Inverse Trigonometric Functions

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Definition

$\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ etc. denote angles or real numbers whose sine is x, whose cosine is x and whose tangent is x, provided that the answers given are numerically smallest available. These are also written as arc sinx, arc cosx etc.

$\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$
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θ

$\sin^{-1}x = \theta$

Sin

$\sin\theta = x$

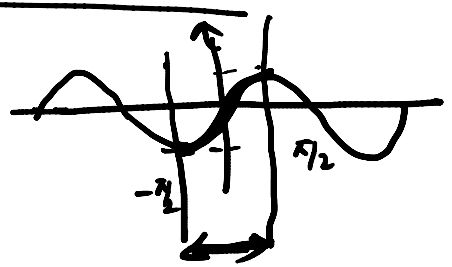
PRINCIPAL VALUES AND DOMAINS OF INVERSE CIRCULAR FUNCTIONS :

Domain Range

(i) $y = \sin^{-1} x$ where $-1 \le x \le 1$; $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and $\sin y = x$.

(ii) $y = \cos^{-1} x$ where $-1 \le x \le 1$; $0 \le y \le \pi$ and $\cos y = x$.

(iii) $y = \tan^{-1} x$ where $x \in R$; $-\frac{\pi}{2} < y < \frac{\pi}{2}$ and $\tan y = x$.



(iv) $y = \text{cosec}^{-1} x$ where $x \le -1$ or $x \ge 1$; $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, $y \ne 0$ and $\text{cosec}\, y = x$. $[-\pi/2, \pi/2] - \{0\}$

(v) $y = \sec^{-1} x$ where $x \le -1$ or $x \ge 1$; $0 \le y \le \pi$; $y \ne \frac{\pi}{2}$ and $\sec y = x$. $[0, \pi] - \{\pi/2\}$

(vi) $y = \cot^{-1} x$ where $x \in R$ $0 < y < \pi$ and $\cot y = x$. $(0, \pi)$

NOTE THAT : **(a)** 1st quadrant is common to all the inverse functions .

(b) 3rd quadrant is **not used** in inverse functions .

(c) 4th quadrant is used in the **CLOCKWISE DIRECTION** i.e. $-\frac{\pi}{2} \le y \le 0$

PRINCIPAL VALUES AND DOMAINS OF INVERSE CIRCULAR FUNCTIONS :

S.No.	Function	Domain	Range
(i)	$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
(ii)	$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
(iii)	$y = \tan^{-1} x$	$x \in R$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$
(iv)	$y = \cot^{-1} x$	$x \in R$	$0 < y < \pi$
(v)	$y = \operatorname{cosec}^{-1} x$	$x \le -1$ or $x \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}, \ y \ne 0$
(vi)	$y = \sec^{-1} x$	$x \le -1$ or $x \ge 1$	$0 \le y \le \pi; \ y \ne \frac{\pi}{2}$

$[-\pi/2, \pi/2]$

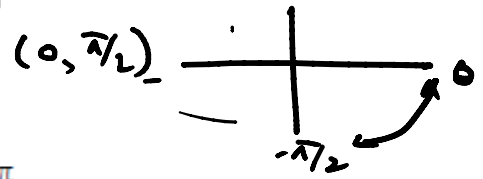
$[0, \pi]$

$(-\pi/2, \pi/2)$

$(0, \pi)$

$[-\pi/2, \pi/2] - \{0\}$

$[0, \pi] - \{\pi/2\}$



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Problems

$\tan^{-1}(x) = \theta$

$x = \tan\theta$

Q) The value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ is equal to

($\pi/4$) ($2\pi/3$) ($-\pi/6$)

(A) $\frac{\pi}{4}$ (B) $\frac{5\pi}{12}$ (C) $\frac{3\pi}{4}$ (D) $\frac{13\pi}{12}$

$\tan^{-1}1 = \theta$

$1 = \tan\theta$

$\theta = \pi/4$

$\cos^{-1}(-1/2) = \theta$

$-1/2 = \cos\theta$

$\cos(\pi - \pi/3) = \cos\theta$

$\frac{2\pi}{3} = \theta$

$45^\circ + 120^\circ - 30^\circ$

$135^\circ \rightarrow \frac{3\pi}{4}$

Problems

Find the value of

(a) $\sin\left(2\sin^{-1}\frac{3}{5}\right)$

(b) $\cos(2\tan^{-1}2) + \sin(2\tan^{-1}3)$

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Problems

3 → 1 eq, 2 eqn

8) x, y, z

$[0, \pi]$ $[0, \pi]$ $[0, \pi]$

If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then the value of $\dfrac{x^{2012} + y^{2012} + z^{2012}}{x^{2011} + y^{2011} + z^{2011}} + \dfrac{6}{}$ is

$\pi \cdot \pi \cdot \pi$

equal to

(A) 0 (B) 1 (C) –1 (D) 2

$\cos^{-1} x = \cos^{-1} y = \cos^{-1} z = \pi$

$x = y = z = -1$

$3 + \left(\frac{6}{-3}\right)$

$3 - 2 = 1$

Properties

P–1

(i) $\sin(\sin^{-1} x) = x \ , \ -1 \le x \le 1$

(ii) $\cos(\cos^{-1} x) = x \ , \ -1 \le x \le 1$

(iii) $\tan(\tan^{-1} x) = x \ , \ x \in R$

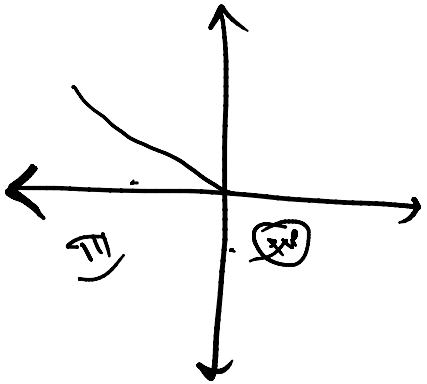
(iv) $\sin^{-1}(\sin x) = x \ , \ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

(v) $\cos^{-1}(\cos x) = x \ ; \ 0 \le x \le \pi$

(vi) $\tan^{-1}(\tan x) = x \ ; \ -\frac{\pi}{2} < x < \frac{\pi}{2}$

$\sin(\sin^{-1} x) = x$

$-1 \le x \le 1$



$\sin^{-1}(\sin x) = x$

$-\pi/2 \le x \le \pi/2$

$\sin^{-1}\left(\sin \frac{3\pi}{4}\right) \neq \frac{3\pi}{4}$

$\sin^{-1}(\sin(\pi - \pi/4))$

$\sin^{-1}(\sin(\pi/4)) = \pi/4$

Problems

Q) $\cos\left(\cos^{-1}\cos\left(\frac{8\pi}{7}\right)+\tan^{-1}\tan\left(\frac{8\pi}{7}\right)\right)$ has the value equal to -

(A) 1 (B) –1 (C) $\cos\frac{\pi}{7}$ (D) 0

$\cos^{-1}\cos\left(2\pi-\frac{6\pi}{7}\right)+\tan^{-1}\tan\left(\pi+\frac{\pi}{7}\right)$

$\cos\left[\cos^{-1}\cos\left(\frac{6\pi}{7}\right)+\tan^{-1}\tan\left(\pi/7\right)\right]$

$\cos\left(\frac{6\pi}{7}+\frac{\pi}{7}\right) \Rightarrow \cos\pi$

Problems

$\sin^{-1}\sin(x)$ $\quad -\frac{\pi}{2} \le x \le \pi/2$ $\qquad \sin\sin^{-1}(x)$ $\quad -1 \le x \le 1$ $\qquad 0 \le \cos^{-1}(x) \le \pi$

9) If $x = \sin^{-1}(\sin 10)$ and $y = \cos^{-1}(\cos 10)$, then $y - x$ is equal to (2019 Main, 9 Jan II)

(a) 0 (b) 10 (c) 7π (d) π

$\sin^{-1}\sin(20)$

$0 \le \cos^{-1} \le \pi$

$x = \sin^{-1}\sin(10)$

$= \sin^{-1}\sin(3\pi - 10)$

$x = (3\pi - 10)$

$y = \cos^{-1}\cos(10)$

$y = \cos^{-1}\cos(4\pi - 10)$

$y = 4\pi - 10$