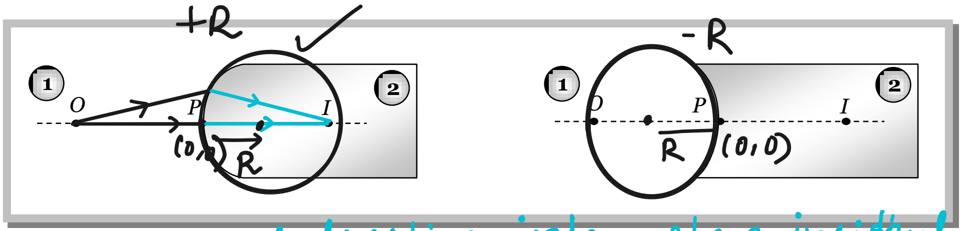
Refraction From Curved Surface



ref. index of that medium where refracted light is present

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

refractive index where incident light is present

μ_1 = Refractive index of the medium from which light rays are coming (from object).

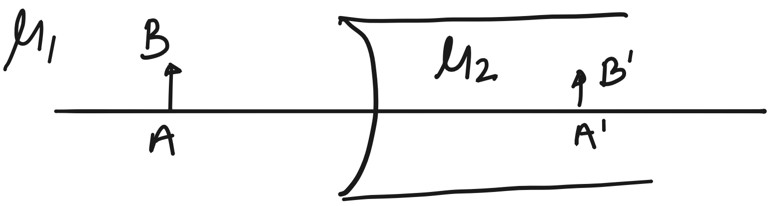
μ_2 = Refractive index of the medium in which light rays are entering.

u = Distance of object, v = Distance of image, R = Radius of curvature

Refraction formula : $\frac{\mu_2 - \mu_1}{R} = \frac{\mu_2}{v} - \frac{\mu_1}{u}$ (use sign convention while solving the problem)

Note : ❑ Real image forms on the side of a refracting surface that is opposite to the object, and virtual image forms on the same side as the object.

❑ Lateral (Transverse) magnification $m = \frac{I}{O} = \frac{\mu_1 v}{\mu_2 u}$.



$$\frac{A'B'}{AB} = \frac{I}{O} = \frac{v}{u}\frac{\mu_1}{\mu_2}$$

Specific Example

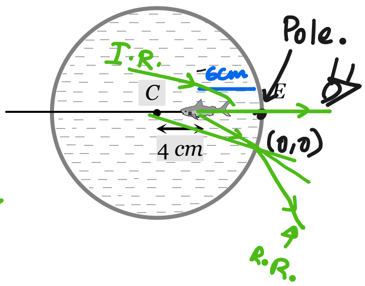
In a thin spherical fish bowl of radius 10 *cm* filled with water of refractive index 4/3 there is a small fish at a distance of 4 *cm* from the centre *C* as shown in figure. Where will the image of fish appears, if seen from *E*

(a) 5.2 *cm* (b) 7.2 *cm* (c) 4.2 *cm* (d) 3.2 *cm*

Solution : (a) By using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

where $\mu_1 = \frac{4}{3}$, $\mu_2 = 1$, $u = -6\,cm$, $v = ?$

On putting values $v = -5.2\,cm$



$u = +6cm$

$R = 10cm$

$\mu_1 = 4/3$

$\mu_2 = 1$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} - \frac{4/3}{-6} = \frac{1 - 4/3}{-10}$$

$$\frac{1}{v} + \frac{2}{9} = +\frac{1}{30}$$

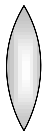
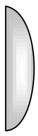
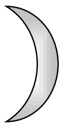
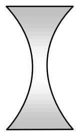
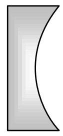
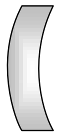
$$\frac{1}{v} = -\frac{1}{30} + \frac{2}{9} = \frac{+6 \mp 40}{180}$$

$$V = \frac{-180}{34}\,cm. = -5.29\,cm.$$

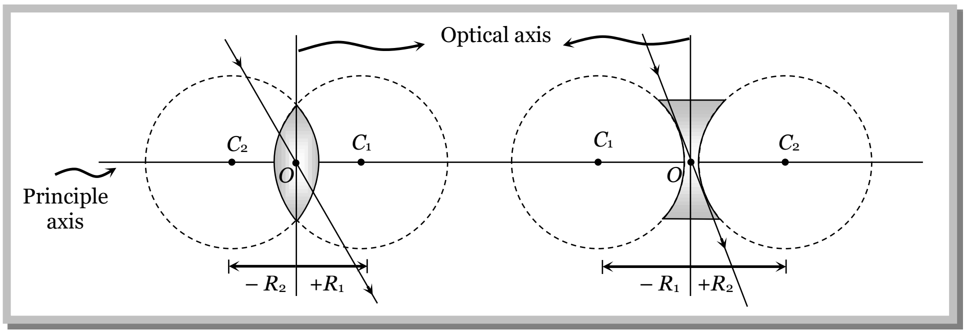
Lens

Lens is a transparent medium bounded by two refracting surfaces, such that at least one surface is spherical.

(1) **Type of lenses**

Convex lens (Converges the light rays)			Concave lens (Diverges the light rays)		
					
Double convex	Plano convex	Concavo convex	Double concave	Plane concave	Convexo concave
Thick at middle			Thin at middle		
It forms real and virtual images both			It forms only virtual images		

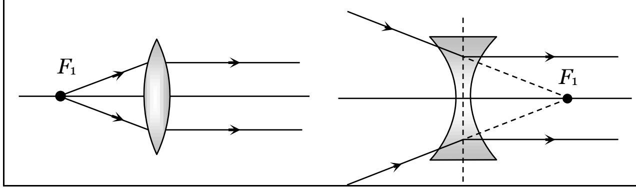
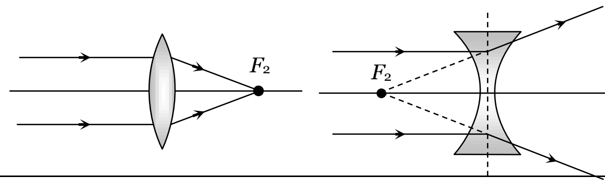
(2) **Some definitions**



C_1, C_2 – Centre of curvature,
R_1, R_2 – Radii of curvature

(i) **Optical centre (*O*) :** A point for a given lens through which light ray passes undeviated (Light ray passes undeviated through optical centre).

(ii) **Principle focus**

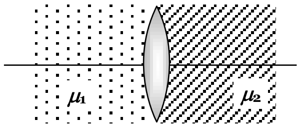
First principle focus	**Second principle focus**
	

Note : ❑ Second principle focus is the principle focus of the lens.

❑ When medium on two sides of lens is same then $|F_1| = |F_2|$.

❑ If medium on two sides of lens are not same then the ratio of two focal lengths

$$\frac{f_1}{f_2} = \frac{\mu_1}{\mu_2}$$



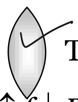
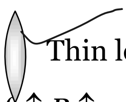
(iii) **Focal length (*f*) :** Distance of second principle focus from optical centre is called focal length

$$f_{\text{convex}} \rightarrow \text{positive}, \ f_{\text{concave}} \rightarrow \text{negative}, \ f_{\text{plane}} \rightarrow \infty$$

(iv) **Aperture :** Effective diameter of light transmitting area is called aperture. Intensity of image $\propto$ (Aperture) ^{2}

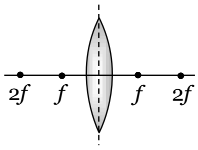
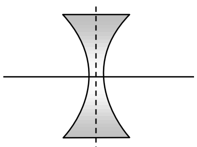
(v) **Power of lens (*P*) :** Means the ability of a lens to converge the light rays. Unit of power is Diopter (*D*).

$$P = \frac{1}{f(m)} = \frac{100}{f(cm)}; \ P_{\text{convex}} \rightarrow \text{positive}, \ P_{\text{concave}} \rightarrow \text{negative}, \ P_{\text{plane}} \rightarrow \text{zero} .$$

Note : ❑  Thick lens  Thin lens

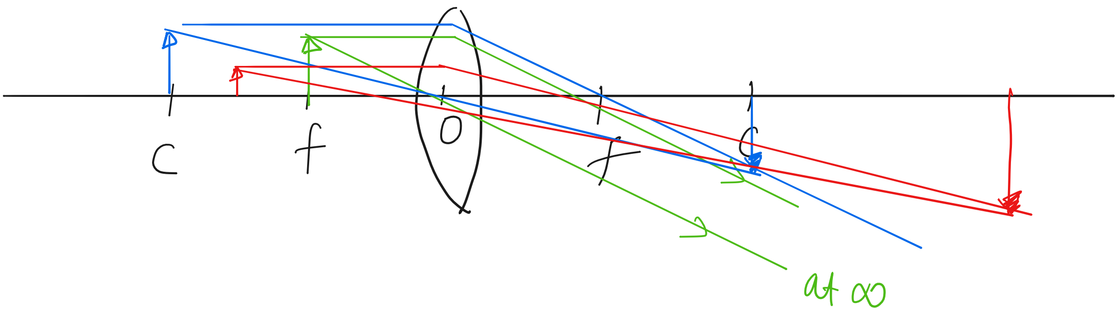
$P\uparrow f\downarrow R\downarrow$ $\qquad$ $P\downarrow f\uparrow R\uparrow$

(3) **Image formation by lens**

Lens	**Location of the object**	**Location of the image**	**Nature of image**		
			Magnification	**Real / virtual**	**Erect / inverted**
Convex	At infinity i.e. $u = \infty$	At focus i.e. $v = f$	$m < 1$ diminished	Real	Inverted
	Away from $2f$ i.e. $(u > 2f)$	Between f and $2f$ i.e. $f < v < 2f$	$m < 1$ diminished	Real	Inverted
	At $2f$ or $(u = 2f)$	At $2f$ i.e. $(v = 2f)$	$m = 1$ same size	Real	Inverted
	Between f and $2f$ i.e. $f < u < 2f$	Away from $2f$ i.e. $(v > 2f)$	$m > 1$ magnified	Real	Inverted
	At focus i.e. $u = f$	At infinity i.e. $v = \infty$	$m = \infty$ magnified	Real	Inverted
	Between optical centre and focus, $u < f$	At a distance greater than that of object $v > u$	$m > 1$ magnified	Virtual	Erect
Concave 	At infinity i.e. $u = \infty$	At focus i.e. $v = f$	$m < 1$ diminished	Virtual	Erect
	Anywhere between infinity and optical centre	Between optical centre and focus	$m < 1$ diminished	Virtual	Erect

Note : ❑ Minimum distance between an object and it's real image formed by a convex lens is $4f$.

❑ Maximum image distance for concave lens is it's focal length.



(4) **Lens maker's formula**

The relation between f, μ, R_1 and R_2 is known as lens maker's formula and it is $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

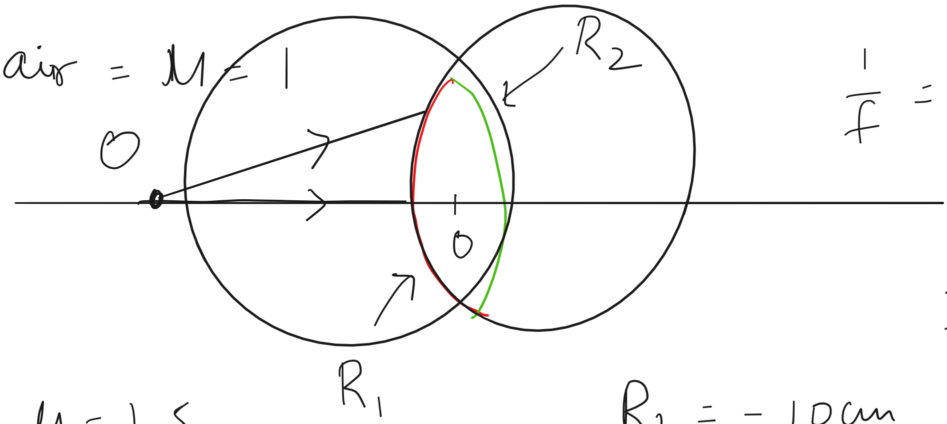
H.W. H.W.

Equiconvex lens	Plano convex lens	Equi concave lens	Plano concave lens
$R_1=R$ and $R_2=-R$ $f=\frac{R}{2(\mu-1)}$ for $\mu=1.5$, $f=R$	$R_1=\infty$, $R_2=-R$ $R_1=\infty$ $f=\frac{R}{(\mu-1)}$ for $\mu=1.5$, $f=2R$	$R_1=-R$, $R_2=+R$ $f=-\frac{R}{2(\mu-1)}$ for $\mu=1.5$ $f=-R$	$R_1=\infty$, $R_2=R$ $f=\frac{R}{2(\mu-1)}$ for $\mu=1.5$, $f=-2R$

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

air $=\mu=1$



$$\frac{1}{f}=(1.5-1)\left(\frac{1}{10}-\left(\frac{-1}{10}\right)\right)$$

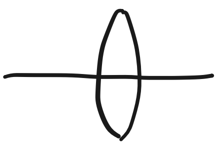
$$\frac{1}{f}=0.5\left(\frac{2}{10}\right)$$

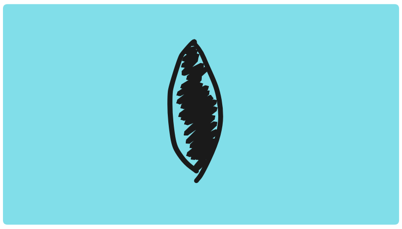
$\mu=1.5$

$R_2=-10cm$

Suppose $R_1=R_2=10cm.$ $R_1=+10cm.$

$f=10cm$





$$\boxed{\frac{f_l}{f_a}=\frac{\frac{\mu_g}{\mu_a}-1}{\frac{\mu_g}{\mu_l}-1}}$$

(5) **Lens in a liquid**

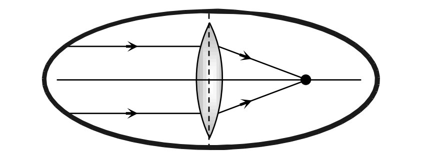
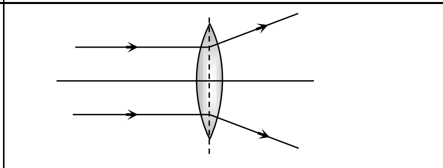
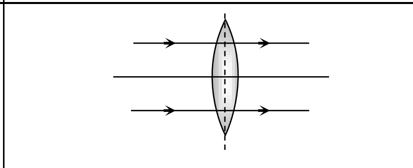
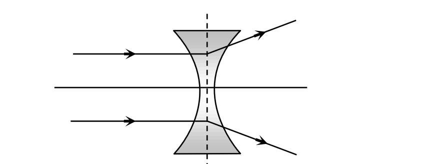
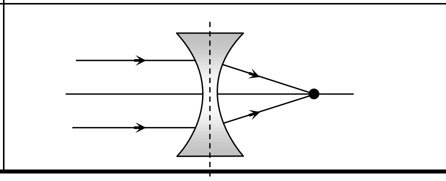
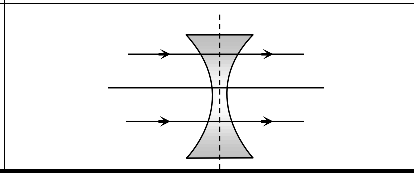
Focal length of a lens in a liquid (f_l) can be determined by the following formula

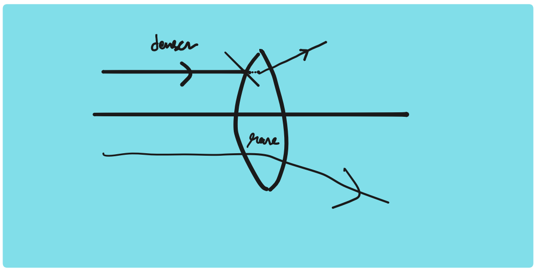
$$\frac{f_l}{f_a} = \frac{({}_a\mu_g - 1)}{({}_l\mu_g - 1)}$$ (Lens is supposed to be made of glass).

Note :
- Focal length of a glass lens $(\mu = 1.5)$ is f in air then inside the water it's focal length is $4f$.
- In liquids focal length of lens increases (↑) and it's power decreases (↓).

(6) **Opposite behaviour of a lens**

In general refractive index of lens $(\mu_L) >$ refractive index of medium surrounding it (μ_M).

$\mu_L > \mu_M$	$\mu_L < \mu_M$	$\mu_L = \mu_M$
		
		



$\mu_M > \mu_L$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

(7) **Lens formula and magnification of lens**

(i) Lens formula : $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$; (use sign convention)

(ii) Magnification : The ratio of the size of the image to the size of object is called magnification.

$$m = \frac{I}{O} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$$

(a) Transverse magnification : $m = \frac{I}{O} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$ (use sign convention while solving the problem)

IIT

(b) Longitudinal magnification : $m = \frac{I}{O} = \frac{v_2 - v_1}{u_2 - u_1}$. For very small object $m = \frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f+u}\right)^2 = \left(\frac{f-v}{f}\right)^2$

Same

(c) Areal magnification : $m_s = \frac{A_i}{A_o} = m^2 = \left(\frac{f}{f+u}\right)^2$, ($A_i$ = Area of image, A_o = Area of object)

$$A_i = m^2 A_o$$

(8) **Relation between object and image speed**

If an object move with constant speed (V_o) towards a convex lens from infinity to focus, the image will move slower in the beginning and then faster. Also $V_i = \left(\frac{f}{f+u}\right)^2 . V_o$

$$V_i = (m)^2 V_o$$

(9) **Focal length of convex lens by displacement method**

(ii) Focal length of the lens $\boxed{f = \dfrac{D^2 - x^2}{4D} = \dfrac{x}{m_1 - m_2}}$ IIT

where $m_1 = \dfrac{I_1}{O}$ and $m_2 = \dfrac{I_2}{O}$

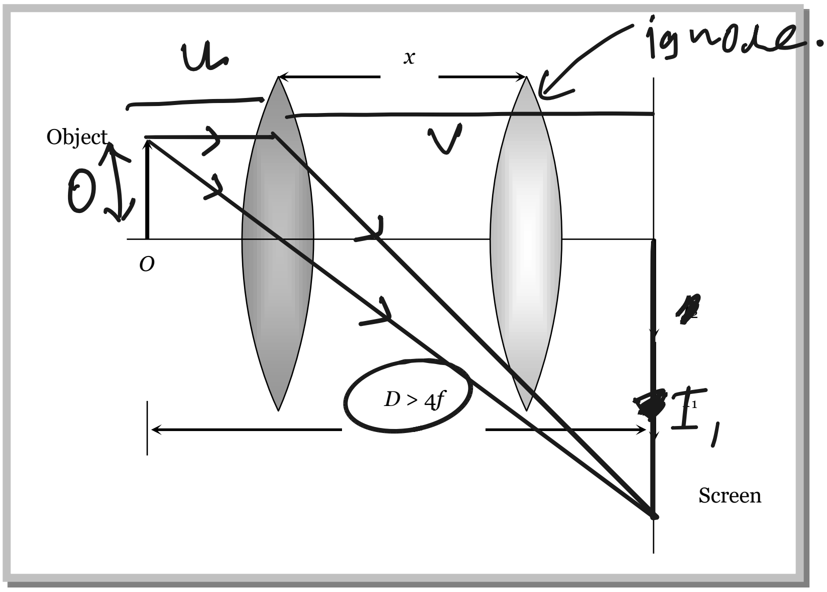
D > 4f

IIT

$f = \dfrac{D^2 - x^2}{4D}$

distance by which lens moved $x = v - u$

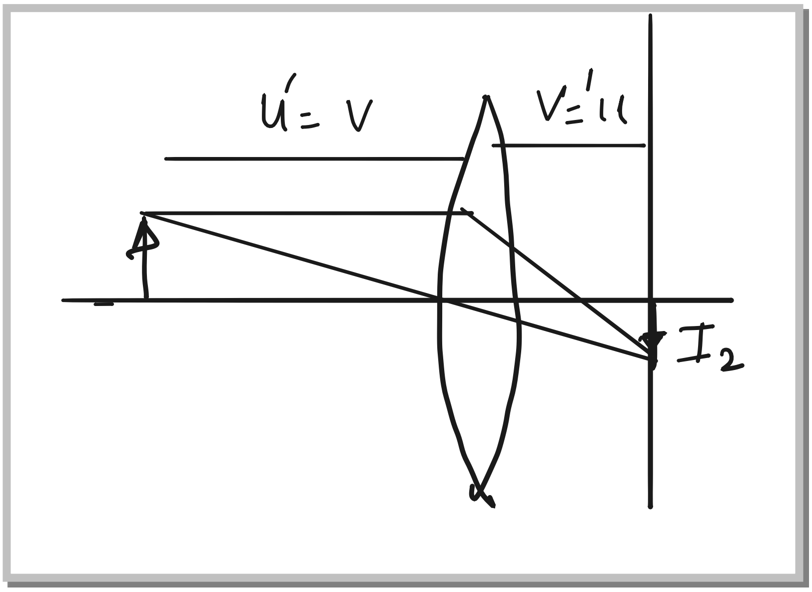
(iii) Size of object $O = \sqrt{I_1 . I_2}$



$m_1 = \dfrac{I_1}{O} = \dfrac{v}{u}$

NEET

$\boxed{O = \sqrt{I_1 I_2}}$



$m_2 = \dfrac{I_2}{O} = \dfrac{v'}{u'} = \dfrac{u}{v}$

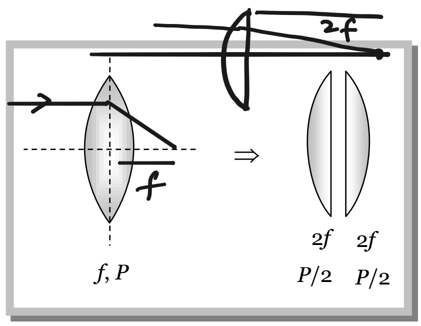
$\dfrac{I_1}{O} \times \dfrac{I_2}{O} = \dfrac{\cancel{v}}{u} \times \dfrac{\cancel{u}}{\cancel{v}}$

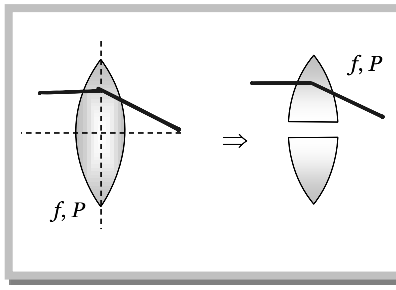
$\boxed{O = \sqrt{I_1 I_2}}$

(10) **Cutting of lens**

(i) A symmetric lens is cut along optical axis in two equal parts. Intensity of image formed by each part will be same as that of complete lens.

(ii) A symmetric lens is cut along principle axis in two equal parts. Intensity of image formed by each part will be less compared as that of complete lens.(aperture of each part is $\frac{1}{\sqrt{2}}$ times that of complete lens)





(11) **Combination of lens**

(i) For a system of lenses, the net power, net focal length and magnification given as follows :

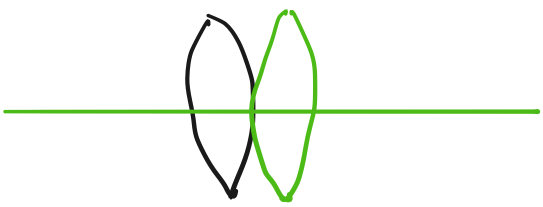
$$P = P_1 + P_2 + P_3 ...., \qquad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ...,$$

$$m = m_1 \times m_2 \times m_3 \times ....$$

(ii) In case when two thin lens are in contact : Combination will behave as a lens, which have more power or lesser focal length.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \quad F = \frac{f_1 f_2}{f_1 + f_2} \quad \text{and} \quad P = P_1 + P_2$$

(iii) If two lens of equal focal length but of opposite nature are in contact then combination will behave as a plane glass plate and $F_{\text{combination}} = \infty$



$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

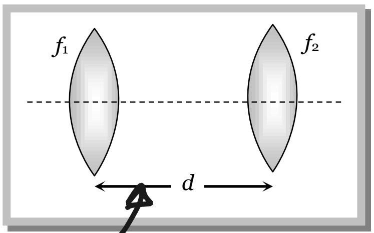
put values with sign.

$P_{eq} = P_1 + P_2$

$f_{convex} = +ve$ concave -ve.

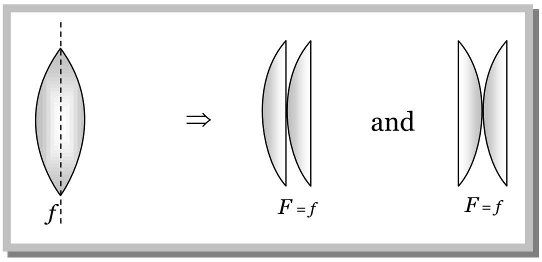
(iv) When two lenses are placed co-axially at a distance d from each other then equivalent focal length (F).

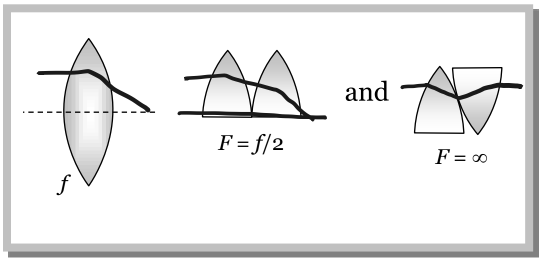
$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2} \quad \text{and} \quad P=P_1+P_2-dP_1P_2$$



$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

(v) Combination of parts of a lens :





(12) **Silvering of lens**

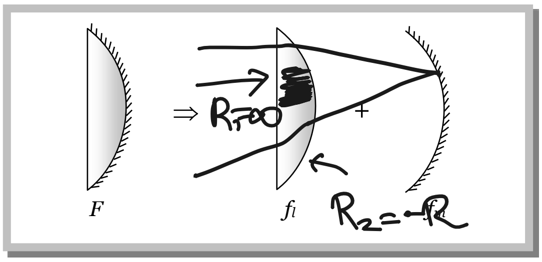
On silvering the surface of the lens it behaves as a mirror. The focal length of the silvered lens is $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$

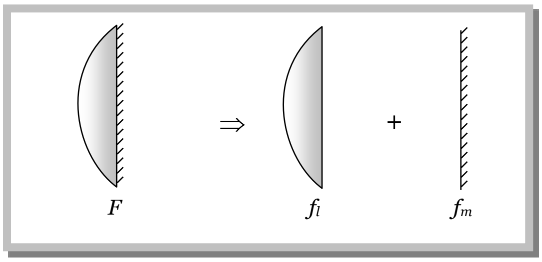
where f_l = focal length of lens from which refraction takes place (twice)

f_m = focal length of mirror from which reflection takes place.

$$\boxed{\frac{1}{f_{eq}} = \frac{2}{f_l} + \frac{1}{f_m}}$$

(i) Plano convex is silvered



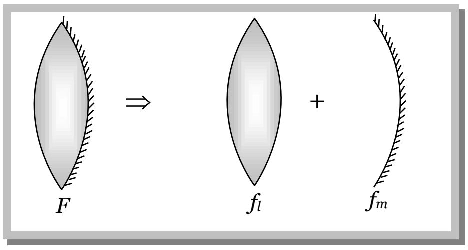


$f_m = \frac{R}{2}$, $f_l = \frac{R}{(\mu-1)}$ so $F = \frac{R}{2\mu}$ $\qquad$ $f_m = \infty$, $f_l = \frac{R}{(\mu-1)}$ so $F = \frac{R}{2(\mu-1)}$

(ii) Double convex lens is silvered

Since $f_l = \frac{R}{2(\mu-1)}$, $f_m = \frac{R}{2}$

So $F = \frac{R}{2(2\mu-1)}$



$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{\infty} + \frac{1}{R}\right)$$

$f_m = \frac{R}{2}$

$$\frac{1}{f_l} = \frac{\mu-1}{R}$$

$$\frac{1}{F_{eq}} = \frac{2}{f_l} + \frac{1}{f_m}$$

$1 \qquad 2(\mu-1) \qquad 2$

Microscope

NEET 100% ! mains imp.

It is an optical instrument used to see very small objects. It's magnifying power is given by

$$m = \frac{\text{Visual angle with instrument } (\beta)}{\text{Visual angle when object is placed at least distance of distinct vision } (\alpha)}$$

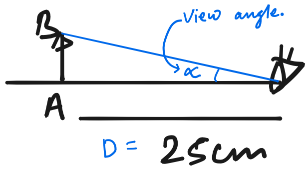
(1) **Simple miscroscope**

(i) It is a single convex lens of lesser focal length.

(ii) Also called magnifying glass or reading lens.

(iii) Magnification's, when final image is formed at D and ∞ (*i.e.* m_D and m_∞)

$$m_D = \left(1 + \frac{D}{f}\right)_{\max} \quad \text{and } m_\infty = \left(\frac{D}{f}\right)_{\min}$$



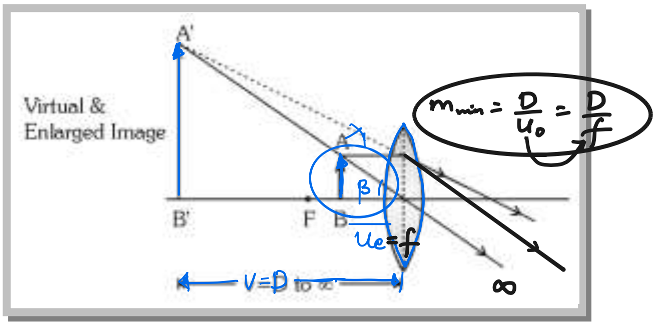
$\tan\alpha = \frac{AB}{D}$

$\simeq \alpha = \frac{AB}{D}$ — (A)

Note : $\boxed{m_{\max} - m_{\min} = 1}$

$m_{max} = 1 + \frac{D}{f}$

$m_{min} = \frac{D}{f}$



$\tan\beta = \frac{A'B'}{D} = \frac{AB}{u_e}$

$\beta = \frac{A'B'}{D} = \frac{AB}{u_e}$.

$m = \frac{\beta}{\alpha} = \frac{\frac{AB}{u_e}}{\frac{AB}{D}} = \frac{D}{u_0} = D\left[\frac{1}{f} + \frac{1}{D}\right] = \boxed{1 + \frac{D}{f}}$ m_{max}

$m = \frac{D}{u_0}$ one more form is there imp. for all instruments

$\frac{1}{f} = \frac{1}{v} - \frac{1}{-u_0}$

$\frac{1}{f} = -\frac{1}{D} + \frac{1}{u_0}$

$v = -D, \; u = -u_0$

$\frac{1}{u_0} = \frac{1}{f} + \frac{1}{D}$

(2) **Compound microscope**

(i) Consist of two converging lenses called objective and eye lens.

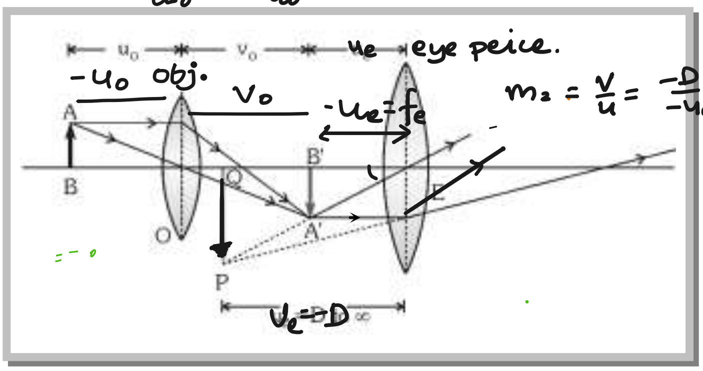
(ii) $f_{\text{eye lens}} > f_{\text{objective}}$ and $(\text{diameter})_{\text{eye lens}} > (\text{diameter})_{\text{objective}}$

(iii) Final image is magnified, virtual and inverted.

(iv) u_0 = Distance of object from objective (o), v_0 = Distance of image $(A'B')$ formed by objective from objective, u_e = Distance of $A'B'$ from eye lens, v_e = Distance of final image from eye lens, f_o = Focal length of objective, f_e = Focal length of eye lens.

Magnification : $m_D = -\dfrac{v_0}{u_0}\left(1+\dfrac{D}{f_e}\right)$

$m_1 = \dfrac{v_0}{-u_0} = -\dfrac{v_0}{u_0}$ — A



$m_2 = \dfrac{v}{u} = \dfrac{-D}{-u_e} = \dfrac{D}{u_e}$

$$m = m_1 \times m_2 = -\frac{v_0}{u_0} \times \frac{D}{u_e}$$

$$\boxed{m_{min} = -\frac{v_0}{u_0} \times \frac{D}{f_e}}$$

$$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

$$= \frac{-v_0}{u_0} D \left[\frac{1}{f_e} + \frac{1}{D}\right]$$

$$m_{max} = -\frac{v_0}{u_0}\left[1 + \frac{D}{f_e}\right]$$

Short cut

$$m = \frac{-L}{f}\left(1 + \frac{D}{f_e}\right)$$

$L = v_0 + u_e$

tube length.

Telescope

By telescope distant objects are seen.

(1) **Astronomical telescope**

(i) Used to see heavenly bodies.

(ii) $f_{\text{objective}} > f_{\text{eyelens}}$ and $d_{\text{objective}} > d_{\text{eye lens}}$.

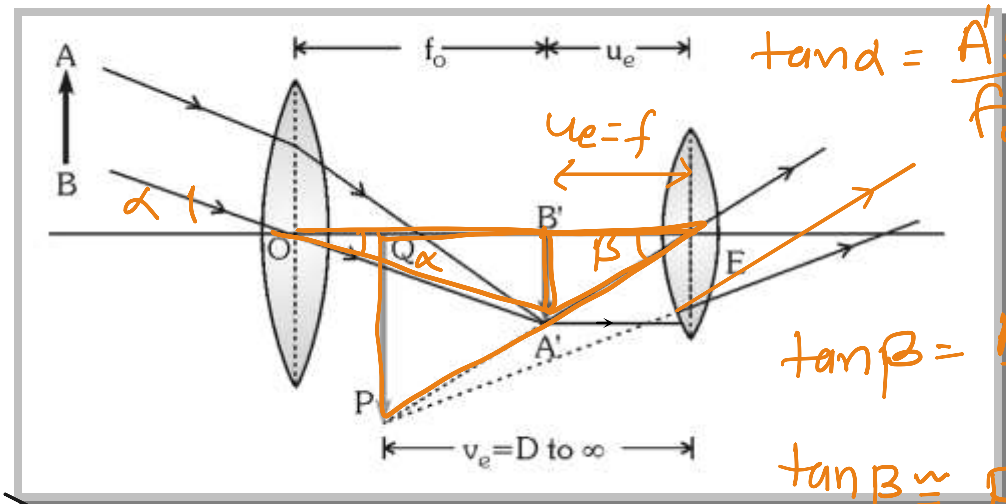
(iii) Intermediate image is real, inverted and small.

(iv) Final image is virtual, inverted and small.

(v) Magnification : $m_D = -\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$ and $m_\infty = -\frac{f_o}{f_e}$

(vi) Length : $L_D = f_0 + u_e = f_0 + \frac{f_e D}{f_e + D}$ and $L_\infty = f_0 + f_e$

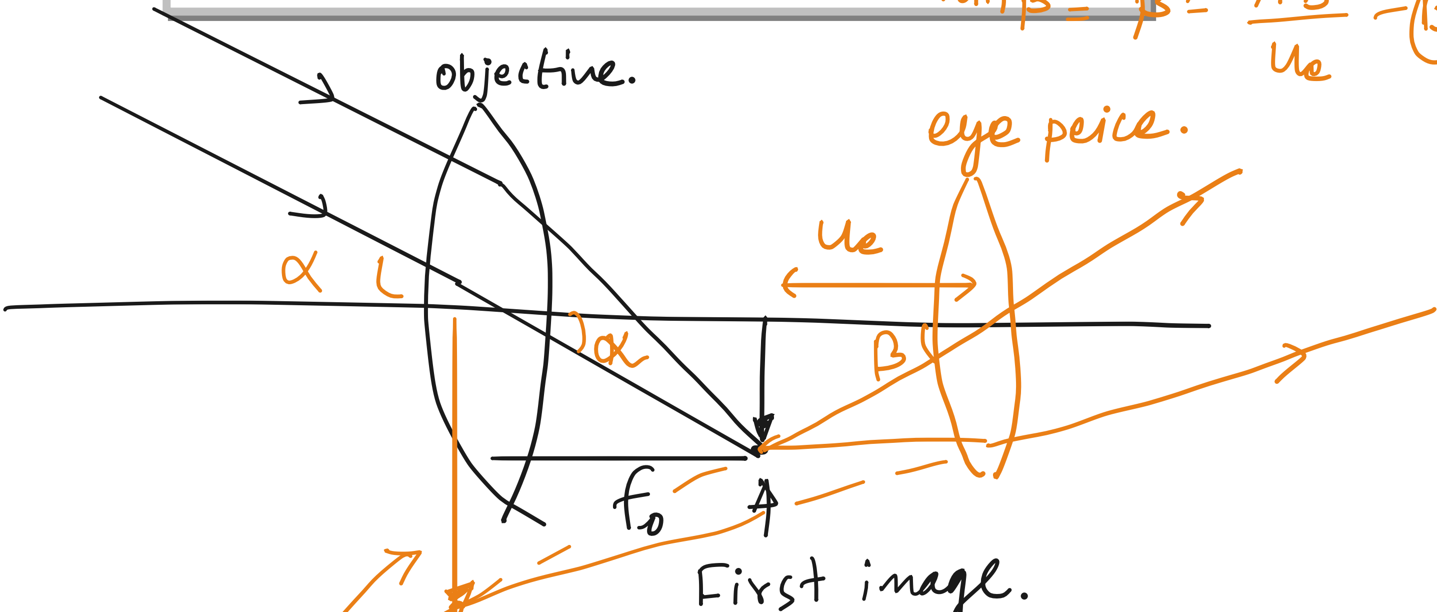
$\tan\alpha \simeq \alpha = \frac{A'B'}{f_o}$ — A



$\tan\alpha = \frac{A'B'}{f_o}$ — A

$\tan\beta = \frac{A'B'}{u_e} = \frac{A''B''}{-D}$

$\tan\beta \simeq \beta = \frac{A'B'}{u_e}$ — B



Final image. $v_e = -D$

tube length. $\rightarrow L = f_o + |u_e|$

$$m = \frac{\beta}{\alpha} = \frac{\frac{\cancel{A'B'}}{u_e}}{\frac{\cancel{A'B'}}{f_o}} = \frac{f_o}{u_e}$$

$\tan\beta =$

$$m = \frac{f_o}{u_e}$$

$$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

NEET

$$m_{max} = f_o\left[\frac{1}{f_e} + \frac{1}{D}\right]$$

$$\boxed{m_{max} = \frac{f_o}{f_e}\left[1 + \frac{f_e}{D}\right]}$$

when Final image forms at ∞

$u_e = f$

$$m = \frac{f_o}{u_e} = \frac{f_o}{f_e}$$

min