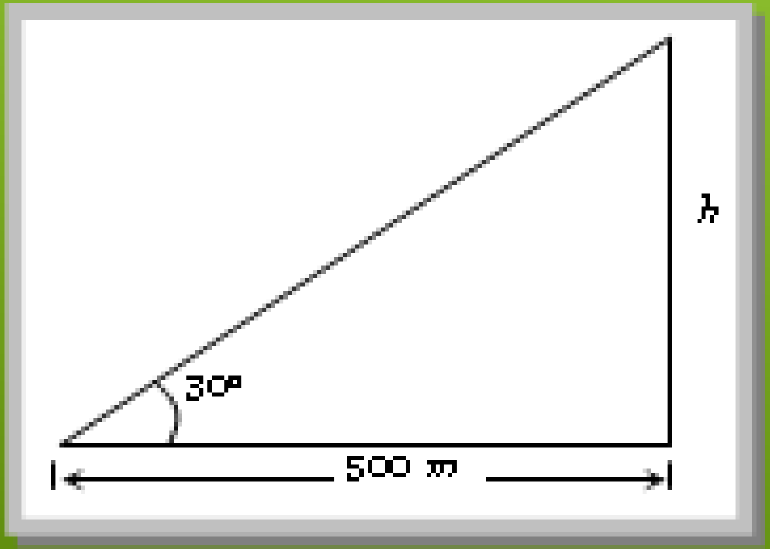
[Kerala (Engg.) 2002]

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{500}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $\frac{1}{500}$

Let the height be h

$$\therefore \tan 30^{o} = \frac{h}{500} \Rightarrow h = \frac{500}{\sqrt{3}}.$$



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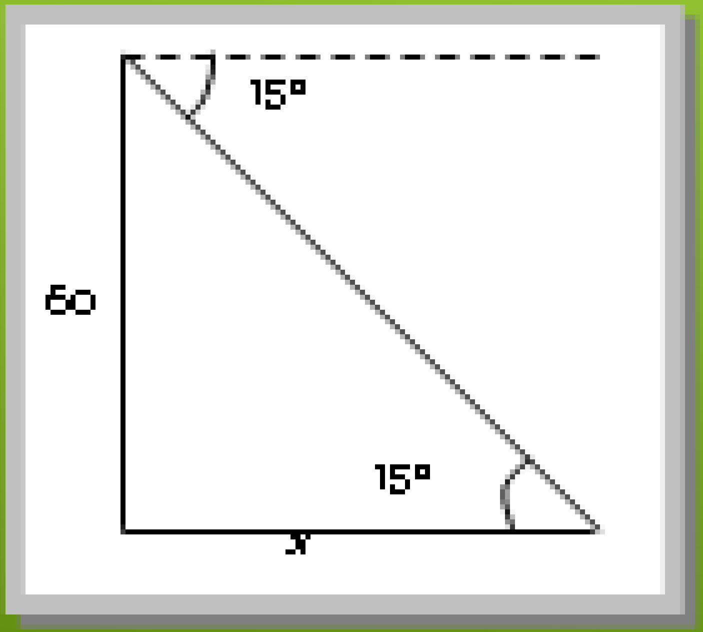
From the top of a light house 60 *metre* high with its base at the sea level the angle of depression of a boat is 15°. The distance of the boat from the foot of the light house is

[MP PET 2001, 1994; IIT 1983; UPSEAT 2000, 1988]

(a) $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) 60\ metre$ (b) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) 60\ metre$

(c) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\ metre$ (d) None of these

Required distance = $60 \cot 15^{o}$



$$= 60\left[\frac{\sqrt{3}+1}{\sqrt{3}-1}\right] \; metre$$

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A person observes the angle of deviation of a building as 30^o.

The person proceeds towards the building with a speed of $25(\sqrt{3}-1)m/hour$.

After 2 *hours*, he observes the angle of elevation as 45^o. The height of the building (in *metres*) is

[UPSEAT 2003]

(a) 100 (b) 50

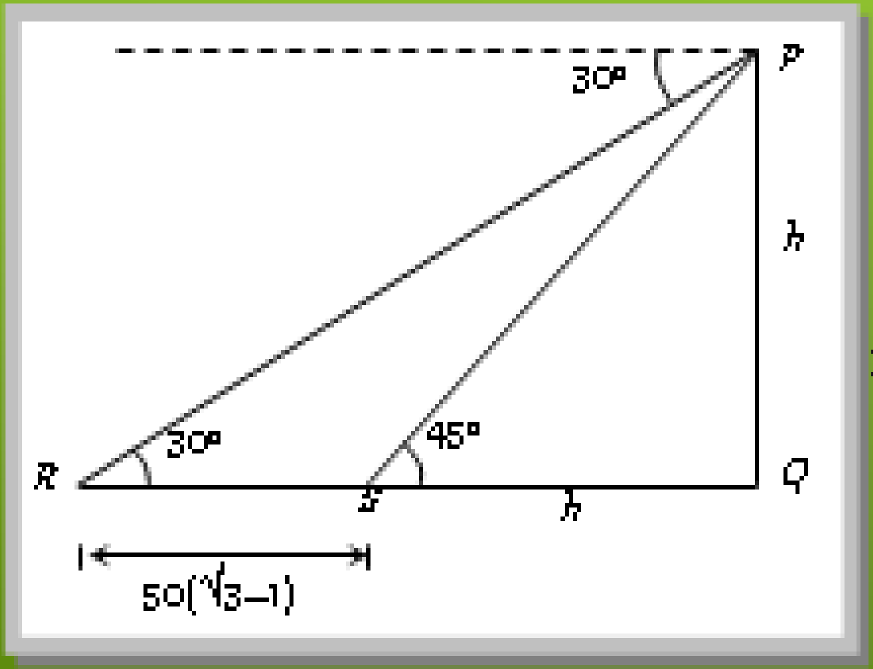
(c) $50(\sqrt{3}+1)$ (d) $50(\sqrt{3}-1)$

$$\text{In } \Delta PQR, \tan 30^o = \frac{PQ}{QR}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{50(\sqrt{3}-1)+h}$$

$$\Rightarrow \quad \sqrt{3}h = 50(\sqrt{3}-1)+h$$

$$\Rightarrow (\sqrt{3}-1)h = 50(\sqrt{3}-1) \Rightarrow h = 50 \text{ metre.}$$



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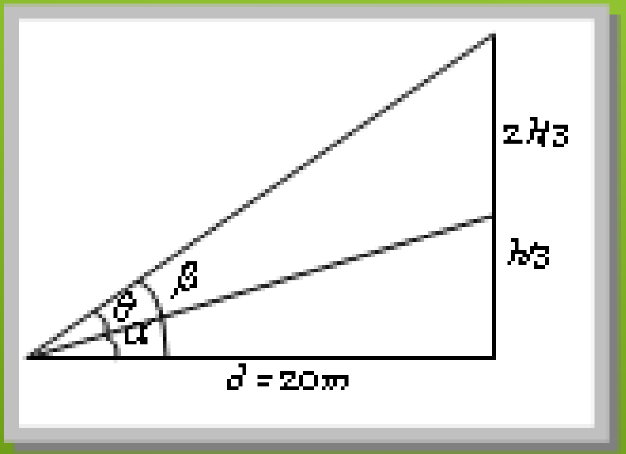
A vertical pole consists of two parts, the lower part being one third of the whole. At a point in the horizontal plane through the base of the pole and distance 20 *metres* from it, the upper part of the pole subtends an angle whose tangent is $\frac{1}{2}$. The possible heights of the pole are [IIT 1964]

(a) $20m$ and $20\sqrt{3}\ m$ (b) $20\ m$ and $60\ m$

(c) $16\ m$ and $48\ m$ (d) None of these

$$\frac{H}{3}\cot\alpha = d \text{ and } H\cot\beta = d \text{ or } \frac{H}{3d} = \tan\alpha \text{ and } \frac{H}{d} = \tan\beta$$



$$\Rightarrow \tan(\beta - \alpha) = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{\frac{H}{d} - \frac{H}{3d}}{1 + \frac{H^2}{3d^2}} \Rightarrow 1 + \frac{H^2}{3d^2} = \frac{4H}{d}$$

$$\Rightarrow H^2 - 4dH + 3d^2 = 0$$

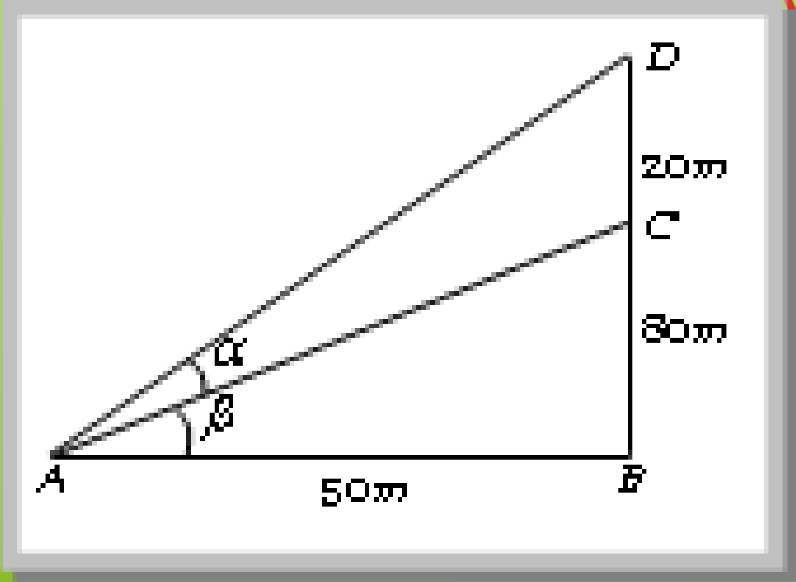
$$\Rightarrow H^2 - 80H + 3(400) = 0 \Rightarrow H = 20 \text{ or } 60\ m.$$

20 *metre* high flag pole is fixed on a 80 *metre* high pillar, 50 *metre* away from it, on a point on the base of pillar the flag pole makes an angle α, then the value of $\tan \alpha$ is **[MP PET 2003]**

(a) $\frac{2}{11}$ (b) $\frac{2}{21}$

(c) $\frac{21}{2}$ (d) $\frac{21}{4}$

Let $\angle BAC = \beta \quad \therefore \tan\beta = \frac{80}{50}$



$$\text{Now } \tan(\alpha+\beta) = \frac{100}{50}$$

$$\Rightarrow \quad \frac{\tan\alpha + \tan\beta}{1-\tan\alpha.\tan\beta} = 2 \quad \Rightarrow$$

$$\Rightarrow \quad \frac{\tan\alpha + \frac{8}{5}}{1-\frac{8}{5}\tan\alpha} = 2 \Rightarrow \tan\alpha = \frac{2}{21}$$

The angular depressions of the top and foot of a chimney as seen from the top of a second chimney, which is 150 m high and standing on the same level as the first are θ and ϕ respectively, then the distance between their tops when $\tan\theta = \frac{4}{3}$ and $\tan\phi = \frac{5}{2}$ is [IIT 1965]

(a) $\frac{150}{\sqrt{3}}$ *metres*

(b) $100\sqrt{3}$ *metres*

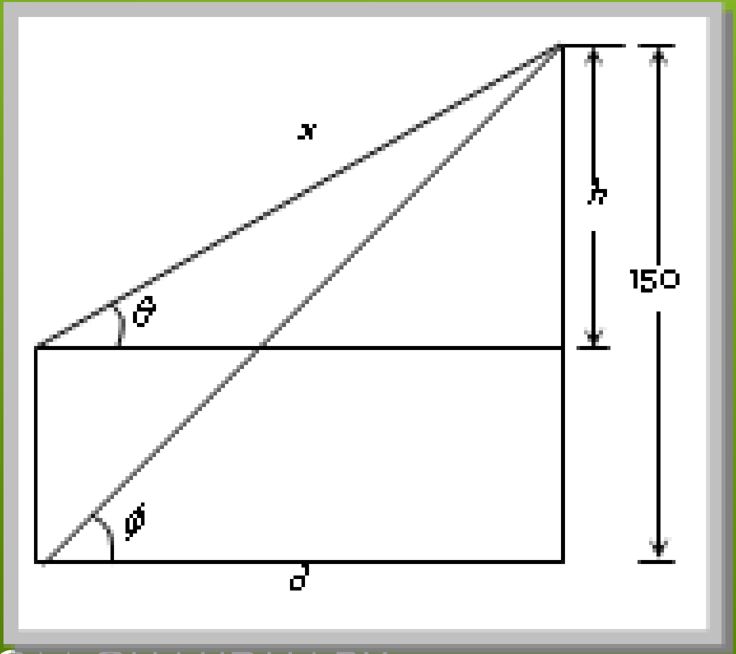
(c) 150 *metres*

(d) 100 *metres*

$$d = 150 \cot \phi = 60\, m$$

$$\text{Also, } h = 60 \tan \theta = 80\, m$$

$$\text{Hence, } x = \sqrt{80^2 + 60^2} = 100\ m.$$



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INVERSE TRIGONOMETRICAL FUNCTIONS

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