Mathematical Reasoning

By
Ankush Garg(B. Tech, IIT Jodhpur)

STATEMENT :

A sentence which is either true or false but cannot be both are called a statement. A sentence which is an exclamatory or a wish or an imperative or an interrogative can not be a statement.

If a statement is true then its truth value is T and if it is false then its truth value is F

For ex.

(i) "New Delhi is the capital of India", a true statement

(ii) "3 + 2 = 6", a false statement

(iii) "Where are you going ?" not a statement beasuse it connot be defined as true or false

Note : A statement cannot be both true and false at a time

SIMPLE STATEMENT :

Any statement whose truth value does not depend on other statement are called simple statement

For ex. (i) "$\sqrt{2}$ is an irrational number" (T) (ii) ("The set of real number is an infinite set") T

COMPOUND STATEMENT :

A statement which is a combination of two or more simple statements are called compound statement

Here the simple statements which form a compound statement are known as its sub statements

For ex.

(i) "If (x is divisible by 2) then (x is even number)" → compound statement

p

(ii) ("ΔABC is equilateral if and only if its three sides are equal")

LOGICAL CONNECTIVES:

The words or phrases which combined simple statements to form a compound statement are called logical connectives.

In the following table some possible connectives, their symbols and the nature of the compound statement formed by them

p , q

S.N.	Connectives	symbol	use	operation
1.	and	$\wedge$	$p \wedge q$	conjunction
2.	or	$\vee$	$p \vee q$	disjunction
3.	not	$\sim$ or $\neg$	$\sim p$ or p'	negation
4.	If .... then	$\Rightarrow$ or $\rightarrow$	$p \Rightarrow q$ or $p \rightarrow q$	Implication or conditional
5.	If and only if (iff)	$\Leftrightarrow$ or $\leftrightarrow$	$p \Leftrightarrow q$ or $p \leftrightarrow q$	Equivalence or Bi-conditional

Double Implication

Double Implication

Explanation :

(i) $p \wedge q \equiv$ statement p and q

($p \wedge q$ is true only when p and q both are true otherwise it is false)

(ii) $p \vee q \equiv$ statement p or q

($p \vee q$ is true if at least one from p and q is true i.e. $p \vee q$ is false only when p and q both are false)

(iii) $\sim p \equiv$ not statement p

($\sim p$ is true when p is false and $\sim p$ is false when p is true)

(iv) $p \Rightarrow q \equiv$ statement p then statement q

($p \Rightarrow q$ is false only when p is true and q is false otherwise it is true for all other cases)

(v) $p \Leftrightarrow q \equiv$ (statement p) if and only if (statement q)

($p \Leftrightarrow q$ is true only when p and q both are true or false otherwise it is false)

Truth Tables

4-Marks

Conjunction (AND)

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR)

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Negation (NOT)

p	(~ p)
T	F
F	T

Conditional (If p then q) → F

T F

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

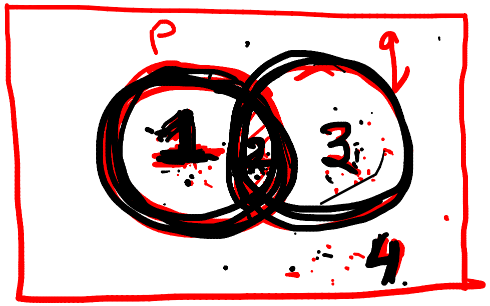
Biconditional ($p \leftrightarrow q$)

p	q	p → q	q → p	(p → q) ∧ (q → p) or p ↔ q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$$\boxed{p \leftrightarrow q \rightarrow (p \rightarrow q) \wedge (q \rightarrow p)}$$

Venn Diagram

(P, q)



Regions → 1, 2, 3, 4

Region

1. AND ($P \wedge q$) → 2
2. OR ($P \vee q$) → 1, 2, 3
3. NOT ($\sim P$) → 3, 4

   ($\sim q$) → 1, 4
4. Implication $\boxed{(P \to q)}$ → 2, 3, 4

   T F
5. Double Implication ($P \leftrightarrow q$) → 2, 4

If $p \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively (2019 Main, 9 April II)

(a) T, T, F (b) T, F, F

(c) F, F, F (d) F, T, T

$$p \Rightarrow (q \vee r) \longrightarrow F$$

$p \rightarrow T$ $\quad$ $(q \vee r) \rightarrow F$

$$q \vee r \longrightarrow F$$

$q \rightarrow F$ $\quad$ $r \rightarrow F$

LOGICAL EQUIVALENCE

Two statements S_1 and S_2 are equivalent if they have identical truth table i.e. the entries in the last column of their truth table are same. If statements S_1 and S_2 are equivalent then we write $\boxed{S_1 \equiv S_2}$

For ex. The truth table for $(p \to q)$ and $(\sim p \vee q)$ given as below

p	q	$(\sim p)$	$p \to q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$(\sim p \vee q)$

$\boxed{S_1 \equiv S_2}$

We observe that last two columns of the above truth table are identical hence compound statements $(p \to q)$ and $(\sim p \vee q)$ are equivalent

i.e. $\boxed{p \to q \equiv \sim p \vee q}$

$$\boxed{(p \to q) \equiv \sim p \vee q}$$