Bohr's Orbits (For Hydrogen and H_2-Like Atoms)

(1) **Radius of orbit**

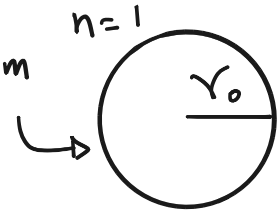
For an electron around a stationary nucleus the electrostatics force of attraction provides the necessary centripetal force

i.e. $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r}$ .... (i)

also $mvr = \frac{nh}{2\pi}$ ....(ii)

H-atom

n=1



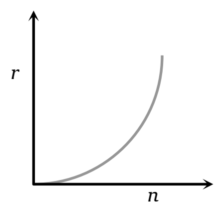
$r = r_0 \frac{n^2}{Z}$ ← no. of orbit

Z → Atomic No.

$r_0 = 0.53$ Å

From equation (i) and (ii) radius of n^{th} orbit

$$r_n = \frac{n^2h^2}{4\pi^2 kZme^2} = \frac{n^2h^2\varepsilon_0}{\pi mZe^2} = 0.53\frac{n^2}{Z}\text{Å} \quad \left[\text{where } k = \frac{1}{4\pi\varepsilon_0}\right]$$



$\Rightarrow r_n \propto \frac{n^2}{Z}$

Note : ❑ The radius of the innermost orbit (n = 1) hydrogen atom (z = 1) is called Bohr's radius a_0 *i.e.* $a_0 = 0.53\,\text{Å}$.

$mvr = \frac{nh}{2\pi}$

$mv\frac{r_0 n^2}{Z} = \frac{nh}{2\pi}$

$V = \frac{Zh}{2\pi m r_0 n}$

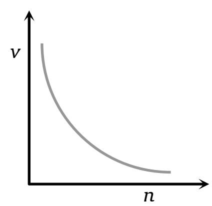
$V = \frac{V_0 Z}{n}$, $V_0 = \frac{c}{137}$

$C = 3 \times 10^8$ m/s

(2) **Speed of electron**

From the above relations, speed of electron in n^{th} orbit can be calculated as

$$v_n = \frac{2\pi kZe^2}{nh} = \frac{Ze^2}{2\varepsilon_0 nh} = \left(\frac{c}{137}\right).\frac{Z}{n} = 2.2\times10^6\frac{Z}{n}\,m/sec$$



where (c = speed of light $3 \times 10^8\ m/s$)

Note : ❑ The ratio of speed of an electron in ground state in Bohr's first orbit of hydrogen atom to velocity of light in air is equal to $\frac{e^2}{2\varepsilon_0 ch} = \frac{1}{137}$ (where c = speed of light in air)

$$\omega = \frac{v}{r} = \frac{\frac{v_0 Z}{n}}{\frac{r_0 n^2}{Z}} = \frac{v_0}{r_0}\frac{Z^2}{n^3}$$

$\omega \propto Z^2/n^3$

(3) **Some other quantities**

For the revolution of electron in n^{th} orbit, some other quantities are given in the following table

Quantity	Formula	Dependency on n and Z
(1) Angular speed $\omega = \frac{v}{r}$	$\omega_n = \frac{v_n}{r_n} = \frac{\pi m z^2 e^4}{2\varepsilon_0^2 n^3 h^3}$	$\omega_n \propto \frac{Z^2}{n^3}$
(2) Frequency $\omega = 2\pi f$,	$\nu_n = \frac{\omega_n}{2\pi} = \frac{m z^2 e^4}{4\varepsilon_0^2 n^3 h^3}$	$\nu_n \propto \frac{Z^2}{n^3}$ $f = \frac{\omega}{2\pi}$, $f \propto \frac{Z^2}{n^3}$
(3) Time period	$T_n = \frac{1}{\nu_n} = \frac{4\varepsilon_0^2 n^3 h^3}{m z^2 e^4}$	$T_n \propto \frac{n^3}{Z^2}$ $T = \frac{1}{f}$, $T \propto \frac{n^3}{Z^2}$
(4) Angular momentum $L = nh/2\pi$ $L \propto n$	$L_n = m v_n r_n = n\left(\frac{h}{2\pi}\right)$	$L_n \propto n$
(5) Corresponding current	$i_n = e\nu_n = \frac{m z^2 e^5}{4\varepsilon_0^2 n^3 h^3}$	$i_n \propto \frac{Z^2}{n^3}$
(6) Magnetic moment M.M. = i Area	$M_n = i_n A = i_n(\pi r_n^2)$ (where $\mu_0 = \frac{eh}{4\pi m}$ = Bohr magneton)	$M_n \propto n$
(7) Magnetic field	$B = \frac{\mu_0 i_n}{2r_n} = \frac{\pi m^2 z^3 e^7 \mu_0}{8\varepsilon_0^3 n^5 h^5}$	$B \propto \frac{Z^3}{n^5}$ NEET

Angular Momentum is independent From atomic No.

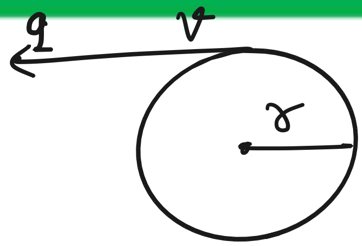
$i = \frac{q}{T} = \frac{e}{T} = ef$

Area $= \pi r^2 = \pi(r_0 n^2)^2$

$i \propto Z^2$

M.M. = iA

Area $\propto \frac{n^4}{Z^2}$ $\frac{n^4}{(Z)}$ n^3 M.M. $\propto \frac{Z^2}{n^3} \times \frac{n^4}{Z^2} \propto n$



$i = \frac{q}{T} = \frac{q}{\frac{2\pi r}{v}} = \frac{qv}{2\pi r}$

$\frac{2\pi r}{T} = v$, $T = \frac{2\pi r}{v}$

$B = \frac{\mu_0 i}{2r}$

$B \propto \frac{i}{r}$ $B \propto \frac{Z^2/n^3}{n^2/Z} \propto \frac{Z^3}{n^5}$

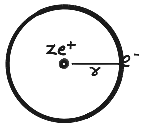
$i \propto \frac{Z^2}{n^3}$ $r \propto \frac{n^2}{Z}$

(4) **Energy**

(i) **Potential energy :** An electron possesses some potential energy because it is found in the field of nucleus potential energy of electron in n^{th} orbit of radius r_n is given by

$U \propto \frac{Z^2}{n^2}$

$$U = k.\frac{(Ze)(-e)}{r_n} = -\frac{kZe^2}{r_n}$$



$U = \frac{k q_1 q_2}{r}$ $r = \frac{r_0 n^2}{Z}$

$U = \frac{k(Ze)(-e)}{r} = -\frac{kZe^2}{r_0 n^2/Z} = -k\frac{Z^2}{n^2}\frac{e^2}{r_0}$

(ii) **Kinetic energy :** Electron posses kinetic energy because of it's motion. Closer orbits have greater kinetic energy than outer ones.

As we know

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v_0 Z}{n}\right)^2$

$$\frac{mv^2}{r_n} = \frac{k.(Ze)(e)}{r_n^2} \Rightarrow$$

$KE \propto \frac{Z^2}{n^2}$

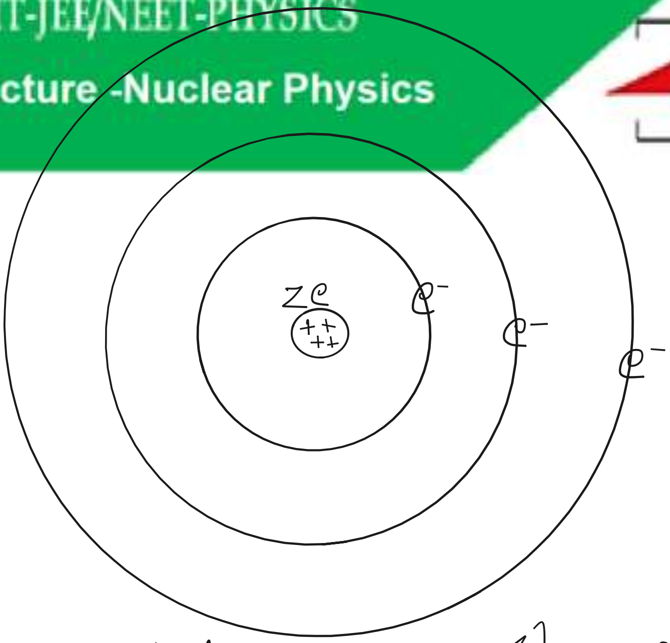
Kinetic energy $K = \frac{kZe^2}{2r_n} = \frac{|U|}{2}$

(iii) **Total energy :** Total energy (E) is the sum of potential energy and kinetic energy *i.e.* $E = K + U$

$\Rightarrow \quad E = -\frac{kZe^2}{2r_n}$ also $r_n = \frac{n^2h^2\varepsilon_0}{\pi mze^2}$.   Hence $E = -\left(\frac{me^4}{8\varepsilon_0^2h^2}\right).\frac{z^2}{n^2} = -\left(\frac{me^4}{8\varepsilon_0^2ch^3}\right)ch\frac{z^2}{n^2}$

$= -R\,ch\frac{Z^2}{n^2} = -13.6\frac{Z^2}{n^2}eV$

total energy $= U + KE = -\left(\frac{me^4}{8\varepsilon_0^2h^2}\right)\frac{Z^2}{n^2} = -13.6\frac{Z^2}{n^2}eV$



$$T.E. = -\frac{13.6\,z^2}{n^2}\,eV$$

Where –ve total energy shows that if we provide this much energy we can eject electron

where $R = \frac{me^4}{8\varepsilon_0^2 ch^3}$ = Rydberg's constant = 1.09×10^7 *per metre*

Note : ❑ Each Bohr orbit has a definite energy

❑ For hydrogen atom ($Z = 1$) $\Rightarrow E_n = -\frac{13.6}{n^2} eV$

❑ The state with $n = 1$ has the lowest (most negative) energy. For hydrogen atom it is $E_1 = -13.6\ eV$.

❑ *Rch* = Rydberg's energy $\simeq 2.17 \times 10^{-18}\ J \simeq 31.6\ eV$.

❑ $E = -K = \frac{U}{2}$.

✓ Total energy = – Kinetic energy

✓ Potential energy = -2·KE

$$T.E = -KE = \frac{U}{2}$$

$$U = P.E. = -27.2\frac{z^2}{n^2}$$

$$TE = -13.6\frac{z^2}{n^2}, \quad KE = 13.6\frac{z^2}{n^2}$$

(5) **Energy level diagram**

The diagrammatic description of the energy of the electron in different orbits around the nucleus is called energy level diagram.

Energy level diagram of hydrogen/hydrogen like atom

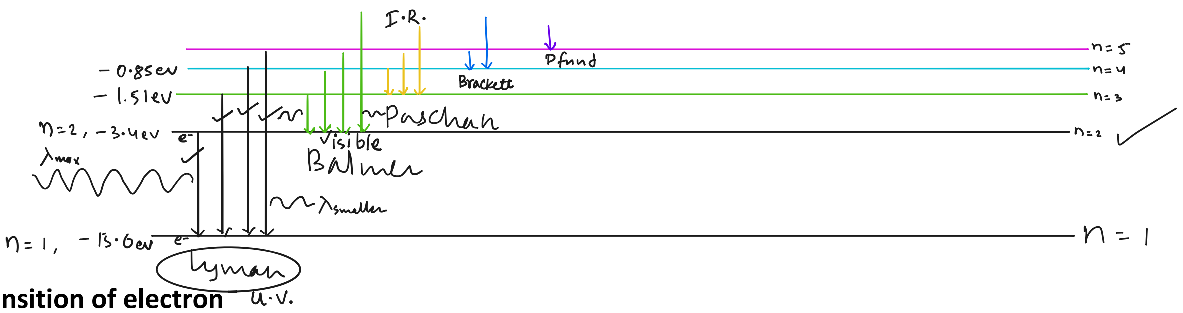
	Principle quantum number	Orbit	Excited state	Energy for H_2 – atom	Energy for H_2 – like atom	Ionisation energy from this level (for H_2 – atom)
- - - - - -	$n = \infty$	Infinite	Infinite	$E_\infty = 0\ eV$	$0\ eV$	$0\ eV$
	$n = 4$	Fourth	Third	$E_4 = -0.85\ eV$	$-0.85\ Z^2$	$+0.85\ eV$
	$n = 3$	Third	Second	$E_3 = -1.51\ eV$	$-1.51\ Z^2$	$+1.51\ eV$
	$n = 2$	Second	First	$E_2 = -3.4\ eV$	$-3.4\ Z^2$	$+3.4\ eV$
	$n = 1$	First	Ground	$E_1 = -13.6\ eV$	$-13.6\ Z^2$	$+13.6\ eV$

Note : ❑ In hydrogen atom excitation energy to excite electron from ground state to first excited state will be $-3.4-(-13.6)=10.2\ eV$.

and from ground state to second excited state it is $[-1.51-(-13.6)=12.09\ eV]$.

❑ In an H_2 atom when e^- makes a transition from an excited state to the ground state it's kinetic energy increases while potential and total energy decreases.

For H atom

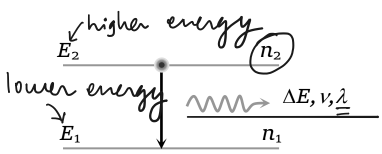


(6) **Transition of electron**

When an electron makes transition from higher energy level having energy $E_2(n_2)$ to a lower energy level having energy E_1 (n_1) then a photon of frequency ν is emitted

(i) **Energy of emitted radiation**

$$\Delta E = E_2 - E_1 = \frac{-Rc\,h\,Z^2}{n_2^2} - \left(-\frac{Rch\,Z^2}{n_1^2}\right) = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$



Emission

(ii) **Frequency of emitted radiation**

$$\Delta E = h\nu \Rightarrow \nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h} = Rc\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$\Delta E = \frac{hc}{\lambda}$

$E_2 - E_1 = \frac{hc}{\lambda}$

$-\frac{13.6\,Z^2}{n_2^2} - \left(-\frac{13.6\,Z^2}{n_1^2}\right) = \frac{hc}{\lambda}$

(iii) **Wave number/wavelength**

$\frac{1}{\lambda} = \frac{+13.6\,Z^2}{hc}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

most imp.

Fastest Formula → $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Wave number is the number of waves in unit length $\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c}$

$$\Rightarrow \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{13.6Z^2}{hc}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

When electron jump from n_2 to n_1

(iv) **Number of spectral lines** : If an electron jumps from higher energy orbit to lower energy orbit it emits raidations with various spectral lines.

If electron falls from orbit n_2 to n_1 then the number of spectral lines emitted is given by

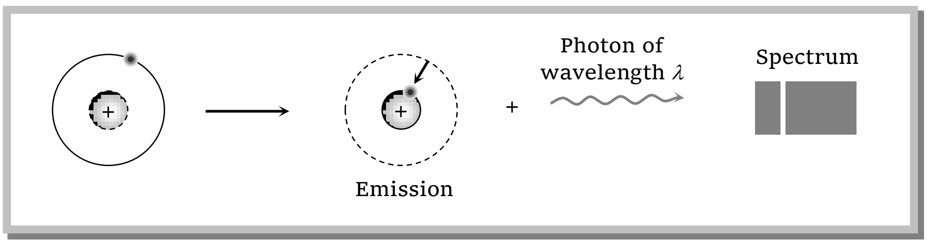
$$N_E = \frac{(n_2 - n_1 + 1)(n_2 - n_1)}{2}$$ IIT /NEET

If electron falls from n^{th} orbit to ground state (*i.e.* $n_2 = n$ and $n_1 = 1$) then number of spectral lines emitted $N_E = \frac{n(n-1)}{2}$

electron jumps From $n_2 = 5$ to $n_1 = 2$.

Hydrogen Spectrum and Spectral Series

$N_E = \frac{(5-2+1)(5-2)}{2} = 6$



Spectral series

The spectral lines arising from the transition of electron forms a spectra series.

(i) Mainly there are five series and each series is named after it's discover as Lymen series, Balmer series, Paschen series, Bracket series and Pfund series.

(ii) According to the Bohr's theory the wavelength of the radiations emitted from hydrogen atom is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

where n_2 = outer orbit (electron jumps from this orbit), n_1 = inner orbit (electron falls in this orbit)

For H atom

maximum wave length of lyman series

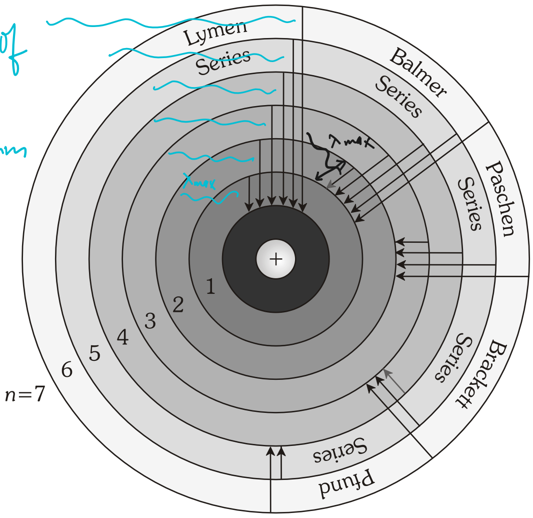
ΔE should be minimum

$n = 2$ to $n = 1$

$\frac{1}{\lambda} = R\left(\frac{1}{1} - \frac{1}{4}\right)$

$\frac{1}{\lambda_{max}} = R\left(\frac{3}{4}\right)$

$\lambda_{max} = \frac{4}{3R}$



Minimum wave length of lyman series gap should be maxi

$n = \infty$, $n = 1$

$\frac{1}{\lambda_{min}} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\frac{1}{\lambda_{min}} = R\left(\frac{1}{1} - \frac{1}{\infty}\right)$

$\lambda_{min} = \frac{1}{R}$

(iii) First line of the series is called first member, for this line wavelength is maximum (λ_{max})

(iv) Last line of the series ($n_2 = \infty$) is called series limit, for this line wavelength is minimum (λ_{min})

maximum wavelength of lyman series

$\frac{1}{\lambda_{max}} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ For H atom

$\frac{1}{\lambda_{max}} = R\left(\frac{1}{4} - \frac{1}{9}\right) = R\left(\frac{5}{36}\right)$

$\lambda_{max} = \frac{36}{5R}$

λ_{min} for Balmer series

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$

$$\lambda_{min} = \frac{4}{R}$$

Nucleus

(1) **Different types of nuclei**

The nuclei have been classified on the basis of the number of protons (atomic number) or the total number of nucleons (mass number) as follows

(i) **Isotopes :** The atoms of element having same atomic number but different mass number are called isotopes.

$$_1H^1,\ _1H^2,\ _1H^3\quad _8O^{16},\ _8O^{17},\ _8O^{18}\quad _2He^3,\ _2He^4\quad _{17}Cl^{35},\ _{17}Cl^{37}\quad _{92}U^{235},\ _{92}U^{238}$$

(ii) **Isobars :** The nuclei which have the same mass number (A) but different atomic number (Z) are called isobars.

$$_1H^3 \text{ and } _2He^3,\quad _6C^{14} \text{ and } _7N^{14},\ _8O^{17} \text{ and } _9F^{17}$$

(iii) **Isotones :** The nuclei having equal number of neutrons are called isotones.

$$_4Be^9 \text{ and } _5B^{10},\ _6C^{13} \text{ and } _7N^{14},\ _8O^{18} \text{ and } _9F^{19},\ _3Li^7 \text{ and } _4Be^8,\ _1H^3 \text{ and } _2He^4$$

(2) **Size of nucleus**

(i) Nuclear radius : Experimental results indicates that the nuclear radius is proportional to $A^{1/3}$, where A is the mass number of nucleus *i.e.* $R \propto A^{1/3}$ $\Rightarrow \boxed{R = R_0 A^{1/3}}$, where $R_0 = 1.2 \times 10^{-15}\ m = 1.2\ fm$.

Radius of Nucleus   Atomic mass   imp

$\frac{R_{Fe}}{R_{Al}} = \frac{\cancel{R_0}}{\cancel{R_0}}\left(\frac{A_{Fe}}{A_{Al}}\right)^{1/3}$

Note : ❑ Heavier nuclei are bigger in size than lighter nuclei.

(ii) Nuclear volume : The volume of nucleus is given by $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A \Rightarrow V \propto A$

$Vol. = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3$

$Vol. = \frac{4}{3}\pi R_0^3 A$

Atomic mass = no. of protons + no. of Neutrons

$A = Z + N$

$\rho = \frac{Mass}{Vol.} = \frac{\cancel{A}}{\frac{4}{3}\pi R_0^3 \cancel{A}} = 2.38 \times 10^{17}\ kg/m^3$

(iii) Nuclear density : Mass per unit volume of a nucleus is called nuclear density.

$$\text{Nuclear density } (\rho) = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi (R_0 A^{1/3})^3}$$

where m = Average of mass of a nucleon (= mass of proton + mass of neutron = $1.66 \times 10^{-27}\ kg$)

and mA = Mass of nucleus

$$\Rightarrow \rho = \frac{3m}{4\pi R_0^3} = 2.38 \times 10^{17}\ kg/m^3$$

Note : ❑ ρ is independent of A, it means ρ is same of all atoms.

❑ Density of a nucleus is maximum at it's centre and decreases as we move outwards from the nucleus.

(3) **Nuclear force**

Forces that keep the nucleons bound in the nucleus are called nuclear forces.

(i) Nuclear forces are short range forces. These do not exist at large distances greater than $10^{-15}\ m$.

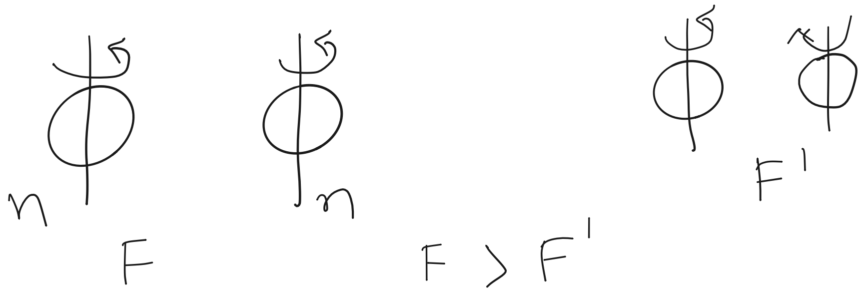
(ii) Nuclear forces are the strongest forces in nature.

(iii) These are attractive force and causes stability of the nucleus.   P – n

(iv) These forces are charge independent.   n – n   P – P

(v) Nuclear forces are non-central force.

(vi) nuclear force depends on spin



$1 \text{ Amu} = \frac{1}{12} \times \text{mass of } 1.\ {}_6C^{12} \text{ atom} = 1.66 \times 10^{-27} \text{ kg}$

(4) **Atomic mass unit (*amu*)**

The unit in which atomic and nuclear masses are measured is called atomic mass unit (*amu*)

1 *amu* (or 1*u*) = $\frac{1}{12}th$ of mass of ${}_6C^{12}$ atom = 1.66×10^{-27} *kg*

Masses of electron, proton and neutrons

Mass of electron (m_e) = 9.1×10^{-31} *kg* = 0.0005486 *amu*,

Mass of proton (m_p) = 1.6726×10^{-27} *kg* = 1.007276 *amu*

Mass of neutron (m_n) = 1.6750×10^{-27} *kg* = 1.00865 *amu*,

Mass of hydrogen atom ($m_e + m_p$) = 1.6729×10^{-27} *kg* = 1.0078 *amu*

Mass-energy equivalence

According to Einstein, mass and energy are inter convertible. The Einstein's mass energy relationship is given by $E = mc^2$

If m = 1 *amu*, $c = 3 \times 10^8$ *m/sec* then E = 931 *MeV* *i.e.*

1 *amu* is equivalent to 931 *MeV* or **1 *amu* (or 1 *u*) = 931 *MeV***

$E = mc^2$
$\downarrow$
atom bomb, H bomb.

(6) **Nuclear stability**

The stability of nucleus is determined by many factors. Few such factors are given below :

(i) Neutron-proton ratio $\left(\frac{N}{Z}\text{Ratio}\right)$

For lighter nuclei, the greatest stability is achieved when the number of protons and neutrons are approximately equal ($N \approx Z$) *i.e.* $\frac{N}{Z} = 1$

He 2 protons $\quad \frac{N}{Z} = 1$

2 Neutrons

Heavy nuclei are stable only when they have more neutrons than protons

For Heavy Nuclei Neutrons > protons then only Nucleus is stable.

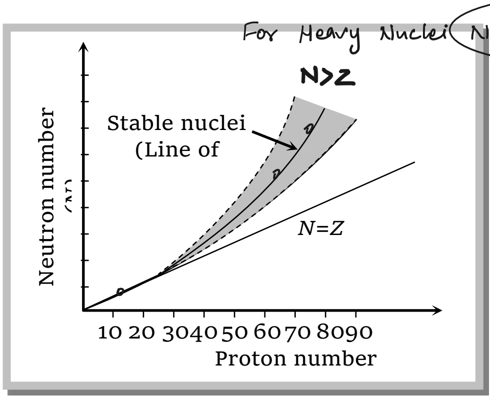
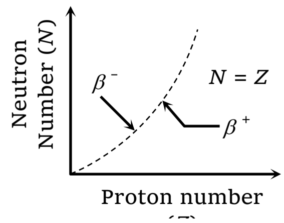


Figure shows a plot of *N* verses *Z* for the stable nuclei. For mass number upto about *A* = 40. For larger value of *Z* the nuclear force is unable to hold the nucleus together against the electrical repulsion of the protons unless the number of neutrons exceeds the number of protons. At *Bi* (*Z* = 83, *A* = 209), the neutron excess in *N* – *Z* = 43. There are no stable nuclides with *Z* > 83.

Note : ❑ The nuclide $_{83}Bi^{209}$ is the heaviest stable nucleus.

❑ A nuclide above the line of stability *i.e.* having excess neutrons, decay through β^- emission (neutron changes into proton). Thus increasing atomic number *Z* and decreasing neutron number *N*. In β^- emission, $\frac{N}{Z}$ ratio decreases.

A nuclide below the line of stability have excess number of protons. It decays by β^+ emission, results in decreasing *Z* and increasing *N*. In β^+ emission, the $\frac{N}{Z}$ ratio increases.



(iii) Binding energy per nucleon : The stability of a nucleus is determined by value of it's binding energy per nucleon. In general higher the value of binding energy per nucleon, more stable the nucleus is

Mass Defect and Binding Energy

(1) **Mass defect (Δm)**

It is found that the mass of a nucleus is always less than the sum of masses of it's constituent nucleons in free state. This difference in masses is called mass defect. Hence mass defect

Δm = Sum of masses of nucleons – Mass of nucleus

$$= \{Zm_p + (A-Z)m_n\} - M = \{Zm_p + Zm_e + (A-Z)m_z\} - M'$$

where m_p = Mass of proton, m_n = Mass of each neutron, m_e = Mass of each electron

M = Mass of nucleus, Z = Atomic number, A = Mass number, M' = Mass of atom as a whole.

Note : ❑ The mass of a typical nucleus is about 1% less than the sum of masses of nucleons.

$$\Delta m = \{ \underline{m_p \times Z + m_N \times N} - M \}$$

atual mass

$A = Z + N$

$N = A - Z$

(3) **Binding energy (B.E.)**

If Δm is mass defect then according to Einstein's mass energy relation

Binding energy = $\Delta m \cdot c^2$ = $[\{m_p Z + m_n(A-Z)\} - M] \cdot c^2$

(This binding energy is expressed in *joule*, because Δm is measured in *kg*)

If Δm is measured in *amu* then binding energy = Δm *amu* = $[\{m_p Z + m_n(A-Z)\} - M]$ *amu* = $\Delta m \times 931$ *MeV*

$$B.E. = \Delta m c^2 = \Delta m \times c^2 \text{ MeV.}$$

amu $\quad c^2 = 931$

(4) **Binding energy per nucleon**

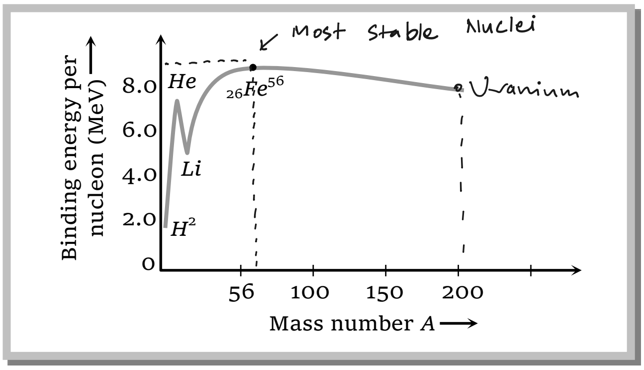
Binding energy per nucleon

$$= \frac{\text{Total binding energy}}{\text{Mass number } (i.e. \text{ total number of nucleons})} = \frac{\Delta m \times 931}{A} \frac{MeV}{Nucleon}$$

Binding energy per nucleon $\propto$ Stability of nucleus

Binding Energy Curve

It is the graph between binding energy per nucleon and total number of nucleons (*i.e.* mass number *A*)



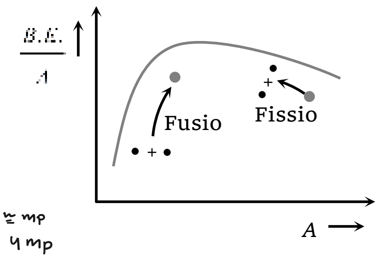
(1) Some nuclei with mass number *A* < 20 have large binding energy per nucleon than their neighbour nuclei. For example $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ and $_{10}Ne^{20}$. These nuclei are more stable than their neighbours.

(2) The binding energy per nucleon is maximum for nuclei of mass number *A* = 56 ($_{26}Fe^{56}$). It's value is 8.8 *MeV* per nucleon.

(3) For nuclei having *A* > 56, binding energy per nucleon gradually decreases for uranium (*A* = 238), the value of binding energy per nucleon drops to 7.5 *MeV*.

Note : ❑ When a heavy nucleus splits up into lighter nuclei, then binding energy per nucleon of lighter nuclei is more than that of the original heavy nucleus. Thus a large amount of energy is liberated in this process (nuclear fission).

❑ When two very light nuclei combines to form a relatively heavy nucleus, then binding energy per nucleon increases. Thus, energy is released in this process (nuclear fusion).



$m_N \simeq m_p$

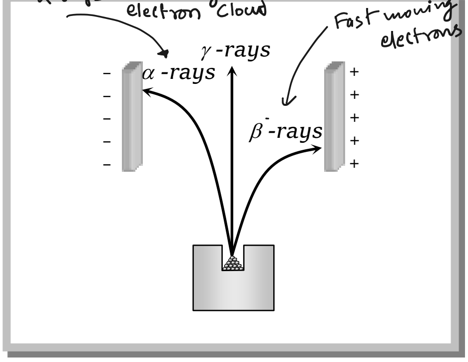
$m_\alpha = 2m_p + 2m_N = 4m_p$

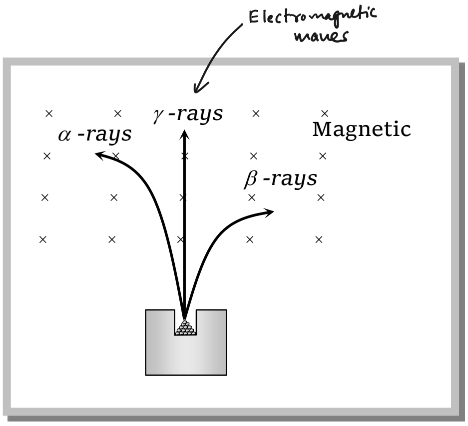
$m_\alpha = 4m_p$, $q = 2e^+$

Nuclear radiatons

$\alpha \longrightarrow {}_2He^{4++}$

α rays are actully He nucleus without electron Cloud





Note :
- Exactly same results were obtained when these radiations were subjected to magnetic field.
- No radioactive substance emits both α and β particles simultaneously. Also γ-rays are emitted after the emission of α or β-particles.
- β-particles are not orbital electrons they come from nucleus. The neutron in the nucleus decays into proton and an electron. This electron is emitted out of the nucleus in the form of β-rays.

$$_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4*} + \alpha \text{ decay} + Q \text{ value}$$

α = ${}_{2}He^{4++}$

$${}_{Z-2}Y^{A-4*} \longrightarrow {}_{Z-2}Y^{A-4} + \gamma \text{ decays.}$$

$$_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + \beta^{-} \text{ decay} + \bar{\nu} + Q \text{ value.}$$

β^{-} = e^{-}

N↓ Z↑
−1 +1

$$n \longrightarrow \text{electron} + \text{Proton} + \text{Antineutrino}$$

electron: e^{-} get out.
Proton: e^{+} still there
Antineutrino: $\bar{\nu}$

$$_{Z}X^{A} \longrightarrow {}_{Z-1}Y^{A} + \beta^{+} \text{ decay} + Q + \nu$$

$$\text{Proton} \longrightarrow \text{Positron} + \text{neutron} + \nu$$

Positron: β^{+} out
neutron: stay.

Properties of α, β and γ-rays

read

Features	α- particles	β- particles	γ- rays
1. Identity	Helium nucleus or doubly ionised helium atom ($_2He^4$)	Fast moving electron $(-\beta^0 \text{ or } \beta^-)$	Photons (E.M. waves)
2. Charge	+ 2*e*	– *e*	Zero
3. Mass 4 m_p (m_p = mass of proton = 1.87×10^{-27}	4 m_p	m_e	Massless
4. Speed	$\approx 10^7$ *m/s*	1% to 99% of speed of light	Speed of light
5. Range of kinetic energy	4 *MeV* to 9 *MeV*	All possible values between a minimum certain value to 1.2 *MeV*	Between a minimum value to 2.23 *MeV*
6. Penetration power (γ, β, α)	1 (Stopped by a paper)	100 (100 times of α)	10,000 (100 times of β upto 30 *cm* of iron (or *Pb*) sheet
7. Ionisation power ($\alpha > \beta > \gamma$)	10,000	100	1
8. Effect of electric or magnetic field	Deflected	Deflected	Not deflected
9. Energy spectrum	Line and discrete	Continuous	Line and discrete
10. Mutual interaction with matter	Produces heat	Produces heat	Produces, photo-electric effect, Compton effect, pair production
11. Equation of decay	$_Z X^A \xrightarrow{\alpha-decay} {}_{Z-2}Y^{A-4} + {}_2He^4$ $_Z X^A \xrightarrow{n_\alpha} {}_{Z'}Y^{A'}$ $\Rightarrow \boldsymbol{n_\alpha} = \dfrac{\boldsymbol{A' - A}}{\boldsymbol{4}}$	$_Z X^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}$ $_Z X^A \xrightarrow{n_\beta} {}_{Z'}X^A$ $\Rightarrow \boldsymbol{n_\beta = (2n_\alpha - Z + Z')}$	$_z X^A \rightarrow {}_z X^a + \gamma$

Radioactive Disintegration

(1) **Law of radioactive disintegration**

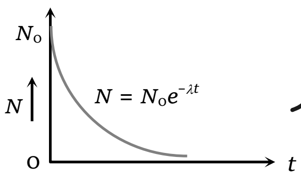
"At any instant the rate of decay of radioactive atoms is proportional to the number of atoms present at that instant" *i.e.* $-\frac{dN}{dt} \propto N \quad \Rightarrow \frac{dN}{dt} = -\lambda N$.

Suppose we have N active Nuclei

$-\frac{dN}{dt} \propto N$

It can be proved that $N = N_0 e^{-\lambda t}$

Note : ❑ λ depends only on the nature of substance. It is independent of time and any physical or chemical changes.



$-\frac{dN}{dt} = \lambda N$

$\int_{N_0}^{N} \frac{dN}{N} = -\int_0^t \lambda dt$

$\log \frac{N}{N_0} = -\lambda t, \quad \frac{N}{N_0} = e^{-\lambda t}$

(2) **Activity**

It is defined as the rate of disintegration (or count rate) of the substance (or the number of atoms of any material decaying per second) *i.e.* $A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$

$N = N_0 e^{-\lambda t}$

where A_0 = Activity of t = 0, A = Activity after time t

$= N_0 - N = N_0 - N_0 e^{-\lambda t}$

$\text{Activity} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$

not imp.

Units of activity (Radioactivity)

It's units are *Becqueral* (*Bq*), Curie (*Ci*) and Rutherford (*Rd*)

1 *Becquerel* = 1 *disintegration/sec*, 1 *Rutherford* = 10^6 *dis/sec*, 1 *Curie* = 3.7×10^{11} *dis/sec*

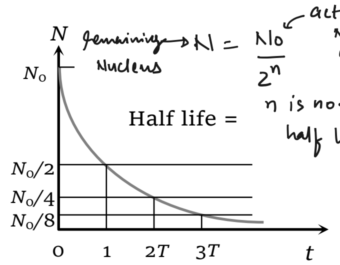
(3) **Half life ($T_{1/2}$)**

Time interval in which the mass of a radioactive substance or the number of it's atom reduces to half of it's initial value is called the half life of the substance.

i.e. if $N = \frac{N_0}{2}$ then $t = T_{1/2}$

Hence from $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda (T_{1/2})} \Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$



remaining Nucleus → $N = \frac{N_0}{2^n}$ ← active Nuclei at t=0

n is no. of half lifes

$N = \frac{N_0}{2}, \quad N' = \frac{N_0}{4}, \quad N'' = \frac{N_0}{8}$

$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$

decay constant.

Time (t)	Number of undecayed atoms (N) (N_0 = Number of initial atoms)	Remaining fraction of active atoms (N/N_0) probability of survival	Fraction of atoms decayed $(N_0 - N)/N_0$ probability of decay
$t = 0$	N_0	1 (100%)	0
$t = T_{1/2}$	$\frac{N_0}{2}$	$\frac{1}{2}$ (50%)	$\frac{1}{2}$ (50%)
$t = 2(T_{1/2})$	$\frac{1}{2} \times \frac{N_0}{2} = \frac{N_0}{(2)^2}$	$\frac{1}{4}$ (25%)	$\frac{3}{4}$ (75%)
$t = 3(T_{1/2})$	$\frac{1}{2} \times \frac{N_0}{(2)} = \frac{N_0}{(2)^3}$	$\frac{1}{8}$ (12.5%)	$\frac{7}{8}$ (87.5%)
$t = 10\,(T_{1/2})$	$\frac{N_0}{(2)^{10}}$	$\left(\frac{1}{2}\right)^{10} \approx 0.1\%$	$\approx 99.9\%$
$t = n\,(N_{1/2})$	$\frac{N}{(2)^2}$	$\left(\frac{1}{2}\right)^n$	$\left\{1 - \left(\frac{1}{2}\right)^n\right\}$

Useful relation

After n half-lives, number of undecayed atoms $N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

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(4) **Mean (or average) life (τ)**

The time for which a radioactive material remains active is defined as mean (average) life of that material.

Other definitions

(i) It is defined as the sum of lives of all atoms divided by the total number of atoms

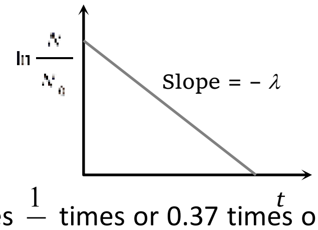
i.e. $\tau = \frac{\text{Sum of the lives of all the atoms}}{\text{Total number of atoms}} = \frac{1}{\lambda}$

(ii) From $N = N_0 e^{-\lambda t} \Rightarrow \frac{\ln \frac{N}{N_0}}{t} = -\lambda$ slope of the line shown in the graph

$t_{mean} = \frac{1}{\lambda}$, $t_{1/2} = \frac{0.693}{\lambda}$

Qustions.

i.e. the magnitude of inverse of slope of $\ln\frac{N}{N_0}$ vs t curve is known as mean life (τ).



(iii) From $N = N_0 e^{-\lambda t}$

If $t = \frac{1}{\lambda} = \tau \Rightarrow N = N_0 e^{-1} = N_0\left(\frac{1}{e}\right) = 0.37 N_0 = 37\%$ of N_0.

i.e. mean life is the time interval in which number of undecayed atoms (N) becomes $\frac{1}{e}$ times or 0.37 times or 37% of original number of atoms. or

It is the time in which number of decayed atoms ($N_0 - N$) becomes $\left(1 - \frac{1}{e}\right)$ times or 0.63 times or 63% of original number of atoms.

(iv) From $T_{1/2} = \frac{0.693}{\lambda} \Rightarrow \frac{1}{\lambda} = \tau = \frac{1}{0.693}.(t_{1/2}) = 1.44\,(T_{1/2})$

i.e. mean life is about 44% more than that of half life. Which gives us $\tau > T_{(1/2)}$

Note : ❑ Half life and mean life of a substance doesn't change with time or with pressure, temperature *etc.*

pls every one attand poll.