THE TRANSPOSE OF A MATRIX : (CHANGING ROWS & COLUMNS)

Let A be any matrix . Then , $A = [a_{ij}]$ of order $m \times n$

$\Rightarrow A^T$ or $A' = [a_{ji}]$ for $1 \le i \le n$ & $1 \le j \le m$ of order $n \times m$

Thus A^T is obtained by changing its rows into column and columns into row.

$$A = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 1 & -1 & 2 \end{bmatrix}_{n\times n}, \quad A^T = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 3 & 2 \end{bmatrix}_{(n\times m)}$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -3 \\ 4 & 5 \end{bmatrix}_{3\times 2} \quad A^T = \begin{bmatrix} 1 & 2 & 4 \\ 2 & -3 & -5 \end{bmatrix}_{2\times 3}$$

Properties of transpose :

If A^T & B^T denote the transpose of A and B,

(a) $(A+B)^T = A^T+B^T$; note that A & B have the same order.

(b) $(A \cdot B)^T = B^T A^T$ (Reversal law) A & B are conformable for matrix product AB

Note : In general : $(A_1 . A_2, \ldots\ldots\ldots\ldots A_n)^T = A_n^T . \ldots\ldots\ldots\ldots\ldots . A_2^T . A_1^T$ (reversal law for transpose)

(c) $(A^T)^T = A$

(d) $(kA)^T = kA^T$, k is a scalar.

$(A \cdot B)^T = B^T \cdot A^T$

$(A+B)^T = A^T + B^T$

A =
B = .

$(A^T)^T = A$

$(2A)^T = 2A^T$

SYMMETRIC & SKEW SYMMETRIC MATRIX : [Square Matrix]

(a) Symmetric matrix :

A square matrix $A = [a_{ij}]$ is said to be, symmetric if $a_{ij} = a_{ji}$ $\forall$ i & j (conjugate elements are equal).

Hence for symmetric matrix $A = A^T$.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & 0 \\ 2 & 0 & -3 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Skew symmetric matrix : $(A = -A^T)$

Square matrix $A = [a_{ij}]$ is said to be skew symmetric if $a_{ij} = -a_{ji}$ $\forall$ i & j (the pair of conjugate elements are additive inverse of each other). For a skew symmetric matrix $A = -A^T$.

diagonal elements = 0

$a_{11}, a_{22}, a_{33} = 0$

$$a_{ij} = -a_{ji}$$

$$a_{ii} = -a_{ii}$$

$$2a_{ii} = 0$$

$$a_{ii} = 0$$

$$\begin{bmatrix} a_{11} & 2 & -3 \\ -2 & a_{22} & 1 \\ 3 & -1 & a_{33} \end{bmatrix}$$

ADJOINT OF A SQUARE MATRIX :

Let $A = [a_{ij}]$ be a square matrix of order n and let C_{ij} be cofactor of a_{ij} in A then the adjoint of A, denoted by adjA, is defined as the transpose of the cofactor matrix.

Then, $\text{adj}A = [C_{ij}]^T \Rightarrow \text{adj}A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

co-factor

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 0 & 4 \\ 2 & 6 & 7 \end{bmatrix}$ find (adj. A).

$$|\text{adj}(A)| = |A|^{3-1} = |A|^2$$

$$C = \begin{bmatrix} -24 & -27 & 30 \\ 4 & 1 & -2 \\ 8 & 11 & -10 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} -24 & 4 & 8 \\ -27 & 1 & 11 \\ 30 & -2 & -10 \end{bmatrix}$$

If $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ then Adj A $= \begin{bmatrix} s & -q \\ -r & p \end{bmatrix}$ e.g. $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ the adj. $A = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

Hence adjoint of a square matrix of order 2 can be easily obtained by interchanging the diagonal elements and changing the signs of the off diagonal elements.

PROPERTIES OF ADJOINT :

Theorem-1 : $A(\text{adj. } A) = (\text{adj. } A).A = |A| I_n$ where A is any square matrix

$|A| \cdot I_n$

Theorem-2 : Let A be a non-singular matrix of order n. Then

$|\text{adj } A| = |A|^{n-1}$ (**Note:** in particular for n = 3 $|\text{adj. } A| = |A|^2$)

Theorem-3 : If A is a non singular square matrix, then

(a) $\text{adj}(\text{adj } A) = |A|^{n-2} A$ (b) $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

PROPERTIES OF ADJOINT :

- $\text{adj}(AB) = (\text{adj } B)(\text{adj } A)$
- $\text{adj } A^T = (\text{adj } A)^T$
- $\text{adj}(KA) = K^{n-1}(\text{adj } A)$, K is a scalar

$\text{adj}(A^T) = (\text{adj}(A))^T$

$n \rightarrow$ order of Matrix A

A

$\text{adj}(3A) = 3^{n-1}\text{adj}(A)$

$n \rightarrow 3$

$= 3^2 \text{adj}(A)$

$= 9 \cdot \text{adj}(A)$

$AI = A$

$\begin{bmatrix} x & y \\ z & w \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$