MODE :

In a frequency distribution the mode is the value of that variate which have the maximum frequency

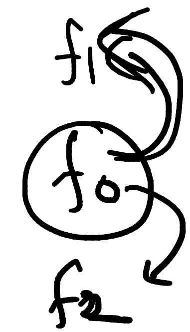
Method for determining mode :

(i) For ungrouped dist. : The value of that variate which is repeated maximum number of times

(ii) For ungrouped freq. dist. : The value of that variate which have maximum frequency.

(iii) For grouped freq. dist. : First we find the class which have maximum frequency, this is model calss

$$\text{Mode} = \ell + \left(\frac{f_0 - f_1}{2f_0 - f_1 - f_2}\right) \times h$$



where ℓ — lower limit of model class

f_0 — freq. of the model class

f_1 — freq. of the class preceeding model class

f_2 — freq. of the class succeeding model class

h — class interval of model class

x_i f_i

1, 2, 2, 3, 2, 4, 5, 5, 3, 1

Mode = 2

RELATION BETWEEN MEAN, MEDIAN AND MODE

$$\text{Mode} = 3\ \text{Median} - 2\ \text{Mean}$$

MEASURES OF DISPERSION :

The dispersion of a statistical distribution is the measure of deviation of its values about the their average (central) value.

(i) Mean deviation (M.D.) : The mean deviation of a distribution is, the mean of absolute value of deviations of variate from their statistical average (Mean, Median, Mode).

If A is any statistical average of a distribution then mean deviation about A is defined as

$$\text{Mean deviation} = \frac{\sum_{i=1}^{n} |x_i - A|}{n} \quad \text{(for ungrouped dist.)}$$

$$\text{Mean deviation} = \frac{\sum_{i=1}^{n} f_i |x_i - A|}{N} \quad \text{(for freq. dist.)}$$

$N = \sum f_i$

Mean deviation about Mean $= \frac{\sum |x_i - \bar{x}|}{n}$

x_i: 5, 6, 7, 8, 9

$\bar{x} = 7$

$\frac{2+1+0+1+2}{5} = \frac{6}{5} = 1.2$

9) If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is

(2019 Main, 12 Jan I)

(a) 50 (b) 30 (c) 51 (d) 31 ✓

$(N = 50)$

$A \to 30$

$$\sum_{1}^{50} (x_i - 30) = 50$$

$x_1, x_2 \, \ldots \, x_{50}$

$\bar{x} = ?$

$$(x_1 - 30) + (x_2 - 30) \ldots (x_{50} - 30) = 50$$

$$(x_1 + x_2 + \ldots x_{50}) = 50 + 30(50) = 50(31)$$

$$\bar{x} = \frac{50 \cdot 31}{50} = 31$$

(iii) **Variance and standard deviation** : The variance of a distribution is, the mean of squares of deviation of variate from their mean. It is denoted by σ^2 or var(x).

The positive square root of the variance are called the standard deviation. It is denoted by σ or S.D.

Hence standard deviation = $+\sqrt{\text{variance}}$

variance $= \frac{\sum(x_i - \bar{x})^2}{n}$

Formulae for variance :

(i) for ungrouped dist. :

$$\sigma_x^2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

variance = a^2

$a_x^2 = \frac{\sum(x_i^2 + \bar{x}^2 - 2x_i\bar{x})}{n}$

$= \frac{\sum x_i^2}{n} + \bar{x}^2 - 2\bar{x}\frac{\sum x_i}{n}$

$= \frac{\sum x_i^2}{n} + \bar{x}^2 - 2\bar{x}^2$

$a^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

$$\sigma_x^2 = \frac{\Sigma x_i^2}{n} - \bar{x}^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2, \quad \text{where } d_i = x_i - a$$

1, 2, 3, 4 – – – 10

$a^2 = \frac{1^2 + 2^2 + 3^2 + \dots + 10^2}{10} - (\bar{x})^2$

Q. Find the variance of first n natural numbers

$S.D = \sqrt{Var}$

1, 2, 3, — — n

$$Var = \frac{\sum x_i^2}{n} - (\bar{x})^2 \qquad \left(\frac{n+1}{2}\right)\left(\frac{n-1}{6}\right) = \frac{n^2-1}{12}$$

$$\bar{x} = \frac{1+2+3 \,--\, n}{n} = \frac{n(n+1)}{2n} = \left(\frac{n+1}{2}\right)$$

$$= \frac{1^2+2^2+3^2 \,--\, n^2}{n} - \left(\frac{n+1}{2}\right)^2 \qquad \left(\frac{n-1}{2}\right)$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2$$

$$= \left(\frac{n+1}{2}\right)\left[\frac{2n+1}{3} - \left(\frac{n+1}{2}\right)\right]$$

9) If the data $x_1, x_2, \ldots, x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2000, then the standard deviation of this data is (2019 Main, 12 April I)

(a) $2\sqrt{2}$ (b) 2 (c) 4 (d) $\sqrt{2}$

$x_1+x_2+x_3+x_4 = 44$

$x_5+x_6 - - x_{10} = 96$

$\bar{x} = \frac{140}{10} = 14$

$x_1^2 + x_2^2 - - - x_{10}^2 = 2000$

$\text{Variance} = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$= \frac{2000}{10} - (14)^2$

$= 200 - 196$

$\sigma^2 = 4 \Rightarrow \sigma = 2$

(ii) For freq. dist. :

$$\sigma_x^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{N}$$

$$\sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2 = \frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2$$

$$\sigma_u^2 = h^2\left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right] \text{ where } u_i = \frac{d_i}{h}$$

(iii) Coefficient of S.D. $= \frac{\sigma}{\bar{x}}$

Coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

[redacted]

Variance $= \frac{\sum (x_i - \bar{x})^2}{n}$

$dx_1 - d\bar{x}$

d^2

Properties

① $x_1, x_2 \ldots x_n =$ a^2 Mean $= \bar{x}$

$(x_1 + d, \; x_2 + d \ldots x_n + d)$

Mean $= \bar{x} + d$

Variance $= a^2$

② $(dx_1, \; dx_2, \ldots dx_n)$

Mean $= d\bar{x}$

Variance $= d^2 \sigma^2$

The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is

(2019 Main, 10 Jan I)

(a) 4 : 9 (b) 6 : 7 (c) 10 : 3 (d) 5 : 8

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 5$$

$$\Sigma x_i = 25$$

$$x_4 + x_5 = 13$$

$36 + 49 = 85$

$81 + 16 = 97$

$$\sigma^2 = 9.2$$

$(1, 3, 8, x_4, x_5)$

$$\frac{\Sigma x_i^2}{n} - 25 = 9.2$$

$$\frac{\Sigma x_i^2}{n} = 34.2$$

$$\Sigma x_i^2 = 171$$

$$1 + 9 + 64 + x_4^2 + x_5^2 = 171$$

$$x_4^2 + x_5^2 = 97$$