Problem Solving Sets & Relations

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Let $S = \{1, 2, 3, \ldots, 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even, is (2019 Main, 12 Jan I)

(a) $2^{50}(2^{50} - 1)$ (b) $2^{50} - 1$

(c) $2^{50} + 1$ (d) $2^{100} - 1$

Total non-empty subset

$S = \{1, 2, \text{---} 100\} \longrightarrow \boxed{2^{100} - 1}$

Non-empty subset of S

$\{2, \ldots \text{---} \}$

$(2^{100} - 1) - (2^{50} - 1)$

$= 2^{100} - 2^{50}$

$\{ \text{all the no. are odd} \}$ → Product of element will be odd

$\{50 \text{ odd}\} \longrightarrow 2^{50} - 1$

In a class of $\boxed{140 \text{ students}}$ numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then, the number of students who did not opt for any of the three courses is

(2019 Main, 10 Jan I)

(a) 42 (b) $\boxed{102}$

(c) 38 (d) 1

$n(M) = 70 \quad (\div 2)$

$n(P) = 46 \quad (\div 3)$

$n(C) = 28 \quad (\div 5)$

$\boxed{n(M \cup P \cup C) = \text{student who opted atleast one course}}$

$n(M \cup P \cup C) = 70 + 46 + 28 - n(P \cap M) - n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)$

$= 70 + 46 + 28 - 23 - 9 - 14 + 4$

$= 70 + 28 + \cancel{46 - 42}$

4

$= 102$

1, 140

$10 + 10 + 3 = 23$

9. If A and B are two sets containing four and two elements, respectively. Then, the number of subsets of the set $A \times B$ each having atleast three elements are (2015 Main)

(a) 219 (b) 256 (c) 275 (d) 510

$A = \{a, b, c, d\}$ — 4

$B = \{1, 2\}$ — 2

$A \times B = \{$ 8 ordered pair $\}$

ϕ,

$\{(a,1)\}, \{(b,2)\}$ — —

$2^8 - [1] - [8] - [{}^8C_2]$

$\boxed{256 - 1 - 8 - 28}$

$256 - 37 = 219$

9) * Let Z be the set of integers. If
$A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$
and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$, is **(2019 Main, 12 Jan II)**

(a) 2^{12} (b) 2^{18} (c) 2^{15} (d) 2^{10}

$A = \{x \in Z : 2^{(x+2)(x-3)(x-2)} = 1\}$

$\rightarrow A = \{2, -2, 3\}$ — 3

$\boxed{x = 2, 3, -2}$

$-2 < 2x < 10$

$\boxed{-1 < x < 5}$

$\rightarrow B = \{0, 1, 2, 3, 4\}$ — 5

$A \times B = \{\underline{15 \text{ order pair}}\}$

2^{15}

If both the mean and the standard deviation of 50 observations $x_1, x_2, \ldots, x_{50}$ are equal to 16, then the mean of $(x_1-4)^2, (x_2-4)^2, \ldots, (x_{50}-4)^2$ is

(2019 Main, 10 April II)

(a) 480 (b) 400 (c) 380 (d) 525

$a^2 = 256$

$$\boxed{\left(\frac{\Sigma x_i^2}{n} - \bar{x}^2\right) = 256}$$

$$\frac{\Sigma x_i^2}{n} = \underline{256 + 256}$$

$x_1, x_2, \ldots x_{50}$

$$\frac{(x_1-4)^2 + (x_2-4)^2 \ldots (x_{50}-4)^2}{50}$$

$$\frac{(x_1^2 + x_2^2 \ldots x_{50}^2) + 4^2(50) - 8[x_1 + x_2 \ldots x_{50}]}{50}$$

$$\Rightarrow \frac{x_1^2 + x_2^2 \ldots x_{50}^2}{50} + 4^2 - \underline{8 \times 16}$$

$$512 + 16 - 128 = 400$$

For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is **(2019 Main, 9 April I)**

(a) $\sim p \wedge \sim q$ (b) $\sim p \vee \sim q$

(c) $p \wedge q$ (d) $p \leftrightarrow q$

$(p \vee \sim p) \wedge (p \vee q)$

$(p \vee q)$

$\sim p \wedge \sim q$