# Determinants

By
Ankush Garg(B. Tech, IIT Jodhpur)

$| \quad | =$ fixed value

## VALUE OF A DETERMINANT :

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} = \underline{a_1 (b_2c_3 - b_3c_2) - b_1 (a_2c_3 - a_3c_2) + c_1 (a_2b_3 - a_3b_2)}$$

If $\begin{vmatrix} x & -6 & 1 \\ 2x & -3 & 4 \\ 0 & 1 & 2 \end{vmatrix} = 0$, find x.

$x(-6-4) + 6(4x) + 1(2x) = 0$

$-10x + 24x + 2x = 0$

$16x = 0$ ✓

$\boxed{x = 0}$

## COFACTOR AND MINORS OF AN ELEMENT :

**Minors** : Minors of an element is defined as the minor determinant obtained by deleting a particular row or column in which that element lies. e.g. in the determinant

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \text{minor of } a_{12} \text{ denoted as } M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ c_{31} & c_{33} \end{vmatrix} \quad \text{and so on}$$

**Cofactor** : It has no separate identity and is related to the cofactor as

$C_{ij} = (-1)^{i+j} M_{ij}$ where ' i ' denotes the row and ' j ' denotes the column.

$(a_{ij})$

$C_{ij} = (-1)^{i+j} M_{ij}$

Q).

$$\begin{vmatrix} 2 & 3 & -1 \\ 2 & -1 & 0 \\ 4 & -2 & 4 \end{vmatrix}$$

$$\boxed{\begin{aligned} C_{11} &= M_{11} \\ C_{12} &= -M_{12} \\ C_{13} &= M_{13} \end{aligned}}$$

$C_{11} = ?\,(M_{11})$

$= (-4)$

$C_{23} = ?$

$= -(-7)$

$= (7)$

$C_{31} = ?\,+M_{31}$

$= (-1)$

## ★ Value of Determinant

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\Delta = \boxed{a_1 C_{11} + a_2 C_{12} + a_3 C_{13}}$$

$$\Delta = \boxed{c_1 C_{31} + c_2 C_{32} + c_3 C_{33}}$$

$$\Delta = \boxed{a_2 C_{12} + b_2 C_{22} + c_2 C_{32}}$$

★ $$\boxed{a_1 C_{21} + a_2 C_{22} + a_3 C_{23} = 0}$$

## PROPERTIES OF DETERMINANTS

unchanged

**P-1:** The value of a determinant remains unaltered, if the rows & columns are inter changed. e.g. if $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = D'$

**P-2:** If any two rows (or columns) of a determinant be interchanged, the value of determinant is changed in sign only. e.g.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ & $D' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$ Then $\boxed{D' = -D.}$

$$\begin{vmatrix} c_1 & b_1 & a_1 \\ c_2 & b_2 & a_2 \\ c_3 & b_3 & a_3 \end{vmatrix} = -D$$

$$\begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix} = D$$

**P-3:** If a determinant has any two rows (or columns) identical , then its value is zero.

e.g. Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then it can be verified that $D = 0$.

$$\begin{vmatrix} 1 & 1 & 3 \\ 1 & 1 & 2 \\ 2 & 2 & -1 \end{vmatrix} = 0$$

**P-4:** If all the elements of any row (or column) be multiplied by the same number, then the determinant is multiplied by that number.

e.g. If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ Then $\boxed{D' = KD}$

$$\begin{vmatrix} 2a_1 & 3b_1 & c_1 \\ 2a_2 & 3b_2 & c_2 \\ 2a_3 & 3b_3 & c_3 \end{vmatrix}$$

$$2 \times 3 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \underline{\underline{6D}}$$

$$D_1 = \begin{vmatrix} Ka_1 & Kb_1 & KC_1 \\ Ka_2 & Kb_2 & KC_2 \\ Ka_3 & Kb_3 & KC_3 \end{vmatrix} = K^3 D$$

**P-5:** If each element of any row (or column) can be expressed as a sum of two terms then the determinant can be expressed as the sum of two determinants, e.g.

$$\begin{vmatrix} a_1+x & b_1+y & c_1+z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\begin{vmatrix} a_1+x & b_1+y & c_1+z \\ a_2+\alpha & b_2+\beta & c_2+\gamma \\ a_3 & b_3 & c_3 \end{vmatrix}$$

# Row & column operations

**P-6:** The value of a determinant is not altered by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row (or column).

e.g. Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_1 + ma_2 & b_1 + mb_2 & c_1 + mc_2 \\ a_2 & b_2 & c_2 \\ a_3 + na_1 & b_3 + nb_1 & c_3 + nc_1 \end{vmatrix}$. Then $\boxed{D'=D}$.

$\boxed{R_1 \to R_1 + 2R_3}$ ,

$\boxed{\begin{array}{l} R_2 \to R_2 - 5R_3 \\ R_1 \to R_1 - 8R_3 \end{array}}$

$$\begin{array}{c} 6 \\ 11 \\ 32 \end{array}\begin{vmatrix} 48 & 54 & 72 \\ 55 & 121 & 88 \\ 32 & 64 & 128 \end{vmatrix} = 6 \times 11 \times 32 \begin{vmatrix} 8 & 9 & 12 \\ 5 & 11 & 8 \\ 1 & 2 & 3 \end{vmatrix}$$

↳ we will try to make 2, 3 zero in a particular Row / column.

$$= \begin{vmatrix} 0 & -7 & -12 \\ 0 & 1 & -7 \\ 1 & 2 & 3 \end{vmatrix}$$

$\boxed{6 \times 11 \times 32 \times 1\,(61)}$

$$\begin{vmatrix} 0 & 0 & 1 \\ - & - & - \\ - & - & - \end{vmatrix}$$