9)

If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$,

then $f(100)$ is equal to **(1999, 2M)**

(a) 0 (b) 1 (c) 100 (d) -100

$$x(x-1)\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3 & (x-2) & (x+1) \end{vmatrix} \qquad \boxed{f(x)=0}$$

$$x^2(x^2-1)\begin{vmatrix} 1 & x & 1 \\ 2 & (x-1) & 1 \\ 3 & (x-2) & 1 \end{vmatrix}$$

$$x^2(x^2-1)\begin{vmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ 3 & (x-2) & 1 \end{vmatrix} = x^2(x^2-1)(0)$$

$$= 0$$

9) Easy

CRamers

If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu, (\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is

(2019 Main, 10 April I)

(a) 7 (b) 12

(c) 10 (d) 9

$\boxed{D = 0}$

$\boxed{D_1 = D_2 = D_3 = 0}$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2, \; C_2 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ -2 & 3-\lambda & \lambda \end{vmatrix} = 0$$

$1((\lambda - 3)) = 0$

$\boxed{\lambda = 3}$

$$D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 6 \\ 1 & 3 & \mu \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & -1 \\ 0 & -1 & 6-\mu \\ 1 & 3 & \mu \end{vmatrix} = 1((\mu - 6) - (1))$$

$= \mu - 7 = 0$

$\boxed{\mu = 7}$

9) The set of all values of λ for which the system of linear equations $x-2y-2z=\lambda x$, $x+2y+z=\lambda y$ and $-x-y=\lambda z$ has a non-trivial solution **(2019 Main, 12 Jan II)**

(a) contains exactly two elements.

(b) contains more than two elements.

(c) is a singleton.

(d) is an empty set.

$x=\frac{D_1}{D}$, $y=\frac{D_2}{D}$, $z=\frac{D_3}{D}$

$\boxed{x=0,\ y=0,\ z=0}$

$\boxed{\text{Homogenous eq}^n}$ $(d_1, d_2, d_3 = 0)$

$x(1-\lambda)-2y-2z=0$

$x+(2-\lambda)y+z=0$

$-x-y-\lambda z=0$

$\boxed{D_1=D_2=D_3=0}$

Non-Trivial sol^n $D=0$ (Infinite)

$D\neq 0$ Trivial sol^n (Unique)

↳ for all Homogenous eq^n Trivial sol^n $(x=0, y=0, z=0)$ is alway there.

↳ $\begin{vmatrix} 1-\lambda & -2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix}=0$

$\boxed{(1-\lambda)^3=0}$

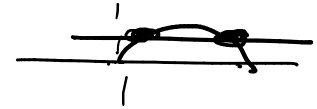
$\boxed{\lambda=1}$

9) The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$$x + 3y + 7z = 0,$$
$$-x + 4y + 7z = 0,$$
$$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$$

has a non-trivial solution, is **(2019 Main, 10 Jan II)** Non-trivia

(a) two (b) three (c) four (d) one



$\boxed{D_1 = D_2 = D_3 = 0}$

$D = 0$ $\quad$ $D \neq 0$ (Trivial)

$$\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2$

$$\to \begin{vmatrix} 0 & 7 & 14 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$$

$$-7(-2 - 7\sin 3\theta) + 14(-\cos 2\theta - 4\sin 3\theta) = 0$$

$$7[2 + 7\sin 3\theta - 2\cos 2\theta - 8\sin 3\theta] = 0$$

$$[2(2\sin^2\theta) - (3\sin\theta - 4\sin^3\theta)] = 0$$

$$4\sin^2\theta - 3\sin\theta + 4\sin^3\theta = 0$$

$$\sin\theta[4\sin^2\theta + 4\sin\theta - 3] = 0$$

$$\sin\theta[4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3] = 0$$

$$2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$$

$$\sin\theta(2\sin\theta - 1)(2\sin\theta + 3) = 0$$

$$\sin\theta = 0, \tfrac{1}{2}$$

9) *

If the system of linear equations

$x - 2y + kz = 1$, $2x + y + z = 2$, $3x - y - kz = 3$

has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is **(2019 Main, 8 April II)**

(a) $3x - 4y - 4 = 0$ (b) $3x - 4y - 1 = 0$

(c) $4x - 3y - 4 = 0$ (d) $4x - 3y - 1 = 0$

$\boxed{x - 2y + \cancel{kz} = 1}$ — (1)

$x = y = z = ?$

$2x + y + z = 2$

$\boxed{3x - y - \cancel{kz} = 3}$ — (2)

$\boxed{4x - 3y = 4}$

9) *

The system of linear equations

$$x + y + z = 2, \qquad 2x + 3y + 2z = 5$$
$$2x + 3y + (a^2 - 1)z = a + 1$$

(2019 Main, 9 Jan I)

(a) has infinitely many solutions for $a = 4$

(b) is inconsistent when $a = 4$

(c) has a unique solution for $|a| = \sqrt{3}$

(d) is inconsistent when $|a| = \sqrt{3}$

Non

3 no

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2-1)z = a+1$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{vmatrix} = D$$

$a^2 = 3$

$a = \pm\sqrt{3}$

$D = 0$ → No solⁿ (Incon)

→ (Infinite) (

D, D_1, D_2, D_3

$$D_3 = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \\ 2 & 3 & a+1 \end{vmatrix} = ?$$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1 & 1 & 2 \\ 0 & 0 & 4-a \\ 2 & 3 & a+1 \end{vmatrix}$$

$-(4-a)(1)$

$(a-4) \neq 0$

If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to **(2014 Main)**

(a) $\alpha\beta$ (b) $\dfrac{1}{\alpha\beta}$ (c) 1 (d) -1

Q2

Let $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$ IIT

r r^2

Show that $\boxed{\sum_{a=1}^{n} \Delta_a = c \in}$ constant.

$\boxed{\Delta_1 + \Delta_2 - - - - \Delta_n = C}$

$$\Delta_a = \begin{vmatrix} 0 & - & - \\ 0 & - & - \\ 0 & - & - \end{vmatrix} + \begin{vmatrix} 1 & - & - \\ 1^2 & - & - \\ 1^3 & - & - \end{vmatrix} + \begin{vmatrix} 2 & - & - \\ 2^2 & - & - \\ 2^3 & - & - \end{vmatrix}$$

$$\frac{n(n+1)(2n+1)}{6}$$

$$\begin{vmatrix} 0+1+2(n-1) & n & 6 \\ 0+1^2 - (n-1)^2 & 2n^2 & 4n-2 \\ 0+1^3 - - (n-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$\begin{vmatrix} \frac{n(n-1)}{2} & n & 6 \\ \frac{(n-1)(n)(2n-1)}{6} & 2n^2 & 4n-2 \\ \left(\frac{n(n-1)}{2}\right)^2 & 3n^3 & 3n^2-3n \end{vmatrix} = \frac{n(n-1)}{12} \begin{vmatrix} 6 & n & 6 \\ 2(2n-1) & 2n^2 & 4n-2 \\ 3n(n-1) & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$= 0$$