Properties

P–2 (i) $\text{cosec}^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$; $\boxed{x \le -1\ ,\ x \ge 1}$

(ii) $\sec^{-1} x = \cos^{-1} \frac{1}{x}$; $\boxed{x \le -1\ ,\ x \ge 1}$

★ (iii) $\cot^{-1} x = \tan^{-1} \frac{1}{x}$; $x > 0$

$= \pi + \tan^{-1} \frac{1}{x}$; $x < 0$

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$

$$\boxed{\text{cosec}^{-1} x = \frac{1}{\sin^{-1} x}}$$

Properties

$\cos^{-1}(-1/2) = \pi - \cos^{-1}(\frac{1}{2})$

$= \pi - \pi/3$

$= 2\pi/3$

P-3 (i) $\sin^{-1}(-x) = -\sin^{-1}x \quad , \quad -1 \le x \le 1$

(ii) $\tan^{-1}(-x) = -\tan^{-1}x \quad , \quad x \in R$

(iii) $\cos^{-1}(-x) = \pi - \cos^{-1}x \quad , \quad -1 \le x \le 1$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x \quad , \quad x \in R$

$\cos(-\theta) = \cos\theta$

$\sin^{-1}(-1/2) = -\sin^{-1}(\frac{1}{2})$

$= -(-\pi/6)$

P-4 (i) $\boxed{\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}}$ $\quad -1 \le x \le 1$

(ii) $\boxed{\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \quad x \in R}$

(iii) $\boxed{\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}}$ $\quad |x| \ge 1$

$x \le -1, \quad x \ge 1$

Problems

Domain

$\boxed{-1 \le x \le 1}$

$\tan^{-1}(x) \rightarrow \mathbb{R}$

$(-\pi/2, \pi/2)$

9) *Find the range of* $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$.

$\pi/2 + [-\pi/4, \pi/4]$

$\boxed{[\pi/4, 3\pi/4]}$

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Properties

P–5 $\boxed{\tan^{-1}x + \tan^{-1}y} = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $x > 0$, $y > 0$ & $\boxed{xy < 1}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$= \pi + \tan^{-1}\frac{x+y}{1-xy}$ where $x > 0$, $y > 0$ & $\boxed{xy > 1}$

$\boxed{\tan^{-1}x - \tan^{-1}y} = \tan^{-1}\frac{x-y}{1+xy}$ where $x > 0$, $y > 0$

Problems

9) If $\tan^{-1}2 + \tan^{-1}4 = \cot^{-1}(\lambda)$ then find λ.

$$\pi + \tan^{-1}\left(\frac{6}{1-8}\right)$$

$$\pi + \tan^{-1}\left(\frac{-6}{7}\right) = \cot^{-1}(\lambda)$$

$$\pi - \tan^{-1}\left(\frac{6}{7}\right) = \cot^{-1}(\lambda)$$

$$\left(\pi - \cot^{-1}\left(\frac{7}{6}\right) = \cot^{-1}\lambda\right)$$

$$\lambda = \cot\left(\pi - \cot^{-1}(7/6)\right)$$

$$= -\cot\left(\cot^{-1}(7/6)\right)$$

$$0 < \frac{7}{6} < \pi$$

$$\lambda = -\cot\left(\cot^{-1}(7/6)\right)$$

$$\boxed{\lambda = -7/6}$$

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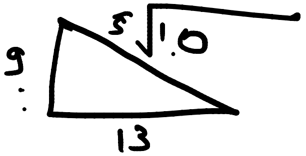
Problems

9) If $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$, $\beta = \tan^{-1}\left(\frac{1}{3}\right)$, where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to (2019 Main, 8 April I)

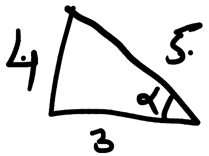
(a) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (b) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

(c) $\tan^{-1}\left(\frac{9}{14}\right)$ (d) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$



$\alpha = \cos^{-1}\left(\frac{3}{5}\right)$

$\cos\alpha = 3/5$ $\tan\alpha = \frac{4}{3}$



$\alpha = \tan^{-1}(4/3)$

$\beta = \tan^{-1}(1/3)$

$\boxed{\tan^{-1}(4/3) - \tan^{-1}(1/3)} = \tan^{-1}\left(\frac{1}{1+\frac{4}{9}}\right)$

$= \tan^{-1}\left(\frac{9}{13}\right)$

Problems

9) Considering only the principal values of inverse functions, the set $A = \left\{x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\right\}$

(2019 Main, 12 Jan I)

(a) is an empty set
(b) is a singleton
(c) contains more than two elements
(d) contains two elements

$\tan^{-1}\left(\frac{2x+3x}{1-6x^2}\right) = \pi/4$

$5x = 1-6x^2$

$6x^2+5x-1=0$

$(x+1)(6x-1)$

$-1, 1/6$

Properties

$\sin(A+B) =$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

PROPERTY-6 :

★ (I) $\sin^{-1}x + \sin^{-1}y = \begin{cases} \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) \text{if } x \ge 0; y \ge 0 \text{ and } x^2 + y^2 \le 1 \\ \pi - \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) \text{if } x \ge 0; y \ge 0 \text{ and } x^2 + y^2 > 1 \end{cases}$

note that $\boxed{x^2 + y^2 \le 1} \Rightarrow \boxed{0 \le \sin^{-1}x + \sin^{-1}y \le \frac{\pi}{2}}$

★ (II) ||| ly we have $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right), x > 0; y > 0$

and $\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right), x > 0, y > 0, x < y$

Problems

9) The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to

(2019 Main, 12 April I)

(a) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$

(b) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$ ✓

(c) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$

(d) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$

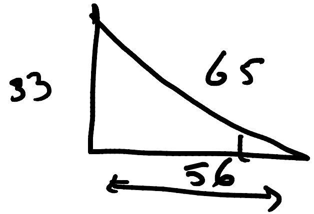
$$\sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$$

$$\sin^{-1}\left(\frac{12}{13}\times\frac{4}{5} - \frac{3}{\cancel{5}}\times\frac{\cancel{5}}{13}\right)$$

$$\left(\frac{48}{65} - \frac{15}{65}\right)$$

$$\sin^{-1}\left(\frac{33}{65}\right) = \cos^{-1}\left(\frac{56}{65}\right)$$

$$= \frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$$



$65 \times 65 \rightarrow 4225$

$4225 - 1089 = 2116$

56

33×33: 99, 99×, 1089

Sin

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Problems

$\sqrt{1-\left(\frac{y}{2}\right)^2}$

9) ★★

If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, where $-1 \le x \le 1$, $-2 \le y \le 2$, $x \le \frac{y}{2}$, then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(2019 Main, 10 April II)

(a) $2\sin^2\alpha$
(b) $4\cos^2\alpha + 2x^2y^2$
(c) $4\sin^2\alpha$
(d) $4\sin^2\alpha - 2x^2y^2$

$\cos^{-1}\left(\frac{xy}{2} + \sqrt{1-x^2}\cdot\frac{\sqrt{4-y^2}}{2}\right) = \alpha$

$\cos^{-1}\left(\frac{xy + \sqrt{1-x^2}\sqrt{4-y^2}}{2}\right) = \alpha$

$\left(\frac{xy + \sqrt{1-x^2}\sqrt{4-y^2}}{2} = \cos\alpha\right)$

$\sqrt{1-x^2}\cdot\sqrt{4-y^2} = 2\cos\alpha - xy$

$(1-x^2)(4-y^2) = 4\cos^2\alpha + \cancel{x^2y^2} - 4xy\cos\alpha$

$(4 - 4x^2 + \cancel{x^2y^2} - y^2) =$

$4x^2 + y^2 - 4xy\cos\alpha = 4 - 4\cos^2\alpha$

$= 4\sin^2\alpha$

SUMMATION OF SERIES :

9) The sum $\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{4n}{n^4 - 2n^2 + 2}\right)$ is equal to

$\boxed{\tan^{-1}(A) - \tan^{-1}(B)} = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$

(A) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{3}$ (B) $4\tan^{-1} 1$ (C) $\frac{\pi}{2}$ (D) $\sec^{-1}\left(-\sqrt{2}\right)$ → $3\pi/4$

$$\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{4n}{n^4-2n^2+2}\right) = \sum_{n=1}^{\infty}\left(\tan^{-1}\left(\frac{4n}{1+(n^2-1)^2}\right)\right)$$

$$\tan^{-1}\left(\frac{4n}{1+(n-1)^2(n+1)^2}\right)$$

$$= \tan^{-1}\left(\frac{(n+1)^2-(n-1)^2}{1+(n-1)^2(n+1)^2}\right)$$

$$= \tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2$$

$\cancel{\tan^{-1}(2)^2 - \tan^{-1}(0)} + \tan^{-1}(3)^2 - \tan^{-1}(1)^2$

$\tan^{-1}(4)^2 - \cancel{\tan^{-1}(2)^2} =$

$\tan^{-1}(5)^2 - \cancel{\tan^{-1}(3)^2}$

$\tan^{-1}\infty = \pi/2$

$\tan^{-1}(n+1)^2$

$\left(\frac{\pi}{2}\right) + \left(\frac{\pi}{2}\right) - \left(\frac{\pi}{4}\right) = \frac{3\pi}{4}$

Problems

The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is

(2019 Main, 10 Jan II)

(a) $\frac{23}{22}$ (b) $\frac{21}{19}$

(c) $\frac{19}{21}$ (d) $\frac{22}{23}$

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End of Lecture