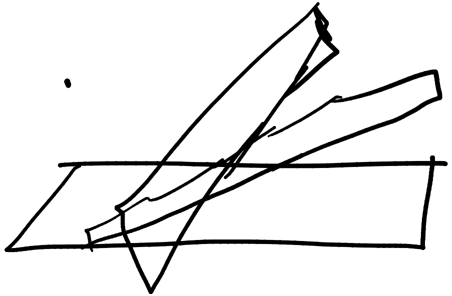
Family of Planes

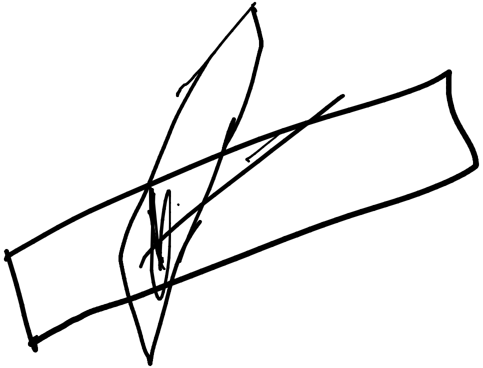
(ix) Planes bisecting the angle between two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2 + b_2y + c_2z + d_2 = 0$ is given by

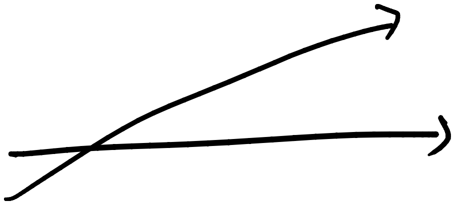
$$\left|\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}\right| = \pm \left|\frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$



Of these two bisecting planes , one bisects the acute and the other obtuse angle between the given planes.

(x) Equation of a plane through the intersection of two planes P_1 and P_2 is given by $\boxed{P_1 + \lambda P_2 = 0}$





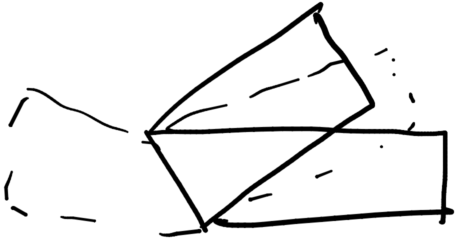
Problems

8)

A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point (2019 Main, 12 April II)

(a) $(1, -4, 1)$ (b) $(1, 4, -1)$

(c) $(2, 4, 1)$ (d) ~~$(2, -4, 1)$~~



$$\left|\frac{2x - y + 2z - 4}{\cancel{\sqrt{4+1+4}}}\right| = \pm\left|\frac{x + 2y + 2z - 2}{\cancel{\sqrt{1+4+4}}}\right|$$

$$2x - y + \cancel{2z} - 4 = x + 2y + \cancel{2z} - 2$$

$$\boxed{x - 3y = 2}$$

$$2x - y + 2z - 4 = -x - 2y - 2z + 2$$

$$\boxed{3x + y + 4z = 6}$$

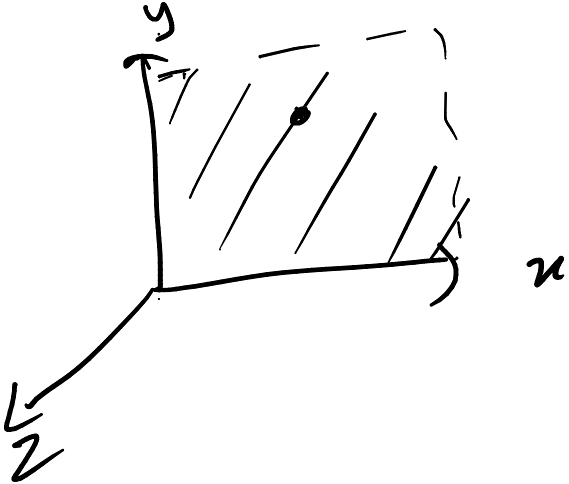
Problems

9

Let P be the plane, which contains the line of intersection of the planes, $x+y+z-6=0$ and $2x+3y+z+5=0$ and it is perpendicular to the XY-plane. Then, the distance of the point $(0, 0, 256)$ from P is equal to (2019 Main, 9 April II)

(a) $63\sqrt{5}$ (b) $205\sqrt{5}$

(c) $\frac{11}{\sqrt{5}}$ (d) $\frac{17}{\sqrt{5}}$



$\boxed{z=0}$

$P_1 + \lambda P_2$

$(x+y+z-6)+\lambda(2x+3y+z+5)=0$

$x(1+2\lambda)+y(1+3\lambda)+z(1+\lambda)-6+5\lambda=0$

$(0)x+(0)y+(1)z=0$

$(1+\lambda)=0$

$\boxed{\lambda=-1}$

$\boxed{-x-2y=11}$

$\frac{11}{\sqrt{1^2+2^2}}$

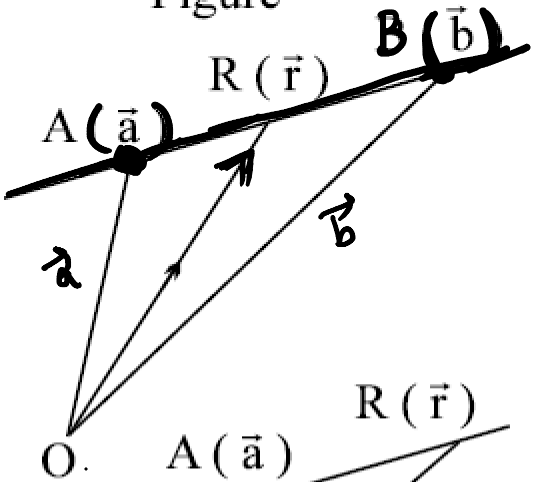
~~$2y+11=0$~~

$\boxed{\vec{r} = \vec{a} + \lambda \vec{b}}$

VECTOR EQUATION OF A LINE :

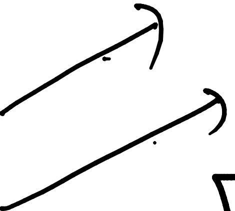
Parametric vector equation of a line passing through two point $A(\vec{a})$ & $B(\vec{b})$ is given by, $\boxed{\vec{r} = \vec{a} + t(\vec{b} - \vec{a})}$ where t is a parameter. If the line passes through the point $A(\vec{a})$ & is parallel to the vector $\vec{b}$ then its equation is, $\boxed{\vec{r} = \vec{a} + t\vec{b}}$

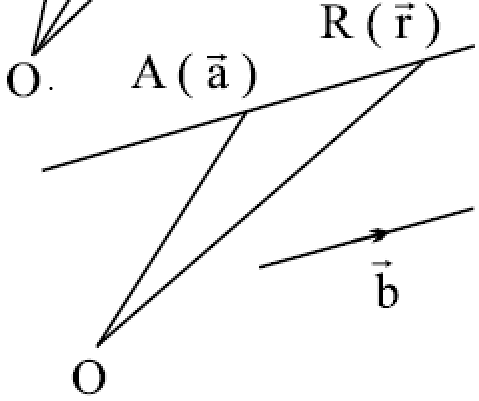
Figure



$$\boxed{\vec{r} = \underbrace{\vec{a}}_{\text{point}} + t\underbrace{(\vec{b} - \vec{a})}_{\text{direction}}}$$

$t \to$ (parameter)





$$\boxed{\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} - 3\hat{j} + \hat{k})}$$

(1,1,1) (2,-3,1)

$\cos\theta =$

Problems

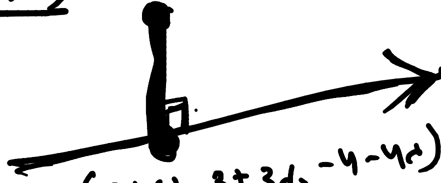
9)

The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point $(2, 3, -4)$ and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is

(2019 Main, 10 April II)

(a) $2\sqrt{13}$ (b) $4\sqrt{3}$ (c) 6 (d) 7

$(-1, 2, 6)$ $\quad -\hat{i}+2\hat{j}+6\hat{k}$



$(2+6\lambda, 3+3\lambda, -4-4\lambda)$

$[(6\lambda+3), 3\lambda+1, -4\lambda-10]$

$$\boxed{\vec{r} = (2\hat{i}+3\hat{j}-4\hat{k}) + \lambda(6\hat{i}+3\hat{j}-4\hat{k})}$$

$[6, 3, -4]$

$(x = 2+6\lambda), (y = 3+3\lambda), (z = -4-4\lambda)$

$0 = 6(6\lambda+3) + 3(3\lambda+1) + (-4)(-4\lambda-10)$

$$\boxed{\lambda = -1}$$

C STRAIGHT LINE IN SPACE

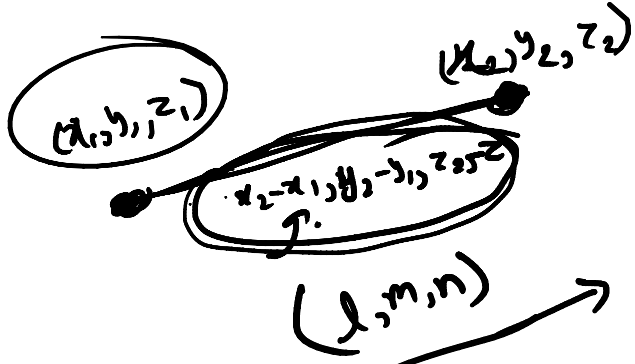
(cartesion form)

(i) ★★ Equation of a line through A (x_1, y_1, z_1) and having direction cosines l, m, n are

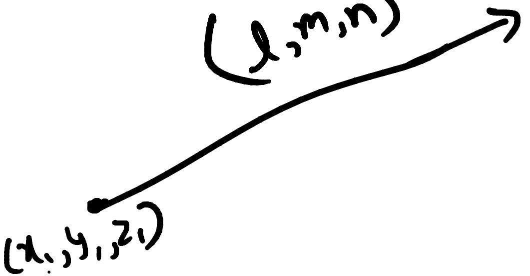
$$\boxed{\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}} = t$$

and the lines through (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$\boxed{\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}}$$



$$x = lt + x_1, \quad y = mt + y_1, \quad z = nt + z_1$$



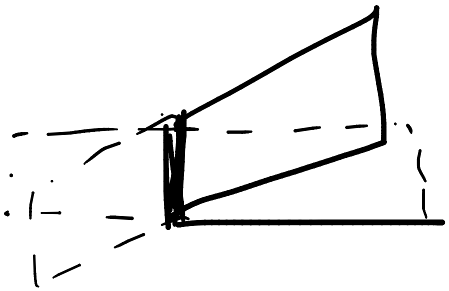
$$x\hat{i} + y\hat{j} + z\hat{k} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + t(l\hat{i} + m\hat{j} + n\hat{k})$$

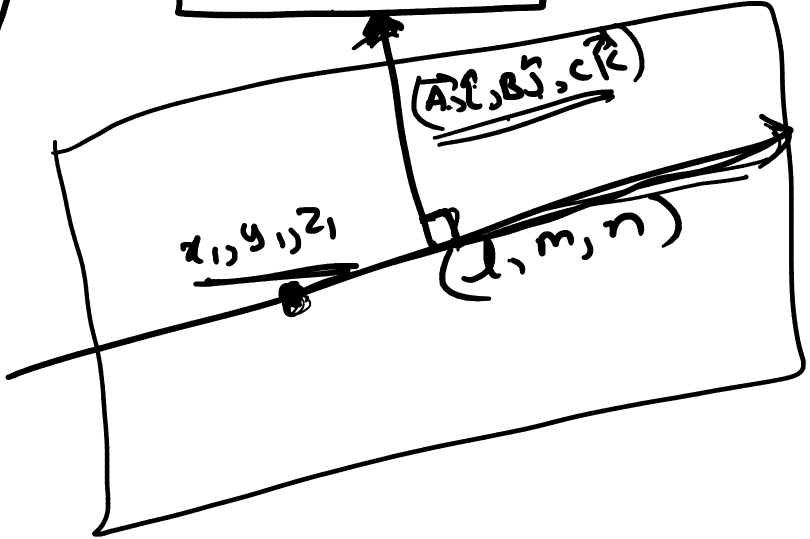
$$\boxed{\vec{r} = \vec{a} + t\vec{b}}$$

(ii) Intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ together represent the unsymmetrical form of the straight line.

(iii) General equation of the plane containing the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ is

$\boxed{A(x - x_1) + B(y - y_1) + C(z - z_1) = 0}$ where $\boxed{Al + bm + cn = 0}$





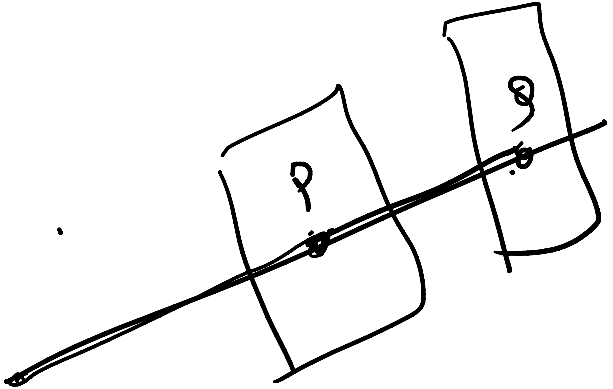
Problems

9)

If the line $\left(\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}\right)$ intersects the plane $2x+3y-z+13=0$ at a point P and the plane $3x+y+4z=16$ at a point Q, then PQ is equal to

(2019 Main, 12 April I)

(a) 14 (b) $\sqrt{14}$ (c) $2\sqrt{7}$ (d) $2\sqrt{14}$



$\boxed{x = 3\lambda+2,\ y = 2\lambda-1,\ z = -\lambda+1}$

$2(3\lambda+2) + 3(2\lambda-1) - (-\lambda+1) + 13 = 0$

$13\lambda = -13$

$\boxed{\lambda = -1}$

$P(-1, -3, 2)$

$3(3\lambda+2) + (2\lambda-1) + 4(-\lambda+1) = 16$

$7\lambda = 7, \ \boxed{\lambda = 1}$

$Q(5, 1, 0)$

$\sqrt{6^2+4^2+2^2} = \sqrt{56}$

Problems

Q) The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x+2y-z-5=0$ and intersecting the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z-2}{-1}$ is (2019 Main, 9 Jan I)

P $\longleftrightarrow (-3\lambda-1, 2\lambda+3, -\lambda+2)$

(a) $\frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$ ✓

(b) $\frac{x+4}{-1}=\frac{y-3}{1}=\frac{z-1}{1}$

(c) $\frac{x+4}{1}=\frac{y-3}{1}=\frac{z-1}{3}$

(d) $\frac{x-4}{2}=\frac{y+3}{1}=\frac{z+1}{4}$

$-3\lambda+3+4\lambda+\lambda-1=0$

$2\lambda+2=0$

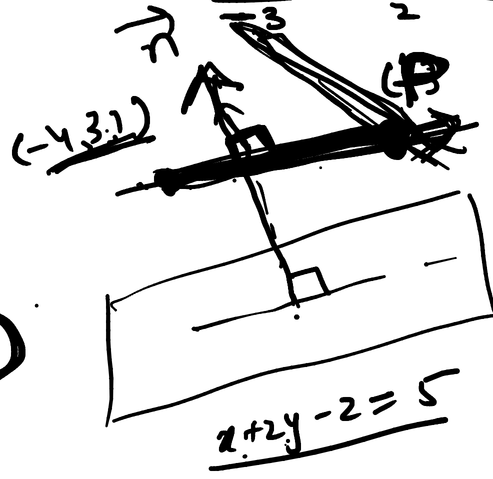
$\boxed{\lambda=-1}$

$(6, -2, 2) \longrightarrow (3, -1, 1)$

$(-3\lambda+3, 2\lambda, -\lambda+1) \longrightarrow$ Dr of Line

$(1, 2, -1) \longrightarrow$ Dr of $\vec{n}$

$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z-2}{-1}$

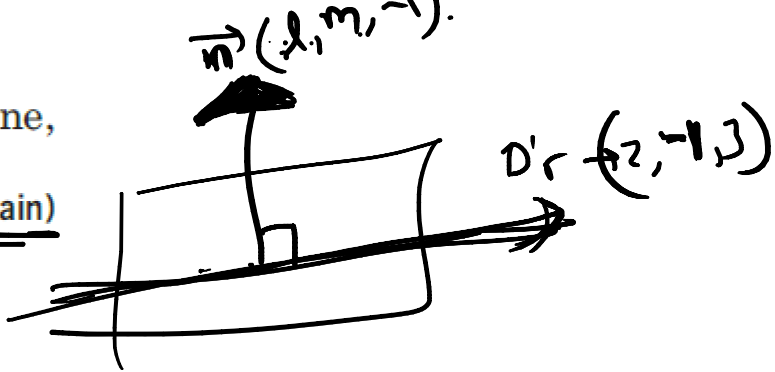


Problems

9 If the line, $\left(\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}\right)$ lies in the plane, $lx+my-z=9$, then $\left(l^2+m^2\right)$ is equal to - (2016 Main)

(a) 26 (b) 18

(c) 5 (d) 2



$3l - 2m + 4 = 9$

$\boxed{3l - 2m = 5}$

$\boxed{2l - m = 3}$

$m = 2l - 3$

$\boxed{m = -1}$

$2l - m - 3 = 0$

$3l - 2(2l - 3) = 5$

$-l = -1$

$\boxed{l = 1}$

End of Lecture