Problem Solving on Determinants

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A value of $\theta \in (0, \pi/3)$, for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0, \text{ is}$$

(2019 Main, 12 April II)

(a) $\frac{\pi}{9}$ (b) $\frac{\pi}{18}$ (c) $\frac{7\pi}{24}$ (d) $\frac{7\pi}{36}$

Row, column operation

75 0

speed
concentration
accuracy

$\boxed{R_2 \rightarrow R_2 - R_3}$, $\boxed{R_1 \rightarrow R_1 - R_3}$

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$\cos\left(\pi - \frac{\pi}{3}\right) = -\frac{1}{2}$

$\frac{2\pi}{3}$

$$1(1+4\cos 6\theta + \sin^2\theta) - 1(-\cos^2\theta) = 0$$

$$1 + 4\cos 6\theta + \underline{\sin^2\theta + \cos^2\theta} = 0$$

$$4\cos 6\theta = -2$$

$$\boxed{\cos 6\theta = -\frac{1}{2}} \Rightarrow \boxed{\overset{3}{\cancel{6}}\theta = \frac{\cancel{2}\pi}{3}}$$

$$\boxed{\theta = \frac{\pi}{9}}$$

9

The sum of the real roots of the equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0, \text{ is equal to}$$

(2019 Main, 10 April II)

~~(a) 0~~ (b) -4

(c) 6 (d) 1

$\boxed{x = 1}$ soln

α, β

$x[-3x(x+2) - 2x(x-3)] + 6[2x+4+3x-9] + (-1)[4x-9x] = 0$

$x[-3x^2 - \cancel{6x} - 2x^2 + \cancel{6x}] + 6[5x-5] + 5x$

$\boxed{-5x^3 + 35x - 30 = 0}$

$ax^3 + bx^2 + cx + d = 0$

$\boxed{\text{Sum of roots} = 0}$

Theory of eqn

9)

Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then, for $y \neq 0$ in **R**,

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$ is equal to

(2019 Main, 9 April I)

(a) $y(y^2-1)$ (b) $y(y^2-3)$ (c) y^3-1 (d) y^3

$\alpha, \beta \longrightarrow$ value ✗

$\rightarrow \boxed{\alpha+\beta=-1}$

$\rightarrow \boxed{\alpha \cdot \beta = 1}$

$R_1 \longrightarrow R_1 + R_2 + R_3$

$$\begin{vmatrix} y+(\alpha+\beta+1) & y+(\alpha+\beta+1) & y+(\alpha+\beta+1) \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$

$$\rightarrow \begin{vmatrix} y & y & y \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$

$C_1 \longrightarrow C_1 - C_2$

$C_2 \longrightarrow C_2 - C_3$

$$\rightarrow \begin{vmatrix} 0 & 0 & y \\ \alpha-y-\beta & y+\beta-1 & 1 \\ \beta-1 & 1-y-\alpha & y+\alpha \end{vmatrix} = y\{(\alpha-\beta-y)(1-y-\alpha) - (y+\beta-1)(\beta-1)\}$$

$\alpha - \cancel{\beta y} - \alpha^2 - \beta + \cancel{\beta y} + y^2 - \alpha^2 + \alpha\beta + \alpha y - y\beta + y - \beta^2 - 1 + 2\beta$

9) ★

If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= (a+b+c)(x+a+b+c)^2$, $x \neq 0$ and $a+b+c \neq 0$, then x is equal to **(2019 Main, 11 Jan II)**

(a) $-(a+b+c)$ (b) $-2(a+b+c)$

(c) $2(a+b+c)$ (d) abc

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$(a+b+c)\begin{vmatrix} 0 & \cancel{a+b+c}\ 1 & \cancel{a+b+c}\ 1 \\ a+c+b & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

$$(a+b+c)\Big\{(a+b+c)(-)\big[(\cancel{a+b+c}\ 1)(c-a-b) - (\cancel{a+b+c}\ 1)(2c)\big]$$

$$+ (a+b+c)^2\big[\cancel{+}(a+b+c)\big]\Big\} = (a+b+c)^3$$

Q)

Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$, where $b > 0$. Then, the minimum value of $\frac{\det(A)}{b}$ is (2019 Main, 10 Jan II)

(a) $-\sqrt{3}$ (b) $-2\sqrt{3}$ (c) $2\sqrt{3}$ (d) $\sqrt{3}$

min Value $= \frac{b^2+3}{b}$ $\sqrt{\cancel{b} \times \frac{3}{\cancel{b}}}$

$= b + \frac{3}{b}$

$\boxed{A.M \geq G.M}$

$\frac{b + 3/b}{2} \geq \sqrt{3}$

$b + 3/b \geq 2\sqrt{3}$

$\begin{vmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{vmatrix} =$

$R_1 \to R_1 - R_3$

$\begin{vmatrix} 1 & 0 & -1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{vmatrix} = 1(b^2+2) - 1(-1)$

$= b^2 + 3$

9* Let $d \in R$, and

$$A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix}; \theta \in [0, 2\pi].$$ If the minimum value of det(A) is 8, then a value of d is

(2019 Main, 10 Jan I)

(a) -5 (b) -7 (c) $2(\sqrt{2}+1)$ (d) $2(\sqrt{2}+2)$

$R_3 \longrightarrow \underline{R_3 + R_1 - 2R_2}$

$$\begin{vmatrix} -2 & 4+d & (\sin\theta-2) \\ 1 & (\sin\theta+2) & d \\ 1 & 0 & 0 \end{vmatrix}$$

$1\left[(d^2+4d)-(\sin^2\theta-4)\right] = |A|$

$\left[(d+2)^2 - \sin^2\theta\right] = |A|$

$(d+2)^2 - \sin^2\theta = 8$

$(d+2)^2 - 1 = 8$

$(d+2)^2 = 9$

$d+2 = \pm 3$

$\boxed{d = 1, -5}$

9)

If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to (2018 Main)

(a) $(-4, -5)$ (b) $(-4, 3)$ (c) $(-4, 5)$ (d) $(4, 5)$

$(5x-4)(x+4)^2$

$(-4+5x)(x-(-4))^2$

$\boxed{R_1 \to R_1 + R_2 + R_3}$

$\begin{vmatrix} 5x-4 & 5x-4 & 5x-4 \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$

$5x-4 \begin{vmatrix} 0 & 1 & 1 \\ x+4 & x-4 & -2x \\ 0 & 2x & x-4 \end{vmatrix}$ $C_1 \to C_1 - C_2$

$(5x-4)(x+4)(-)(x-4-2x)$

$(5x-4)(x+4)(x+4)$