THE TRANSPOSE OF A MATRIX: (CHANGING ROWS & COLUMNS)

Let A be any matrix. Then, $A = [a_{ij}]$ of order $m \times n$ \Rightarrow A^T or A' = [a_i] for $1 \le i \le n$ & $1 \le j \le m$ of order $n \times m$ Thus A^T is obtained by changing its rows into column and columns into row.

$$A = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 1 & -1 & 2 \end{bmatrix}_{n \times n}, \quad A^{T} = \begin{bmatrix} 2 & -1 \\ 3 & 2 & -1 \\ 1 & 3 & 2 \end{bmatrix}_{(n \times m)}$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -3 \\ 4 & 5 \end{bmatrix}_{3\times2}$$

$$A^{T} = \begin{bmatrix} 2 & -1 \\ 3 & 2 & -1 \\ 1 & 3 & 2 \end{bmatrix} \begin{pmatrix} n \times m \end{pmatrix}$$

$$A^{T} = \begin{bmatrix} 1 & 2 & 4 \\ 2 & -3 & -5 \end{bmatrix}_{2\times3}$$

Properties of transpose :

If A^T & B^T denote the transpose of A and B,

- (a) $(A+B)^T = A^T + B^T$; note that A & B have the same order.
- **(b)** $(\underline{A} \ \underline{B})^T = \underline{B}^T \ \underline{A}^T$ (Reversal law) A & B are conformable for matrix product AB

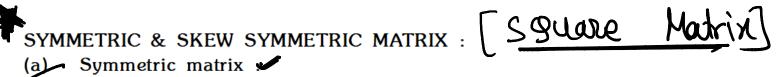
Note: In general: $(A_1, A_2, \dots, A_n)^T = A_n^T \cdot \dots \cdot A_n^T \cdot A_n^T$ (reversal law for transpose)

$$(A^T)^T = A$$

 $(kA)^T = kA^T$, k is a scalar.

$$(2A)^{T} = 2A^{T}$$

$$(\theta^{\tau})^{\tau} = A$$



A square matrix $\underline{A} = [a_{ij}]$ is said to be, symmetric if $a_{ij} = a_{ji} \forall i \& j$ (conjugate elements are equal).

Hence for symmetric matrix
$$A = A^T$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{22} & a_{23} & a_{33} \\ a_{33} & a_{32} & a_{33} \end{pmatrix}$$

Skew symmetric matrix : $(A = -A^7)$

Square matrix $A = [a_{ij}]$ is said to be skew symmetric if $a_{ij} = -a_{ji} \forall i \& j$ (the pair of conjugate elements are additive inverse of each other). For a skew symmetric matrix $A = -A^T$.

diagonal elements =
$$0$$

$$\begin{bmatrix}
q_{11} & = 0 \\
q_{22} & = 0
\end{bmatrix}$$

ADJOINT OF A SQUARE MATRIX :

Let $A = [a_{ij}]$ be a square matrix of order n and let C_{ij} be cofactor of a_{ij} in A then the adjoint of A, denoted by adjA is defined as the transpose of the cofactor matrix.

Then,
$$adjA = [C_{ij}]^T \Rightarrow adjA = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{23} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{5} & \frac{2}{0} & \frac{3}{4} \\ \frac{1}{2} & \frac{2}{6} & \frac{3}{7} \end{bmatrix} \text{ find (adj. A)} \qquad | adj(A) | = |A|^{3-1} = |A|^{2}$$

$$C = \begin{bmatrix} -\frac{1}{2}4 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & +\frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{2} \end{bmatrix} \qquad | adj(A) | = \begin{bmatrix} -\frac{1}{2}4 & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{2}7 & \frac{1}{3}7 & -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} &$$

If
$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$
 then $Adi A = \begin{bmatrix} s & -q \\ -r & p \end{bmatrix}$ e.g. $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ he $a\underline{dj}$. $A = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

Hence adjoint of a square matrix of order 2 can be easily obtained by interchanging the diagonal elements and changing the signs of the off diagonal elements.

PROPERTIES OF ADJOINT:

Theorem-1:
$$A (adj. A) = (adj. A).A = |A| I_n$$
 where A is any square matrix

(A) $\cdot 1_n$

Theorem-2: Let A be a non-singular matrix of order n. Then
$$|adj A| = |A|^{n-1} \quad (Note: \text{ in particular for } n = 3 \mid adj. A \mid = |A|^2)$$

Theorem-3: If A is a non singular square matrix, then

(a) adj
$$(adj A) = |A|^{n-2} A$$
 (b) $|adj (adj A)| = |A|^{(n-1)^2}$

PROPERTIES OF ADJOINT: