## Determinants



By Ankush Garg(B. Tech, IIT Jodhpur) VALUE OF A DETERMINANT :

$$\mathbf{D} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & c_3 \end{vmatrix} = a_1 (b_2 c_3 - b_3 c_2) - b_1 (a_2 c_3 - a_3 c_2) + c_1 (a_2 b_3 - a_3 b_2)$$

If 
$$\begin{vmatrix} x & -6 & 1 \\ 2x & -3 & 4 \\ 0 & 1 & 2 \end{vmatrix} = 0$$
, find x  
 $x(-6 - 4) + 6(4x) + 1(2x) = 0$   
 $- 10x + 24x - 12x = 0$   
 $16x = 0$ 

## **COFACTOR AND MINORS OF AN ELEMENT :**

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<u>Minors</u> : Minors of an element is defined as the minor determinant obtained by deleting a particular row or column in which that element lies. e.g. in the determinant

$$\mathbf{D} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \text{minor of } a_{12} \text{ denoted as } \mathbf{M}_{12} = \begin{vmatrix} a_{21} & a_{23} \\ c_{31} & c_{33} \end{vmatrix} \quad \text{and so on}$$

**Cofactor** : It has no separate identity and is related to the cofactor as  $C_{i i} = (-1)^{i+j} M_{i i}$  where 'i' denotes the row and 'j' denotes the column.  $C_{11} = (-1)^{(+1)} H_{11}$ (a(j) 2 - 1 $C_{11} = 2(M_{11})$  $C_{31} = 7 + M_{31}$  $C_{23} = ?$ =- (-7)

Value of Determinant  
Ly 
$$a_1 a_2 a_3$$
  
 $\Delta = \begin{bmatrix} a_1 & a_1 \\ b_1 & b_2 & b_3 \\ \Rightarrow & c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} a_1 & c_{11} + a_2 & c_{12} + a_3 & c_{13} \\ A = \begin{bmatrix} c_1 & c_{31} + c_2 & c_{32} + c_3 & c_{33} \\ A = \begin{bmatrix} a_2 & c_{12} + b_2 & c_{22} + c_2 & c_{32} \\ a_1 & c_{21} + a_2 & c_{22} + a_3 & c_{23} \end{bmatrix} = 0$ 

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## PROPERTIES OF DETERMINANTS

**P-1:** The value of a determinant remains unaltered, if the rows & columns are

inter changed. e.g. if 
$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = D'$$

**P-2:** If any two rows (or columns) of a determinant be interchanged, the value of determinant is changed in sign only. e.g.

Let 
$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \& D = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$
 Then  $D' = -D$ .

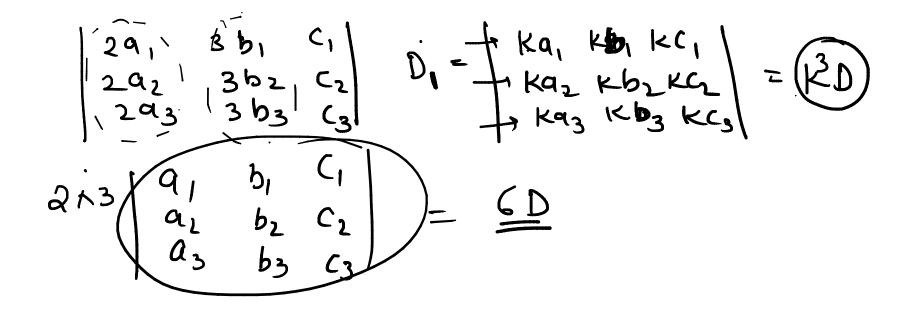
$$\begin{vmatrix} C_{1} & Q_{1} & b_{1} \\ C_{2} & Q_{2} & b_{2} \\ C_{3} & Q_{3} & b_{3} \end{vmatrix} = \mathbf{D}$$

P-3: If a determinant has any two rows (or columns) identical, then its value is zero.

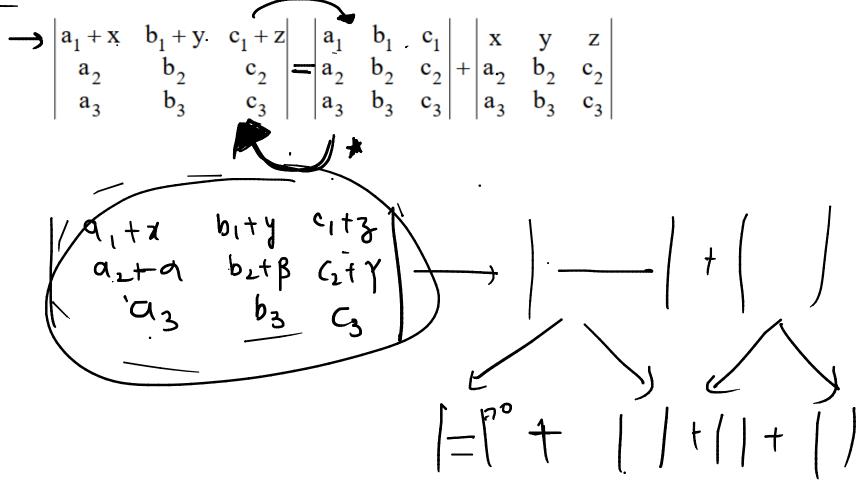
e.g. Let 
$$\mathbf{D} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$
 then it can be verified that  $\mathbf{D} = \mathbf{0}$ .

P-4: If all the elements of any row (or column) be multiplied by the same number, then the determinant is multiplied by that number.

e.g. If 
$$\underline{D} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$
 and  $\underline{D'} = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$  Then  $\underline{D'} = KD$ 



P-5: If each element of any row (or column) can be expressed as <u>a sum of two terms then</u> the determinant can be expressed as the sum of two determinants. e.g.



## Row & column operations

The value of a determinant is <u>not altered</u> by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row (or column).

