PHYSICS

JEE and NEET CRASH COURSE

Calorimetry, Thermal Expansion and Heat Transfer

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Heat

The energy that is being transferred between two bodies or between adjacent parts of a body as a result of temperature difference is called heat. Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist.

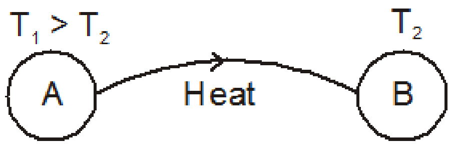
Once it is transferred, it becomes the internal energy of the receiving body. It should be clearly understood that the word "heat" is meaningful only as long as the energy is being transferred.

Thus, expressions like "heat in a body" or "heat of a body" are meaningless.

S.I. unit of heat energy is joule (J). Another common unit of heat energy is calorie (cal).

1 calorie = 4.18 joules.

1 calorie : The amount of heat needed to increase the temperature of 1 gm of water from 14.5 to 15.5 °C at one atmospheric pressure is 1 calorie.



Specific Heat

Specific heat of substance is equal to heat gain or released by that substance to raise or fall its temperature by 1° C for a unit mass of substance.

$$s = \frac{\Delta Q}{m \Delta T}$$

$$\boxed{\Delta Q = m\ s\ \Delta\ T}$$

Specific heat of water : S = 4200 J/kg°C = 1000 cal/kg°C = 1 Kcal/kg°C = 1 cal/gm°C

Specific heat of steam = specific heat of ice = half of specific heat of water $\simeq$ 0.5 cal/gm °C

Heat Capacity or Thermal Capacity

Heat capacity of a body is defined as the amount of heat required to raise the temperature of that body by 1°.

$\Delta Q = C \Delta T$

If 'm' is the mass and 's' the specific heat of the body, then
Heat capacity = m s.
Units of heat capacity in: CGS system is, **cal °C^{-1}**; SI unit is **JK^{-1}**

$\Delta Q = m s \Delta T$

$C = m s$

Phase Change and Latent Heat

Heat required for the change of phase or state,
Q = mL ,   L = latent heat.

Latent heat (L): The heat supplied to a substance which changes its state at constant temperature is called latent heat of the body.

Latent heat of Fusion (L_f): The heat supplied to a substance which changes it from solid to liquid state at its melting point and 1 atm. pressure is called latent heat of fusion. Latent heat of fusion of ice is 80 kcal/kg

Latent heat of vaporization (L_v): The heat supplied to a substance which changes it from liquid to vapour state at its boiling point and 1 atm. pressure is called latent heat of vaporization. Latent heat of vaporization of water is 540 kcal kg^{-1}.

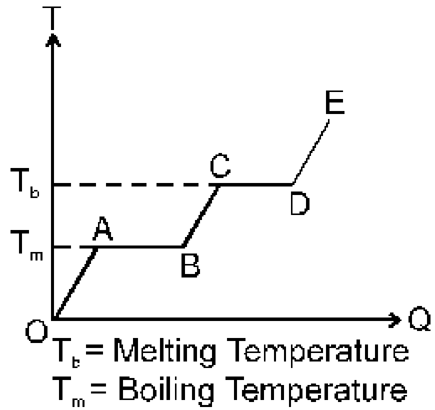
Latent heat of ice : L = 80 cal/gm = 80 Kcal/kg = 4200 × 80 J/kg

Latent heat of steam : L = 540 cal/gm = 540 Kcal/kg = 4200 × 540 J/kg

L_{ice} = 80 Cal/gm L_{water} = 540 Cal/gm

Graph of Phase Change

The given figure, represents the change of state
by different lines
OA – solid state , AB – solid + liquid state (Phase change)
BC – liquid state , CD – liquid + vapour state (Phase change)
DE – vapour state



ice-water mixture = $0^\circ C$
water-steam " = $100^\circ C$

Calorimetry

The branch of thermodynamics which deals with the measurement of heat is called calorimetry.

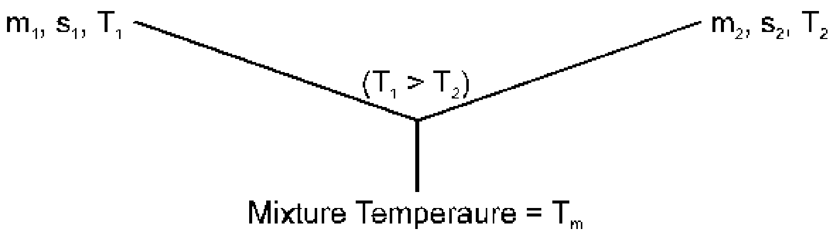
Law of Mixture (no phase change)

When two substances at different temperatures are mixed together, then exchange of heat continues to take place till their temperatures become equal. This temperature is then called final temperature of mixture. Here,

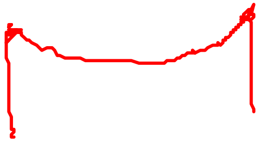
Heat taken by one substance = Heat given by another substance

$$\Rightarrow \quad m_1 s_1 (T_1 - T_m) = m_2 s_2 (T_m - T_2)$$

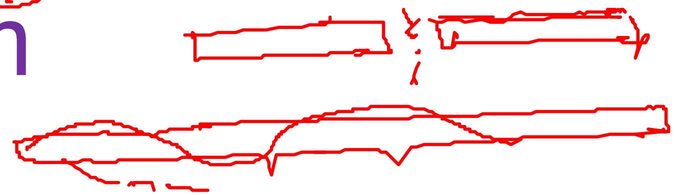
$$\Delta Q = m s \Delta T$$



$$T_m = \frac{m_1 s_1 T_1 + m_2 s_2 T_2 + m_3 s_3 T_3}{m_1 s_1 + m_2 s_2 + m_3 s_3}$$

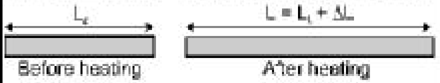
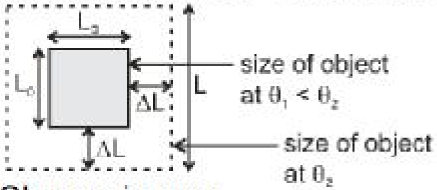


$R_1 \uparrow \quad R_2 \uparrow \quad (R_2 - R_1)\uparrow$



Thermal Expansion

Most materials expand when their temperature is increased.
Thermal expansion is like a photographic enlargement

LINEAR EXPANSION	SUPERFICIAL OR AREAL EXPANSION	VOLUME OR CUBICAL EXPANSION
 Change in length $\Delta L = \alpha L_0 \Delta T$ Final Length $L = L_0 (1 + \alpha \Delta T)$ $\Delta L \propto L$ $\propto \Delta T$ coeff. of linear exp.	 Change in area $\Delta A = \beta A_0 \Delta T$ Final area $A = A_0 (1 + \beta \Delta T)$ coeff. of areal or superficial exp.	Change in volume $\Delta V = \gamma V_0 \Delta T$ Final volume $V = V_0 (1 + \gamma \Delta T)$ coeff. of cubical or volumetric exp.

RELATION BETWEEN α , β AND γ (solid)

$\beta = 2\alpha$
$\gamma = 3\alpha$

(i) For isotropic solids: $\alpha : \beta : \gamma = 1 : 2 : 3$ or $\dfrac{\alpha}{1} = \dfrac{\beta}{2} = \dfrac{\gamma}{3}$

(ii) For non-isotropic solid $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$. Here α_1 , α_2 and α_3 are coefficient of linear expansion in X, Y and Z direction.

Variation of Time Period of Pendulum Clocks

$T = 2\pi\sqrt{\frac{L}{g}}$ at temperature θ.

For small percentage change in L

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta L}{L} = \frac{1}{2}\alpha\,\Delta\,\theta$$ temp

Gain or loss in time in duration of 't' in

$$\mathbf{\Delta t = \frac{1}{2}\alpha\Delta\theta\ t}$$

If T is the correct time then

(a) $\theta < \theta_0$, $T' < T$ clock becomes fast and gain time

(b) $\theta > \theta_0$, $T' > T$ clock becomes slow and loose time

$\Delta L = \alpha L \Delta T$

Variation of Density with Temperature

As we known that mass = volume × density .

Mass of substance does not change with change in temperature so with increase of temperature, volume increases so density decreases and vice-versa.

$$d = \frac{d_0}{(1+\gamma\Delta T)}.$$

For solids values of γ are generally small so we can write $d = d_0\,(1 - \gamma\,\Delta T)$ (using binomial expansion).

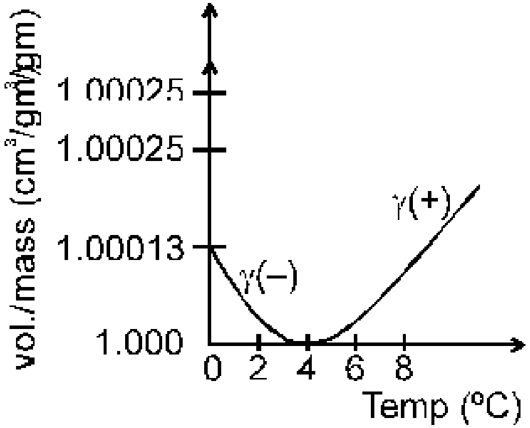
$$d_0 = \frac{m_0}{V_0}$$

$$d = \frac{m_0}{V_0(1+\gamma\Delta T)} = \frac{d_0}{1+\gamma\Delta T}$$

$$\simeq d_0[1-\gamma\Delta T)$$

Anomalous Expansion of water

For water density increases from 0 °C to 4 °C so γ is negative and for 4 °C to higher temperature γ is positive.
At 4 °C density is maximum.
This anamolous behaviour of water is due to presence of three types of molecules i.e. H_2O, $(H_2O)_2$ and $(H_2O)_3$ having different volume/mass at different temperatures.



Comparison between Different Temperature Scales

$$\frac{K-273}{100} = \frac{C}{100} = \frac{F-32}{180} = \frac{N-LSP}{USP-LSP}$$

(water - steam mix)

USP

L

L.S.P

(water ice mix)

	Celsius	Kelvin	Fahrenheit	New
USP	+100	373	212°	U.S.P
	C	K	F	N
LSP	0	273	32°	L.S.P

$\frac{x}{l} = \text{const}$

$\frac{C-0}{100-0} = \frac{K-273}{373-273}$

$= \frac{F-32}{212-32}$

$= \frac{N-LSP}{USP-LSP}$

Heat Transfer

Heat is energy in transit which flows due to temperature difference; from a body at higher temperature to a body at lower temperature. This transfer of heat from one body to the other takes place through three routes.

(i) Conduction
(ii) Convection
(iii) Radiation

Conduction

The process of transmission of heat energy in which heat is transferred from one particle of the medium to the other, but each particle of the medium stays at its own position is called conduction.

Consider a slab of face area A, Lateral thickness L, whose faces have temperatures T_H and T_C ($T_H > T_C$).

The amount of heat crossing the area A of the slab at position x in time t is given by

Power

$$\boxed{\frac{dQ}{dt} = -KA\frac{dT}{dx}}$$

K = thermal conductivity of the material,

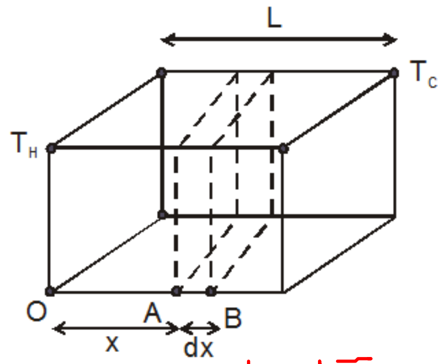
$\left(\frac{dT}{dx}\right)$ = temperature gradient

The (–) sign shows heat flows from high to low temperature (ΔT is a –ve quantity)

-ve

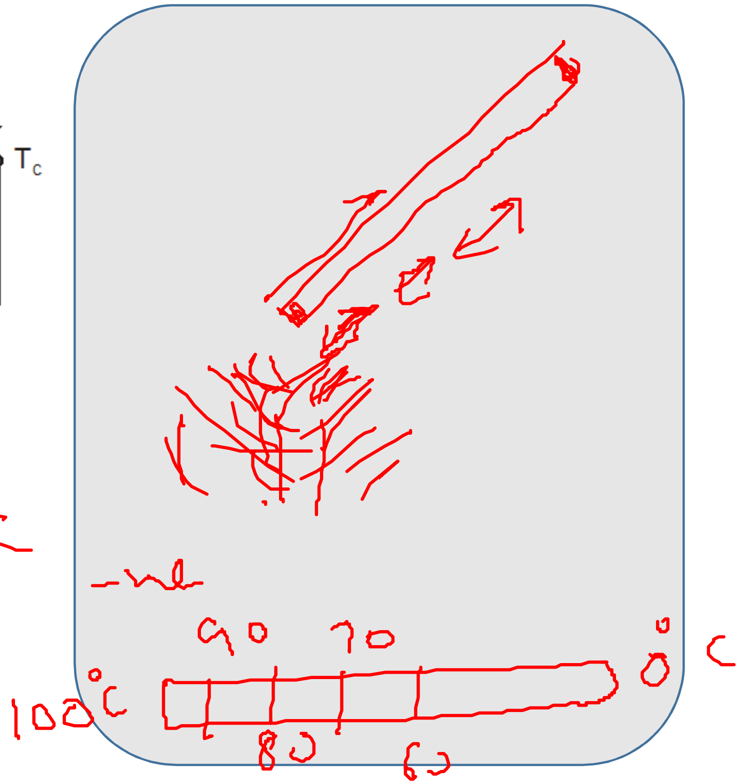
At steady state

$$\frac{Q}{t} = KA\left(\frac{T_H - T_C}{L}\right)$$



T T+dT

Q



Thermal Resistance to Conduction

For a slab of cross-section A, Lateral thickness L and thermal conductivity K,

Thermal resistance, $R = \dfrac{L}{KA}$

In terms of R, the amount of heat flowing though a slab in steady-state (in time t)

$$\frac{Q}{t} = \frac{(T_H - T_C)}{R} = \frac{KA(T_H - T_c)}{L}$$

If we name $\dfrac{Q}{t}$ as thermal current i_T

then, $i_T = \dfrac{T_H - T_C}{R}$

This is mathematically equivalent to OHM's law, with temperature playing the role of electric potential. Hence results derived from OHM's law are also valid for thermal conduction.

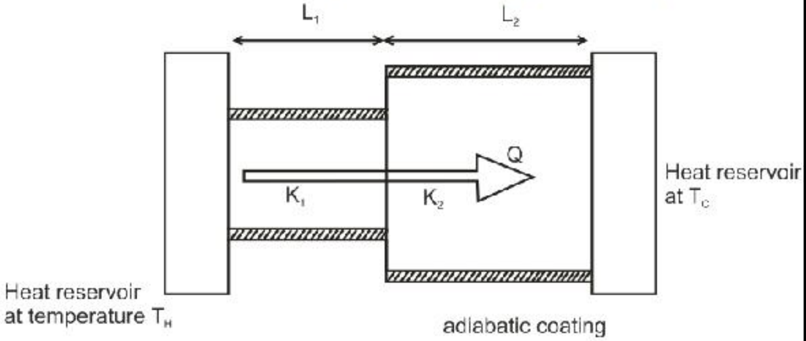
Elec. res.

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A}$$

$$I = \frac{V_1 - V_2}{R}$$

Slabs in Parallel and Series

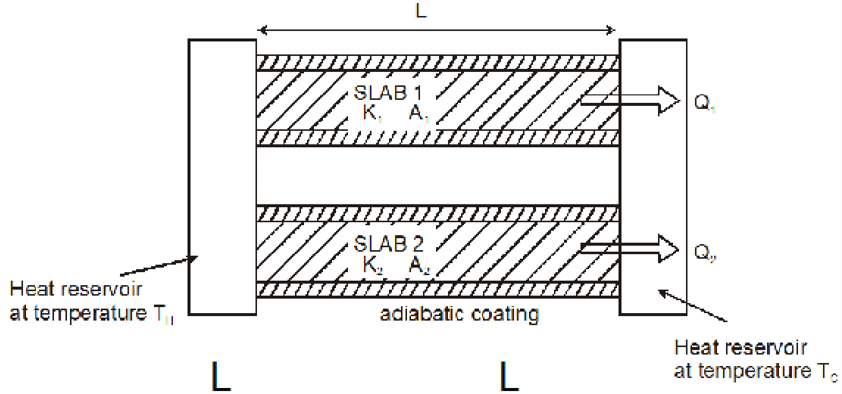
Slabs in series (in steady state)



$$R_1 = \frac{L_1}{K_1A_1}, \qquad R_2 = \frac{L_2}{K_2A_2}$$

$$i = \frac{T_H - T_C}{R} \qquad R = R_1 + R_2 + R_3 + \ldots\ldots$$

Slabs in parallel (in steady state)



$$R_1 = \frac{L}{K_1A_1}, \qquad R_2 = \frac{L}{K_2A_2}$$

$$i = \frac{T_H - T_C}{R_{eq}}, \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Convection

When heat is transferred from one point to the other through actual movement of heated particles, the process of heat transfer is called convection. In liquids and gases, some heat may be transported through conduction. But most of the transfer of heat in them occurs through the process of convection. Convection occurs through the aid of earth's gravity.

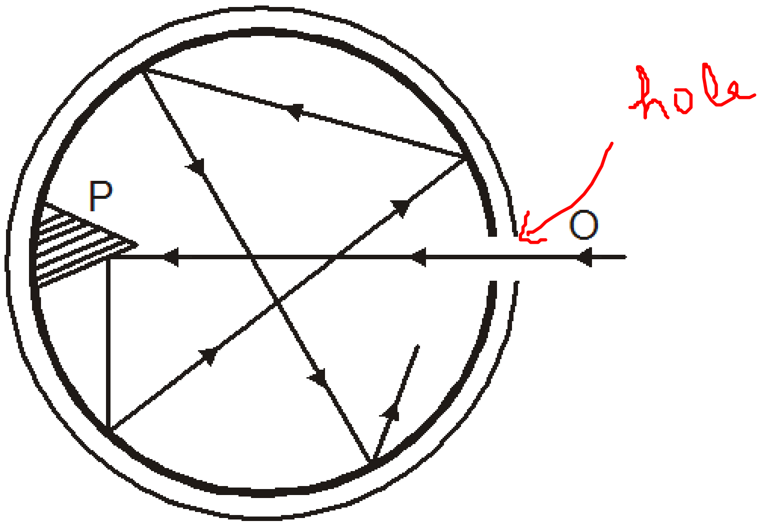
Radiation

The process of the transfer of heat from one place to another place without the requirement of medium is called radiation. The term radiation used here is another word for electromagnetic waves.

Perfectly black body and black body radiation

A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and therefore appears black whatever be the colour of the incident radiation.

(FERY'S BLACK BODY)



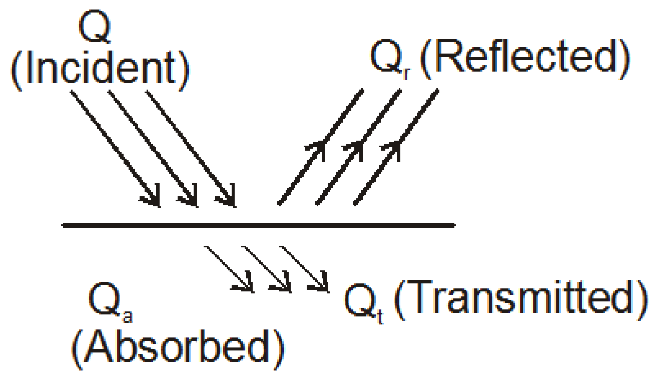
will act as perfect black body

Absorption, reflection and emission of radiations

$$Q = Q_r + Q_t + Q_a$$

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}$$

$$1 = r + t + a$$



where r = reflecting power , a = absorptive power
and t = transmission power.

(i) $r = 0$, $t = 0$, $a = 1$, perfect black body

(ii) $r = 1$, $t = 0$, $a = 0$, perfect reflector

(iii) $r = 0$, $t = 1$, $a = 0$, perfect transmitter

Absorptive Power, Emissive Power and Emissivity

Absorptive power :

In particular absorptive power of a body can be defined as the fraction of incident radiation that is absorbed by the body.

$$a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

Unitless, dimensionless

As all the radiations incident on a black body are absorbed, a = 1 for a black body.

Emissive power:

Energy radiated per unit time per unit area along the normal to the area is known as emissive power.

$$E = \frac{Q}{\Delta A \, \Delta t}$$

(Notice that unlike absorptive power, emissive power is not a dimensionless quantity).

Emissivity:

$$e = \frac{\text{Emissive power of a body at temperature } T}{\text{Emissive power of a black body at same temperature } T} = \frac{E}{E_0}.$$

$$e = \frac{E_{body}}{E_{blackbody}}$$

at same temp.
T

Kirchoff's Law

For the radiation of a given wavelength at the same temperature

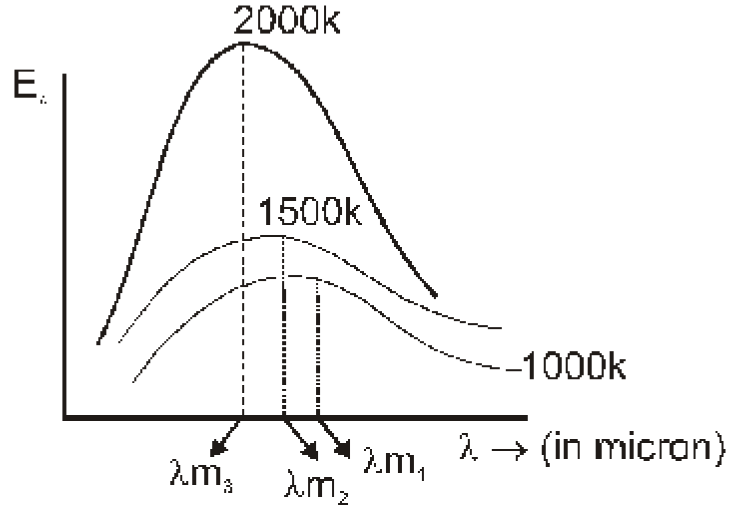
$$\frac{E(body)}{E\ (black\ body)} = a(body)$$

Hence we can conclude that good emitters are also good absorbers.

For anybody $\boxed{e = a}$

Nature of thermal radiations : (Wien's displacement law)

$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m T = b$$



This is called Wien's displacement law.

Here b = 0.282 cm-K, is the Wien's constant.

10^{-6}

$E = \frac{hc}{\lambda}$

$T \uparrow \quad E \uparrow$

$\lambda \downarrow$

Stefan-Boltzmann's law

STEFAN-BOLTZMANN'S LAW

According to this law, the amount of radiation emitted per unit time from an area A of a black body at absolute temperature T is directly proportional to the fourth power of the temperature.

$$P = \sigma A T^4$$ for black body

where σ is Stefan's constant $= 5.67 \times 10^{-8}$ W/m^{2}K^4

A body which in not a black body absorbs and hence emits less radiation then

For such a body, $u = e\sigma A T^4$

where e = emissivity (which is equal to absorptive power) which lies between 0 to 1

With the surroundings of temperature T_0 , net energy radiated by an area A per unit time..

$$\Delta u = u - u_0 = e\sigma A\,(T^4 - T_0^4)$$

$P_{emit} = \sigma e A T^4$

$P_{absorb} = \sigma e A T_0^4$

Newton's law of Cooling

For small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed.

$\frac{dQ}{dt} \propto (\theta - \theta_0)$, where θ and θ_0 are temperature corresponding to object and surroundings.

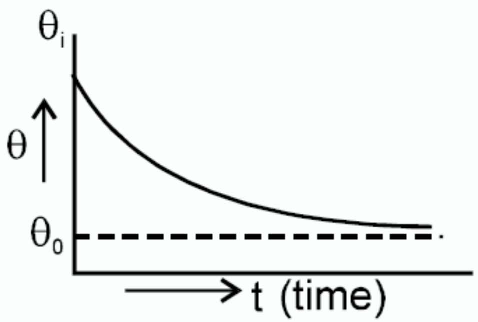
From above expression, $\boxed{\frac{d\theta}{dt} = -k(\theta - \theta_0)}$, where $k = \frac{4e\sigma\theta_0^3}{mc}A$

surface area

specific heat

$(\theta_f - \theta_0) = (\theta_i - \theta_0)\, e^{-kt}$

$\Rightarrow \theta_f = \theta_0 + (\theta_i - \theta_0)\, e^{-kt}$



$-\frac{d\theta}{dt} = K(\theta - \theta_0)$

Approximate method for applying Newton's law of cooling

$$\left\langle \frac{d\theta}{dt} \right\rangle = -k(<\theta> - \theta_0)$$

If θ_i & θ_f be initial and final temperature of the body then,

$$<\theta> = \frac{\theta_i + \theta_f}{2}$$

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\frac{\Delta\theta}{\Delta t} = -k(\theta_{avg} - \theta_0)$$

$$\frac{\theta_f - \theta_i}{\Delta t} = -k\left(\frac{\theta_i + \theta_f}{2} - \theta_0\right)$$

Example

A body at temperature 40°C is kept in a surrounding of constant temperature 20°C. It is observed that its temperature falls to 35°C in 10 minutes. Find how much more time will it take for the body to attain a temperature of 30°C.

Sol.

for the interval in which temperature falls from 40 to 35°C

$$<\theta> = \frac{40+35}{2} = 37.5^{\circ}C$$

$$\Rightarrow \quad \frac{(35^{\circ}C - 40^{\circ}C)}{10(\text{min})} = -K(37.5^{\circ}C - 20^{\circ}C)$$

$$\Rightarrow \quad K = \frac{1}{35}(\text{min}^{-1})$$

for the interval in which temperature falls from 35°C to 30°C

$$<\theta> = \frac{35+30}{2} = 32.5^{\circ}C$$

$$\Rightarrow \quad \frac{(30^{\circ}C - 35^{\circ}C)}{t} = -(32.5^{\circ}C - 20^{\circ}C)$$

$$\Rightarrow \quad \text{required time,} \quad t = \frac{5}{12.5} \times 35\,\text{min} = 14\text{ min}$$

$\theta_0 = 20^{\circ}C$

$40^{\circ}C \rightarrow 35^{\circ}C$ 10 min.

$\frac{35-40}{10} = -K(37.5 - 20)$ $\Rightarrow K = \frac{1}{35}$

$\frac{1}{\cancel{2}} = K\left(\frac{35}{\cancel{2}}\right)$

$35^{\circ}C \rightarrow 30^{\circ}C$ $t = ?$

$\frac{30-35}{\Delta t} = -\frac{1}{35}(32.5 - 20)$