

Matrices

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Definition : Rectangular array of m n numbers enclosed by a pair of brackets

e.g. $A = \begin{bmatrix} 2 & 3 & 7 \\ 1 & -1 & 5 \end{bmatrix}$. Unlike determinants it has no value and are subjected to certain rules of operations.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

or

$$\Rightarrow \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

$m \rightarrow \text{rows}$

$n \rightarrow \text{columns}$

Order = $m \times n$

Abbreviated as : $A = [a_{ij}]$ $1 \leq i \leq m$; $1 \leq j \leq n$, i denotes the row and j denotes the column is called a matrix of order $m \times n$. The elements of a matrix may be real or complex numbers. If all the elements of a matrix are real, the matrix is called real matrix.

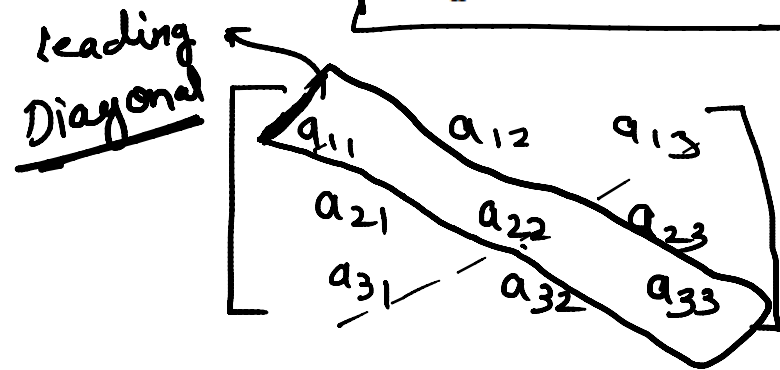
→ Square Matrix : (Order n)

If number of rows = number of columns \Rightarrow a square matrix. A real square matrix all whose elements are positive is called a positive matrices. Such matrices have application in mechanics and economics.

$$\text{Order} = \textcircled{n \times n}$$

The elements $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$ are called Diagonal Elements. The line along which the diagonal elements lie is called "Principal or Leading" diagonal.

The quantity $\sum a_{ii} = \text{trace of the matrix}$ written as, $(t_r) A = t_r(A)$



— Square Matrix

$$\textcircled{a_{ii}}$$

$$a_{11} + a_{22} + a_{33} = \text{trace of Matrix}(A)$$

New Matrix =

Square Matrix

(Triangular Matrix)

✓ Upper triangular matrix

$$\text{if } a_{ij} = 0 \quad \forall i > j$$

$$A = \begin{bmatrix} \times & \times & \times \\ 0 & \times & \times \\ 0 & 0 & \times \end{bmatrix};$$

Lower triangular matrix

$$\text{if } a_{ij} = 0 \quad \forall i < j$$

$$A = \begin{bmatrix} \times & 0 & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{bmatrix}$$

Diagonal Matrix

atleast one, $a_{ii} \neq 0$ and $a_{ij} = 0$ if $i \neq j$

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

abbreviated as dia ($d_1, d_2, d_3, \dots, d_n$)

Scalar matrix

if $d_1 = d_2 = d_3, \dots = a \neq 0$

$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

Unit matrix

if $d_1 = d_2 = d_3, \dots = 1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

(Identity Matrix)

3 \rightarrow order of matrix

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

EQUALITY OF MATRICES :

The matrices $\underline{A} = [a_{ij}]$ & $B = [b_{ij}]$ are equal if,

- (i) both have the same order. (ii) $a_{ij} = b_{ij}$ for each pair of i & j .

Hence two matrices are equal if and only if one is a duplicate of the other.

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

ALGEBRA OF MATRICES :

Addition : $A + B = [a_{ij} + b_{ij}]$ where A & B are of the same type. (same order)

If A and B are square matrices of the same type then, $t_r(A + B) = t_r(A) + t_r(B)$

① **Addition of matrices is commutative**

i.e. $A + B = B + A$ where (A and B) must have the same order

$$\begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 0 & -2 \end{bmatrix} \checkmark$$

MULTIPLICATION OF A MATRIX BY A SCALAR: Constant No. $\rightarrow 2, 3,$

$$\text{If } A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} ; \quad \underline{kA} = \begin{bmatrix} ka & kb & kc \\ kb & kc & ka \\ kc & ka & kb \end{bmatrix} \text{ i.e. } \underline{k(A+B)} = \underline{kA} + \underline{kB}$$

$$A = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}, \quad 3A = \begin{bmatrix} 9 & 6 \\ -3 & 6 \end{bmatrix}$$

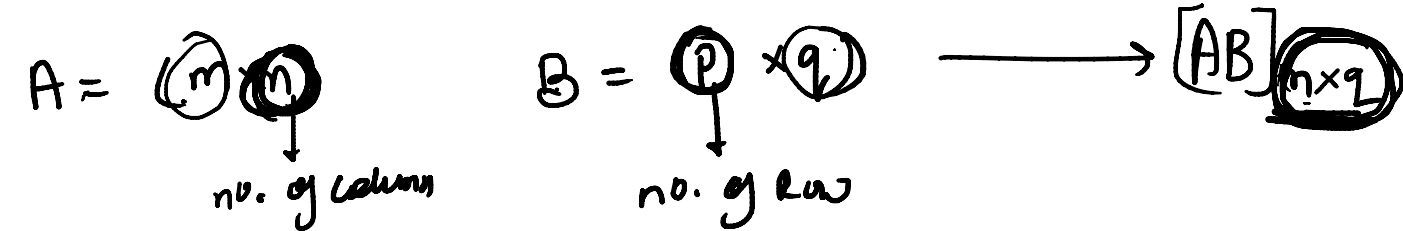
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MULTIPLICATION OF MATRICES : (ROW BY COLUMN)

AB exists if, $A = m \times n$ & $B = n \times p$
 $\quad \quad \quad \underline{2 \times 3} \quad \quad \quad \underline{3 \times 3}$ $3 \times \textcircled{3} \quad \textcircled{3} \times 3$

A B is matrix of 2×3

Note that, AB exists, but BA does not $\Rightarrow \boxed{AB \neq BA} \quad \times$



If $\boxed{n = p}$, then A.B will exist

- C.B ✓

B.C ✓

$$(\underline{A+B})^2 = \underline{A^2 + B^2 + AB + BA}$$

$$(\underline{A+B})(\underline{A+B}) = A^2 + \textcircled{BA + AB} + B^2$$