Matrices

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Definition **:** Rectangular array of m n numbers enclosed by a pair of brackets e.g. $A = \begin{bmatrix} 2 & 3 & 7 \\ 1 & -1 & 5 \end{bmatrix}$ (2×3). Unlike determinants it has no value and are subjected to certain rules of operations.

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \ldots\ldots & a_{mn} \end{bmatrix} \quad \text{or} \quad \begin{pmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \ldots\ldots & a_{mn} \end{pmatrix}$$

m → rows

n → columns

$\boxed{\text{order} = m \times n}$

Abbreviated as : $A = [a_{ij}]$ $1 \le i \le m$; $1 \le j \le n$, i denotes the row and j denotes the column is called a matrix of order $m \times n$. The elements of a matrix may be real or complex numbers. If all the elements of a matrix are real, the matrix is called real matrix.

Square Matrix : (Order n)

If number of rows = number of columns $\Rightarrow$ a square matrix. A real square matrix all whose elements are positive is called a positive matrices. Such matrices have application in mechanics and economics.

Order = $n \times n$

The elements a_{11}, a_{22}, a_{33}, ... a_{mn} are called **Diagonal Elements**. The line along which the diagonal elements lie is called **"Principal or Leading"** diagonal.

The quantity $\boxed{\Sigma\, a_{ii} = \text{trace of the matrix written as, } (t_r)A = t_r(A)}$

leading Diagonal

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Square Matrix

a_{ii}

$a_{11} + a_{22} + a_{33}$ = trace of Matrix(A)

Null Matrix =

Square Matrix

- (Triangular Matrix)
  - Upper triangular matrix

    $\boxed{\text{if } a_{ij} = 0 \ \forall\, i > j}$

    $$A = \begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{bmatrix} ;$$

  - Lower triangular matrix

    $\boxed{\text{if } a_{ij} = 0 \ \forall\, i < j}$

    $$A = \begin{bmatrix} x & 0 & 0 \\ x & x & 0 \\ x & x & x \end{bmatrix}$$

- Diagonal Matrix

  atleast one, $a_{ii} \neq 0$ and $a_{ij} = 0$ if $i \neq j$

  $$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \qquad \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

  abbreviated as dia $(d_1, d_2, d_3 \ldots\ldots d_n)$

  - Scalar matrix

    if $d_1 = d_2 = d_3 \ldots\ldots = a \neq 0$

    $$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

  - Unit matrix (Identity Matrix)

    if $d_1 = d_2 = d_3 \ldots\ldots = 1$

    $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

    3 → order of Matrix

1 0 0

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

EQUALITY OF MATRICES :

The matrices $A = [a_{ij}]$ & $B = [b_{ij}]$ are equal if ,

(i) both have the same order. (ii) $a_{ij} = b_{ij}$ for each pair of i & j .

Hence two matrices are equal if and only if one is a duplicate of the other.

$$\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

ALGEBRA OF MATRICES :

Addition : $A + B = [a_{ij} + b_{ij}]$ where A & B are of the same type . (same order)

If A and B are square matrices of the same type then, $t_r (A + B) = t_r (A) + t_2 (B)$

1 **Addition of matrices is commutative ,**

i.e. $A + B = B + A$ where A and B must have the same order

$$\begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 0 & -2 \end{bmatrix}$$

MULTIPLICATION OF A MATRIX BY A SCALAR :

Constant

NO. $\to$ 2, 3,

If $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$; $kA = \begin{bmatrix} ka & kb & kc \\ kb & kc & ka \\ kc & ka & kb \end{bmatrix}$ i.e. $k(A + B) = kA + kB$

$A = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$, $3A = \begin{bmatrix} 9 & 6 \\ -3 & 6 \end{bmatrix}$

MULTIPLICATION OF MATRICES : (ROW BY COLUMN)

AB exists if, $A = m \times n$ & $B = n \times p$

2×3 3×3

3×3 3×3

A B is matrix of 2×3

Note that, AB exists, but BA does not $\Rightarrow$ $\boxed{AB \neq BA}$ ✗

$A = m \times n$ $B = p \times q$ $\longrightarrow$ $[AB]_{m \times q}$

no. of column no. of row

If $\boxed{n = p}$, then A·B will exist

$\boxed{\begin{matrix} C \cdot B \checkmark \\ B \cdot C \checkmark \end{matrix}}$

$$(A+B)^2 = A^2 + B^2 + AB + BA$$

$$(A+B)(A+B) = A^2 + BA + AB + B^2$$