Sets & Relations

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SETS

A set is a well-defined collection of distinct objects.

ROASTER FORM

In this form, we list all the member of the set within braces (curly brackets) and separate these by commas.

SET BUILDER FORM

In this form, we write a property rile which gives us al the elements belonging to the particular set.

THE EMPTY SET

A set which has no element is called the null set or empty set and is denoted by ϕ .
The number of elements of a set A is denoted as n (A) and n (ϕ) = 0 as it contains no element.

Singleton set : A set consisting of a single element is called a singleton set.

Order of a finite set : The number of elements in a finite set A is called the order of this set and denoted n(A).

It is also called cardinal number of the set.

e.g. A = {a, b, c, d} n(A) = 4

EQUAL SET

Given two sets A and B, if every element of A is also an element of B and if every element of B is also an element of A, then the sets A and B are said to be equal.

Clearly, the two sets have exactly the same elements.

SUBSETS

A set A is said to be a subset of the set B if each element of the set A is also the element of the set B. The symbol used is '$\subseteq$' i.e. $A \subseteq B$

A is not a subset of B then we write $A \nsubseteq B$

Proper subset : If A is a subset of B and $A \neq B$ then A is a proper subset of B, and we write $A \subset B$

Note-1 : Every set is a subset of itself i.e. $A \subseteq A$ for all A

Note-2 : Empty set ϕ is a subset of every set.

Note-3 : Clearly $N \subset W \subset Z \subset Q \subset R \subset C$

Note-4 : The total number of subsets of a finite set containing n elements is 2^n

POWER SET

The set of all subsets of a given set A is called the power set A and is denoted by P (A).

e.g. Let A = {1, 2} then P(A) = {{ }, {1}, {2}, {1, 2}}

Universal set : A set consisting of all possible elements which occur in the discussion is called a Universal set and is denoted by U

Note : All sets are contained in the universal set

e.g. If A = {1, 2, 3}, B = {2, 4, 5, 6}, C = {1, 3, 5, 7} then

U = {1, 2, 3, 4, 5, 6, 7} can be taken as the Universal set.

Union of sets

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once.

The symbol '∪' is used to denote the *union*.
Symbolically, we write A ∪ B *and usually read as* ***'****A union B****'***.

Intersection of sets

The intersection of sets A and B is the set of all elements which are common to both A and B. The symbol '∩' is used to denote the *intersection*.

The intersection of two sets A and B is the set of all those elements which belong to both A and B. Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

DIFFERENCE OF TWO SETS

The difference of two sets A and B is the set of all those elements of A which are not elements of B. It is denoted by

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

Complement of a Set

Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A.

Symbolically, we write A′ to denote the complement of A with respect to U

$A' = \{x : x \in U \text{ and } x \notin A\}$. Obviously $A' = U - A$

CARDINAL NUMBER OF SETS

1. If A, B, C are finite sets and U be the finite universal set then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Q. If X and Y are two sets such that $X \cup Y$ has 50 elements, X has 28 elements and Y has 32 elements, how many elements does $X \cap Y$ have ?

2. If A, B, C are finite sets and U be the finite universal set then

$$n(A - B) = n(A) - n(A \cap B)$$

3. If A, B, C are finite sets and U be the finite universal set then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

CARTESIAN PRODUCT:

The Cartesian product of two sets A, B is a non-void set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. This is denoted by $A \times B$

$\therefore \quad A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$

$A = \{1, 2\}, \quad B = \{a, b\}$

$A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$

RELATION:

Every non-zero subset of $A \times B$ defined a relation from set A to set B.

If R is a relation from $A \rightarrow B$

$R : \{(a, b) \mid (a, b) \in A \times B \text{ and } a\, R\, b\}$

IDENTITY RELATION:

A relation defined on a set A is said to be an Identity relation if each & every element of A is related to itself & only to itself.

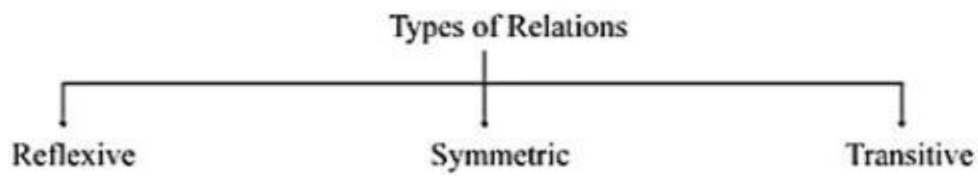
A relation defined on the set of natural nos. is

aRb $\Rightarrow$ $a = b$ where a & $b \in N$

$R = \{(1, 1), (2, 2), (3, 3), ...\}$

R is an Identity relation

CLASSIFICATION OF RELATIONS:



<u>Reflexive</u>: A relation defined on a set A is said to be an Identity relation if each & every element of A is related to itself.

i.e. if $(a, b) \in R$ then $(a, a) \in R$. However if there is a single ordered pair of $(a, b) \in R$ such $(a, a) \notin R$ then R is not reflexive.

A relation defined on (set of natural numbers)

$aRb \Rightarrow$ 'a' divides 'b' $a, b \in N$

R would always contain (a, a) because every natural number divides itself and hence it is a reflexive relation.

Symmetric: A relation defined on a set is said to be symmetric if aRb $\Rightarrow$ bRa.
If $(a, b) \in R$ then (b, a) must be necessarily there in the same relation.

EXAMPLES:

A relation defined on the set of lines.

(1) aRb $\Rightarrow$ $a \parallel b$

It is a symmetric relation because if line is || to 'b' then the line 'b' is || to 'a'.

where $(a, b) \in L$ {L is a set of || lines}

(2) $L_1 R L_2$ $\Rightarrow$ $L_1 \perp L_2$ It is a symmetric relation

$L_1, L_2 \in L$ {L is a set of lines}

(3) aRb $\Rightarrow$ 'a' is borther of 'b' is not a symmetric relation as 'a' may be sister of 'b'.

(4) aRb $\Rightarrow$ 'a' is a cousin of 'b'. This is a symmetric relation.

Transitive: A relation on set A is said to be transitive if aRb and bRc implies aRc

i.e. $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$

Here a, b, c need not be distinct.

EXAMPLES:

(1) A relation R defined on a set of natural numbers N with rule aRb $\Rightarrow$ $a < b$

R: $\{(1, 2), (1, 1)\}$.

In this relation a, b, c are not distinct but it is transitive. It is neither reflexive nor symmetric as (2, 1) is missing. Minium number of ordered pair that must be added to make it reflexive, symmetric and transitive is 2 i.e. (2, 1) and (2, 2).

(2) Only Transitive $R = \{(x, y) \mid x < y, \quad x \in N, y \in N\}$

Only Symmetric $R = \{(x, y) \mid x + y = 10, \quad x \in N, y \in N\}$

EQUIVALENCE RELATION:

If a relation is Reflexive, Symmetric and Transitive then it is said to be an equivalence relation.