PHYSICS

JEE and NEET Crash Course

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Problem Solving Class

(Kinetic Theory of Gasses and Thermodynamics)

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P-Q2442

At a certain temperature, the *r.m.s.* velocity for O_2 is 400 m/sec. At the same temperature, the *r.m.s.* velocity for H_2 molecules will be

(A) 100 m/sec (B) 25 m/sec

(C) 1600 m/sec (D) 6400 m/sec

$$v_{rms} = \sqrt{\frac{3RT}{M_0}} \propto \sqrt{\frac{T}{M_0}} \propto \frac{1}{\sqrt{M_0}}$$

$$\frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4$$

$$v_{H_2} = 1600\ m/s$$

P-Q2442-Solution

Ans [C]

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{RMS} \propto \frac{1}{\sqrt{M}} \quad \because T \text{ is Constant}$$

$$\Rightarrow \frac{(v_{rms})_{O_2}}{(v_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} \Rightarrow \frac{400}{(v_{rms})_{H_2}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

$$\Rightarrow (v_{rms})_{H_2} = 1600\ m/s\ .$$

P-Q2463

A mixture contains 1 mole of the helium $\left(C_p = 2.5\,R, C_v = 1.5\,R\right)$ and 1 mole of hydrogen $\left(C_p = 3.5\,R, C_v = 2.5\,R\right)$. Calculate the value of γ for the mixture.

(A) 1.83

(B) 1.5

(C) 2.4

(D) 1

$$\gamma_{mix.} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{v_1} + n_2 C_{v_2}} = \frac{2.5R + 3.5R}{1.5R + 2.5R}$$

$$= \frac{6R}{4R} = 1.5$$

P-Q2463

Ans [B]

$$C_p = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$$

$$= \frac{2.5\,R + 3.5\,R}{1+1} = 3R$$

$$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

$$= \frac{1.5\,R + 2.5\,R}{1+1} = 2\,R$$

$$\gamma = \frac{C_P}{C_V} = \frac{3R}{2R} = 1.5$$

For mixture C_V and C_P will be distributed in the ratio of moles of gases.

$$\gamma_{mix} = \frac{C_P(mixture)}{C_V(mixture)}$$

P-Q2464

An ideal gas is taken through a process in which the pressure and the volume are changed according to the equation P=kV. the molar heat capacity of the gas for the process is

(A) $C = C_v + \frac{R}{2}$.

(B) $C = C_v - \frac{R}{2}$.

(C) $C = \frac{R}{2} - C_v$

(D) $C = C_v + \mathrm{R}$

P-Q2464

Ans [A]

$$p = kV \Rightarrow pV^{-1} = Constant$$

Comparing to $pV^{x} = Constant$ (Polytropic process)

$$x = -1$$

$$C = C_V + \frac{R}{1 + 1} = C_V + \frac{R}{2}$$

For a polytropic process: $C = C_V + \frac{R}{1-x}$

P-Q2441

If the molecular weight of two gases are M_1 and M_2, then at a temperature the ratio of root mean square velocity v_1 and v_2 will be

(A) $\sqrt{\frac{M_1}{M_2}}$

(B) $\sqrt{\frac{M_2}{M_1}}$

(C) $\sqrt{\frac{M_1 + M_2}{M_1 - M_2}}$

(D) $\sqrt{\frac{M_1 - M_2}{M_1 + M_2}}$

$v_{rms} = \sqrt{\frac{3RT}{M_0}} \propto \frac{1}{\sqrt{M_0}}$

$\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}$

P-Q2441-Solution

Ans [B]

$$v_{\text{rms}} = \sqrt{3\,\text{RT}/M}$$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}$$

$$v_{RMS} \propto \frac{1}{\sqrt{M}}$$

P-Ques

During an experiment an ideal gas is found to obey an additional law VP^2 = constant. The gas is initially at temp T and volume V. What will be the temperature of the gas when it expands to a volume 2V?

(A) $T' = \sqrt{4}\ T$

(B) $T' = \sqrt{2}\ T$ ✓

(C) $T' = \sqrt{5}\ T$

(D) $T' = \sqrt{6}\ T$

$$VP^2 = \text{const}$$

$$V\frac{T^2}{V^2} = \text{const}$$

$$\frac{T^2}{V} = \text{const}$$

$$T^2 \propto V$$

$$P = \frac{nRT}{V} \propto \frac{T}{V}$$

$$T \propto \sqrt{V}$$

P-Sol

Ans [B]

According to the given problems

VP^2 = constant

From the gas law

PV = nRT

$\Rightarrow \quad \left(\frac{k}{\sqrt{V}}\right)V = nRT \qquad \Rightarrow \quad |\sqrt{V} = \left(\frac{nR}{K}\right)T$

$\therefore \quad \sqrt{\frac{V_1}{V_2}} = \left(\frac{T_1}{T_2}\right)$, i.e, $\sqrt{\frac{V}{2V}} = \frac{T}{T'}$

$\Rightarrow \quad T' = \sqrt{2}\ T$

P-Ques

Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300K. The piston A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30K, then the rise in temperature of the gas in B is.

(A) 30K (B) 18K

(C) 50K (D) 42K

Isobaric

Isochoric

$\Delta Q = nC\Delta T$

$\Delta Q_B = nC_v \Delta T_B$

$\Delta Q_A = nC_p \Delta T_A$

$\Delta Q_B = \Delta Q_A$

$nC_v \Delta T_B = nC_p \Delta T_A$

$\frac{5}{2} R \Delta T_B = \frac{7}{2} R \times 30$

$\Delta T_B = 42K$

P-Sol

Ans [D]

For cylinder A

$dQ = nC_P dT_1$

for cylinder B

$dQ = nC_v dT_2$

$\Rightarrow$ $nC_P dT_1 = nC_v dT_2$

From (I) and (II)

$$c_v dT_2 = (c_v + R)30$$

$$\therefore \quad dT_2 = \frac{(c_v + R)30}{c_v}$$

For diatomic gas $c_v = \frac{5}{2}R$

$$\therefore \quad dT_2 = 42K.$$

P-Ques

An ideal gas ($\gamma = 1.5$) is expanded adiabatically. How many times has the gas to be expanded to reduce the root mean square velocity of molecules 2.0 times

(A) *4 times*

(B) *16 times*

(C) *8 times*

(D) *2 times*

$$PV^{\gamma} = \text{const}$$

$$PV = nRT$$

$$P \propto \frac{T}{V}$$

$$\frac{T}{V} V^{\gamma} = \text{const}$$

$$v_{rms} = \sqrt{\frac{3RT}{M_0}} \propto \sqrt{T}$$

$$v'_{rms} = \frac{v_{rms}}{2}$$

$$T' = \frac{T}{4}$$

$$TV^{\gamma-1} = \text{const}$$

$$TV^{1/2} = \frac{T}{4} \cdot (V')^{1/2}$$

$$4V^{1/2} = (V')^{1/2}$$

$$16V = V'$$

P-Sol

Ans [B]

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\therefore \quad V_{rms} \propto \sqrt{T}$$

V_{rms} is to reduce two times i.e, temperature of the gas will have to reduce four times or

$$\frac{T'}{T} = \frac{1}{4}$$

During adiabatic process

$$TV^{\gamma-1} = T'V'^{\gamma-1}$$

$$\text{or, } \frac{V'}{V} = \left(\frac{T}{T'}\right)^{\frac{1}{\gamma-1}} = (4)^{\frac{1}{1.5-1}} = 4^2 = 16$$

$$\therefore \quad V' = 16V$$

P-Ques

Heat is supplied to a diatomic gas at constant pressure. The ratio of $\Delta Q : \Delta U : \Delta W$ ***is***

(A)	***5:3:2***	***(B)***	***5:2:3***
(C) ✓	***7:5:2***	***(D)***	***7:2:5***

$$\Delta W = nR\Delta T = W$$

$$\Delta U = \frac{f}{2} nR\Delta T = \frac{f}{2} W = \frac{5}{2} W$$

$$\Delta Q = \Delta U + \Delta W = \frac{7}{2} W$$

$$\Delta Q : \Delta U : \Delta W = \frac{7}{2} : \frac{5}{2} : 1$$

$$= 7 : 5 : 2$$

P-Sol

Ans [C]

$$\Delta Q = nC_P dT = \frac{7}{2}nR(\Delta T), \qquad \left[C_P = \frac{7}{2}R\right]$$

$$\Delta U = nC_V \Delta T = \frac{5}{2}nR\Delta T, \qquad \left[C_V = \frac{5}{2}R\right]$$

and $\Delta W = \Delta Q - \Delta U = nR\Delta T$

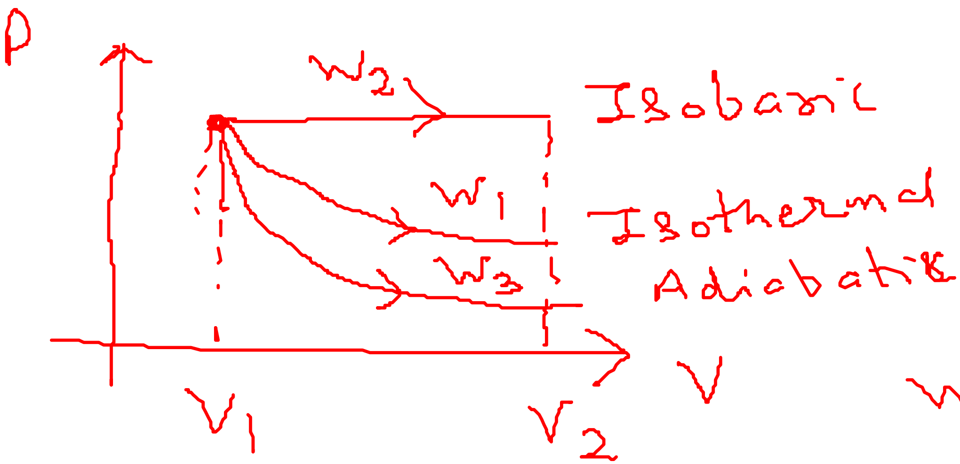
$\therefore \ \Delta Q : \Delta U : \Delta W = 7 : 5 : 2$

P-Ques

Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic. Then

(A) $W_2 > W_1 > W_3$ (B) $W_2 > W_3 > W_1$

(C) $W_1 > W_2 > W_3$ (D) $W_1 > W_3 > W_2$



$PV^m = \text{const}$

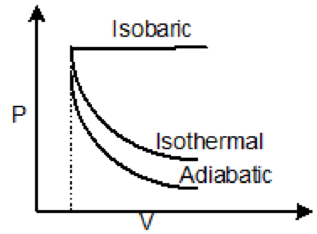
$m\uparrow$ graph $\downarrow$

if $V\uparrow$

$W_2 > W_1 > W_3$

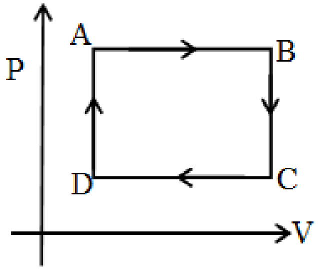
P-Sol

Ans [A]

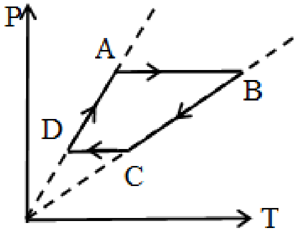


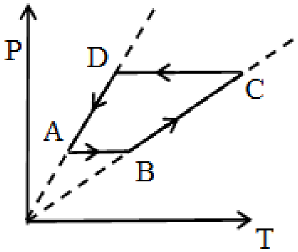
P-Ques

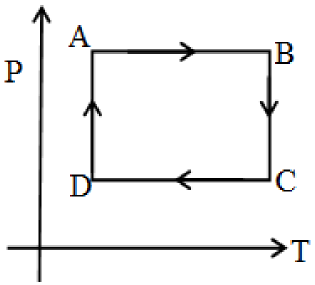
Cyclic process of an ideal gas is shown in figure.

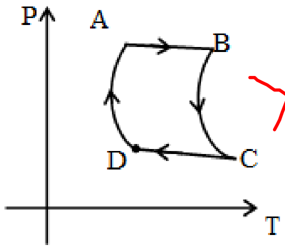


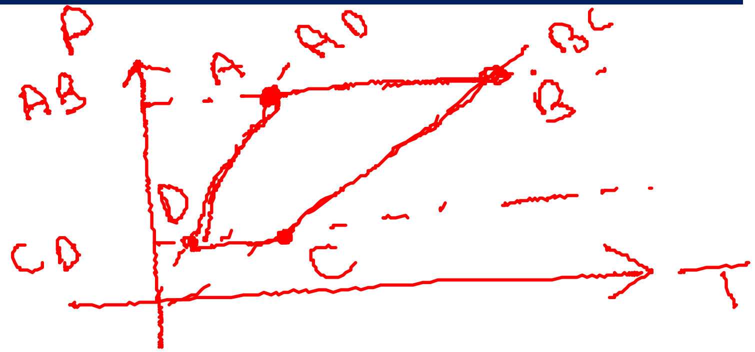
The above process in the P-T coordinates is given as

(A)


(B)


(C)


(D)




V = const

PV = nRT

$P \propto T$

$P = \left(\frac{nR}{V}\right)T$

P-Sol

Ans [A]

P-Question

The equation of states for a gas is given by $PV = nRT + \alpha V$, where n is the number of moles and α is a positive of one mole of the gas contained in a cylinder are T_0 and P_0 respectively. The work done by the gas when its temperature doubles isobaric ally will be:

1) $\frac{P_0T_0R}{P_0-\alpha}$ 2) $\frac{P_0T_0R}{P_0+\alpha}$ 3) $P_0T_0R\ln 2$ 4) P_0T_0R

$$PV - \alpha V = nRT$$

$$(P-\alpha)V = RT$$

$$V = \frac{RT}{P-\alpha}$$

$$W = \int P\,dV$$

$$\frac{dV}{dT} = \frac{R}{P-\alpha}$$

$$dV = \left(\frac{R}{P-\alpha}\right)dT$$

$$W = \int P\left(\frac{R}{P-\alpha}\right)dT$$

$$= \frac{P_0R}{P_0-\alpha}\int_{T_0}^{2T_0} dT = \frac{P_0T_0R}{P_0-\alpha}$$

P-Solution

Ans [1]

$$PV = nRT + \alpha V \Rightarrow PV - \alpha V = nRT$$

$$V(P - \alpha) = \text{nRT}$$

$$\Rightarrow V = \frac{\text{nRT}}{(P - \alpha)} \qquad \therefore \text{dV} = \frac{\text{nR}}{(P - \alpha)}\text{dT}$$

workdone at constant pressure

$$\text{dW} = \text{PdV}$$

$$\int dW = \text{P}\int dV = P\,\frac{nR}{P - \alpha}\int_{T_0}^{2T_0} dT$$

$$\text{W} = \frac{PnR}{P-\alpha}(2T_0 - T_0)$$

Where n = 1 mole T = T_0 *and* $P = P_0$

$$\text{W} = \frac{P_0(1)R}{P_0-\alpha}T_0$$

$$W = \frac{P_0RT_0}{P_0 - \alpha}$$

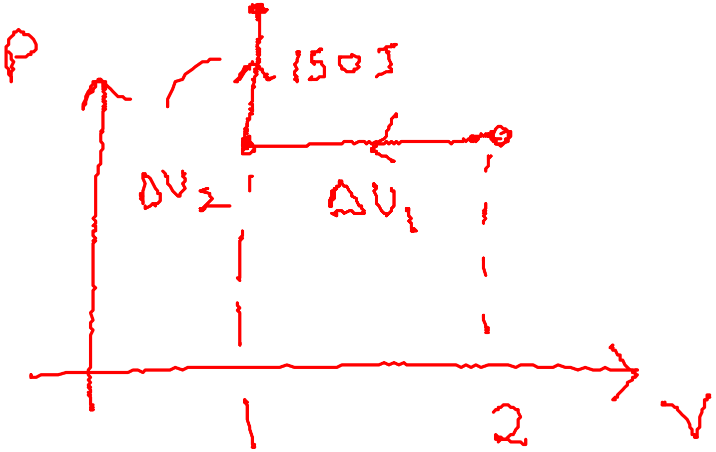
P-Question

A gas is compressed from a volume of 2 m^3 to volume of 1 m^3 at a constant pressure of 100 N/m^2. Then it is heated at constant volume by supplying 150 J of energy. As a result the internal energy of the gas

1) increased by 250 J
2) decreased by 250 J
3) increased by 50 J
4) decreased by 50 J

Isochoric

$\Delta Q = \Delta U + W$



$\Delta U = \Delta U_1 + \Delta U_1$

$= \frac{f}{2} W + 150$

$= \frac{3}{2}(-100) + 150$

$\Delta U = q - W$

$=$

$W_1 = P\Delta V$

$= 100(1-2)$

$= -100$

P-Solution

Ans [1]

$$During\ compression\ of\ constant\ pressure\ internal\ energ$$
$$increased\ by\ du_1 = P\Delta v = 100\ (2-1) = 100\ \text{J}$$

$$H\ \ at\ supplied\ at\ constant\ v\ \ lume\ \ dW = 0$$

$$d\quad = d\quad + dW$$

$$d\quad = d\quad + 0$$

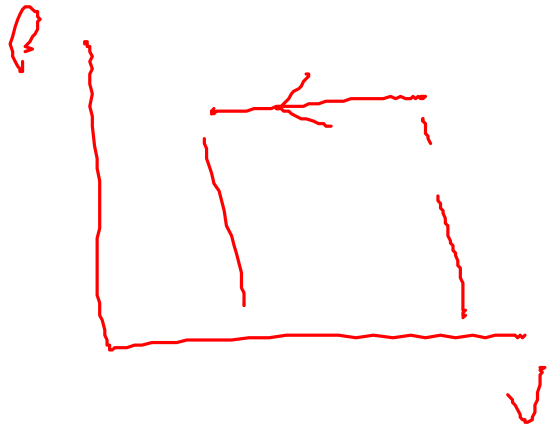
dU= d

$$du_2 = 1\ \ 0\ \text{J}$$

$$T\ \ e\ internal\ energ\ \ \ of\ t\ \ e\ gas$$
$$d\quad = du_1 + du_2 = 100 + 1\ \ 0 = 2\ \ 0$$

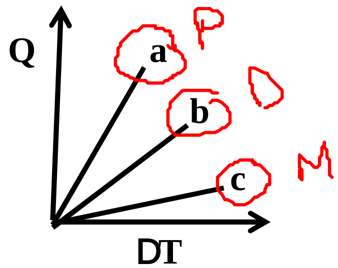
$PV = nRT$

$V \propto T$



P-Question

Figure shows the variation in temperature (DT) with the amount of heat supplied (Q) in an isobaric process corresponding to a monoatomic (M),diatomic (D) and a polyatomic (p) gas. The initial state of all gases are the same and the scales for the two axes coincide. Ignoring vibrational degrees of freedom, the lines a , b and c respectively correspond to



1) P, D and M ✓
2) M, D and P
3) D, M and P
4) P, M and D

$\Delta Q = nC_p \Delta T$

slope $\propto C_p$

$C_p = C_v + R$

$= \frac{f}{2}R + R$

$f\uparrow$ $C_p\uparrow$

slope$\uparrow$

P-Solution

Ans [1]

$$d \quad = n \quad _v dT$$

$$Q = n \quad _v \Delta T$$

$$n \quad _v = \frac{Q}{\Delta T} \longrightarrow (1)$$

$$S \ ope \ of \ t \quad e \ strig \quad t \ line = \frac{Q}{\Delta T} \rightarrow (2)$$

$$Eq \quad ation \ (2) = e \quad uation \ (1)$$

$$S \ ope = nc_v$$

$$S \ ope \ of \ a \ monoatomic \ gas \ (M) = n \ \frac{3R}{2} \rightarrow C$$

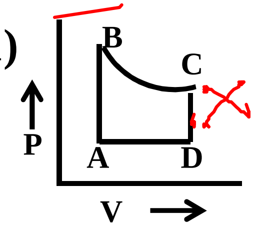
$$S \ ope \ of \ a \ diatomic \ gas \ (D) = n \ \frac{5}{2} \rightarrow b$$

$$S \ ope \ of \ a \ pol \quad atomic \ gas \ (P) = n \ 3R \rightarrow b$$

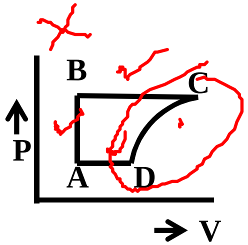
P-Question

A certain amount of gas is taken through a cyclic process (ABCDA) that has two isobars, one isochore and isothermal . The cycle can be represented on a p-v indicator diagram as:

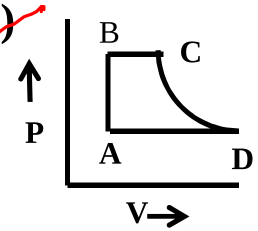
1)



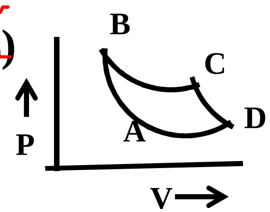
2)

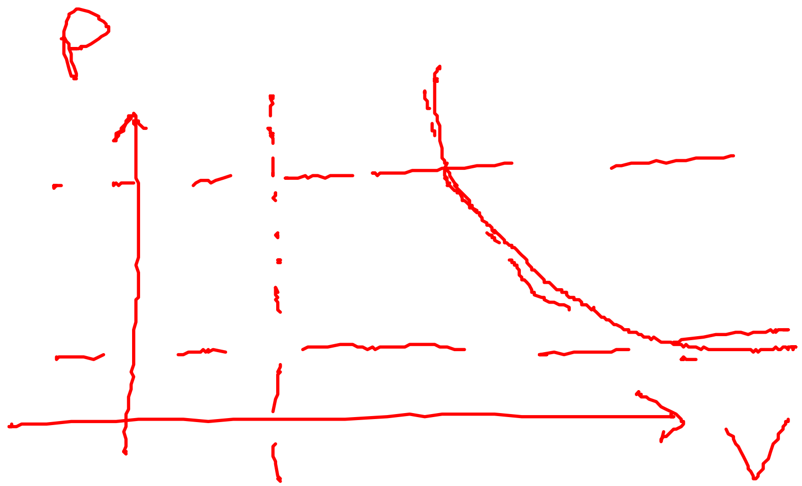


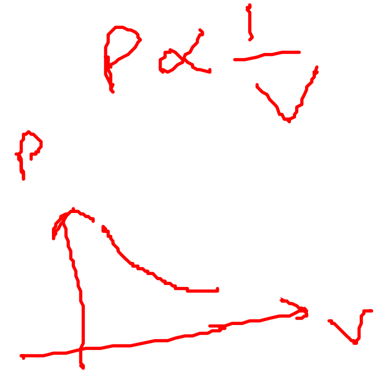
3)



4)







P-Solution

Ans [3]

In Option (1): ***one iso aric process and t o isoc oric process***

In Option (2): ***T o iso aric processes and one isoc oric process***

But C → D curve is not correct

C → D curve neither isothermal nor adiabatic

In option (3): ***T o iso aric processes and one isoc oric processes***

C→ D : Either isobaric or adiabatic process

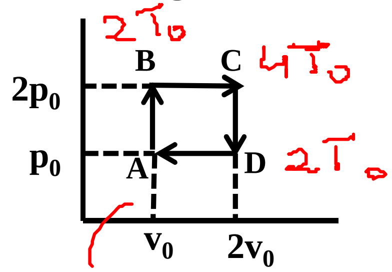
In option (4): ***T o adia atic and t o isot ermal processes***

P-Question

Helium gas goes through a cycle ABCD (consisting of two icochoric and isobaric lines) as shown in figure. efficiency of this cyclic is nearly:(Assume the gas to be close to ideal gas)

1) 15.4% **2) 9.1%**

3) 10.5% **4) 12.5%**



$PV = nRT$

$\eta = \frac{W}{Q_1}$

$\Delta Q = \Delta U + W$

$W = P_0 V_0$

$Q_1 = Q_{AB} + Q_{BC}$

$= n C_v \Delta T_{AB} + n C_p \Delta T_{BC}$

$= n \frac{3}{2} R T_0 + n \frac{5}{2} R 2T_0$

$= \frac{13 nRT_0}{2} = \frac{13}{2} P_0 V_0$

$\eta = \frac{W}{Q_1} = \frac{P_0 V_0}{\frac{13}{2} P_0 V_0} = \frac{2}{13}$

$\eta = \frac{2}{13} \times 100 = \frac{200}{13}$

$= 15.4\%$

P-Solution

Ans [1]

$$W_{net} = (2P_0 - V_0)(2V_0 - V_0) = P_0V_0$$

$H\ \ at\ gi\ \ en\ to\ t\ \ e\ gas$:

$$Q_B\ \ = n\ \ _p\ \mathrm{dt} = \mathrm{n}\frac{5}{2}\ \mathrm{Rdt} = \frac{5}{2}\ \mathrm{nRdT} = \frac{5}{2}\ 2P_0\ V_0 = 5\ P_0\ V_0 \rightarrow (1)$$

$$Q_{AB} = n\ \ _p\ \mathrm{dt} = \frac{3}{2}\ nRdt = \frac{3}{2}\ P_0\ V_0$$

$$Q_{in} = Q_{BC} + Q_{AB} = 5\ P_0V_0 + \frac{3}{2}\ P_0V_0$$

$$\mathrm{efficiency} = \frac{w\ \ r\ \ \ done\ by\ gas}{h\ \ at\ gi\ \ en\ to\ t\ \ e\ gas} \times 100$$

$$= \frac{P_0V_0}{P_0V_0\left[\frac{3}{2} + 5\right]} \times 100\% = 1\ \ .4\%$$

P-Question

At what temperature is the rms velocity of a hydrogen molecule equal to that of an oxygen molecule at 47^{0}C.......

1) 80K

2) -73K

3) 3K

4) 20K

P-Solution

Ans [4]

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{(V_{rms})_{O_2}}{(V_{rms})_{H_2}} = \sqrt{\frac{T_{O_2}}{T_{H_2}}\frac{M_{H_2}}{M_{O_2}}}$$

$$\frac{1}{1} = \sqrt{\frac{(47\quad 2\quad 3)}{T_{H_2}}\frac{2}{32}}$$

$$T_{H_2} = \frac{320}{1} = 20$$

P-Question

The temperature of an ideal gas in increased from 120K to 480K. If at 120K the root mean square velocity of the gas molecules is v, at 480k it becomes.......

1) 4V **2) 2V**

3) V/2 **4) V/4**

$$v_{rms} = \sqrt{\frac{3RT}{M_0}} \propto \sqrt{T}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{480}{120}} = \sqrt{4} = 2$$

$$v_2 = 2v_1 = 2v$$

P-Solution

Ans [2]

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{(V_{rms})_2}{(V_{rms})_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{(V_{rms})_2}{V} = \sqrt{\frac{48}{120}} = \sqrt{4}$$

$$(V_{rms})_2 = 2V$$

P-Question

Root mean square velocity of a particle is ‘v’ at pressure ‘p’. If pressure is increased two times keeping temperature constant, the r.m.s velocity becomes....

1) 2v

2) 3v

3) 0.5v

4) v

$$v_{rms} = \sqrt{\frac{3RT}{M_0}} = \sqrt{\frac{3P}{\rho}}$$

P-Solution

Ans [4]

Root mean square velocity does not depend on pressure

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

PV= nRT

PV= RT If n=1 mole

$$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$$

At constant temperature P is constant .

V_{rms} **rema ns t e same**.