If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where, I is 3×3 identity matrix, then the ordered pair (a, b) is equal to **(2015 Main)**

(a) $(2, -1)$ (b) $(-2, 1)$

(c) $(2, 1)$ (d) $(-2, -1)$

$a + \cancel{2b} + 4 = 0$ — (1)

$2a + 2 - \cancel{2b} = 0$ — (2)

$3a + 6 = 0$

$\boxed{a = -2}$

$\boxed{b = -1}$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & a+4+2b \\ & & 2a+2-2b \\ & & \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Q)

If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3×3 matrix A, then the sum of all values of α for which $\det(A) + 1 = 0$, is

(2019 Main, 12 April I)

(a) 0 (b) -1 (c) ~~1~~ (d) 2

$\boxed{B = A^{-1}}$

$|B| = |A^{-1}|$

$|B| = \frac{1}{|A|}$

$\boxed{|B| = -1}$ ✓

$\boxed{\det(A) = -1}$

$\begin{vmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{vmatrix}$

$2(-5-\alpha) - 1(15 - 2\alpha^2) = -1$

$-10 - 2\alpha - 15 + 2\alpha^2 = -1$

$2\alpha^2 - 2\alpha - 24 = 0$

$\Rightarrow \boxed{\alpha^2 - \alpha - 12 = 0}$

9) Let A and B be two invertible matrices of order 3×3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to **(2019 Main, 11 Jan II)**

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{1}{16}$ (d) 16

$\Rightarrow \boxed{|AB| = |A||B|}$

$\Rightarrow \boxed{|B^T| = |B|}$

$|A^{-1}| = \frac{1}{|A|}$

$|ABA^T| = 8$

$|A||B||A^T| = 8$

$\boxed{|A|^2 |B| = 8}$

$|A|^3 = 64$

$\boxed{|A| = 4}$

$|AB^{-1}| = 8$

$|A||B^{-1}| = 8$

$\boxed{\frac{|A|}{|B|} = 8}$

$|B| = \frac{1}{2}$

$|BA^{-1}B^T|$

$\frac{|B||A^{-1}||B^T|}{}$

$\frac{|B||B^T|}{|A|}$

$\frac{|B|^2}{|A|} = ?$

$\frac{1}{4 \times 4} = \frac{1}{16}$

9) If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \dfrac{\pi}{12}$, is equal to **(2019 Main, 9 Jan I)**

(a) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ 1

$A^{-50} = (A^{-1})^{50}$ (1)

$= (A^{50})^{-1}$ (2)

$\dfrac{50\pi}{12} = \left(4\pi + \dfrac{2\pi}{12}\right)$

$A^2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

$A^{50} = \begin{bmatrix} \cos 50\theta & -\sin 50\theta \\ \sin 50\theta & \cos 50\theta \end{bmatrix}$

$(A^{50})^{-1} = \begin{bmatrix} \cos 50\theta & \sin 50\theta \\ -\sin 50\theta & \cos 50\theta \end{bmatrix}$

$= \begin{bmatrix} \cos \pi/6 & \sin \pi/6 \\ -\sin \pi/6 & \cos \pi/6 \end{bmatrix}$

$= \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$

$|A| \neq 0$

9) Let M and N be two 3×3 non-singular skew-symmetric matrices such that $MN = NM$. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

(a) M^2 (b) $-N^2$ (c) $-M^2$ (d) MN (2011)

$M^T = -M$

$N^T = -N$

$\boxed{MN = NM}$

$(AB)^{-1} = B^{-1}A^{-1}$

$NN^{-1} = I$

$AI = \cancel{A}$

$M^2N^2(M^TN)^{-1}(MN^{-1})^T$

$M^2N^2\,N^{-1}(M^T)^{-1}((N^{-1})^T \cdot M^T)$

$M^2N(M^T)^{-1}(N^T)^{-1}(M^T)$

$M^2N(-M)^{-1}(-N)^{-1}(M^T)$

$M^{-1} \cdot N^{-1} \cdot M^T$

$M^2N(N.M)^{-1} \cdot M^T$

$M^2N(MN)^{-1} \cdot M^T$

$M^2NN^{-1}M^{-1}M^T$

$= -M^2$

If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to (2017

(a) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (b) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

(c) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (d) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix} = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix}$$

$$\begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix} = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\boxed{\text{adj} = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}}$$

9) If A is a 3×3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then BB^T is equal to (2014 Main)

(a) $I + B$ (b) I (c) B^{-1} (d) $(B^{-1})^T$

$AA^T = A^T A$

$B = A^{-1} A^T$

$BB^T = (A^{-1}A^T)(A^{-1}A^T)^T$

$= (A^{-1}A^T)((A^T)^T \cdot (A^{-1})^T$

$= (A^{-1}A^T)(A \cdot (A^T)^{-1})$

$= A^{-1}A\,A^T(A^T)^{-1}$

$= (I) \cdot I$

$= I$