PHYSICS

JEE and NEET CRASH COURSE

Viscosity, Elasticity and Surface Tension

SAFALTA.COM
An Initiative by अमर उजाला

By, Ritesh Agarwal, B. Tech. IIT Bombay

Viscosity

The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity'.

Viscous Force, Velocity Gradient and Coefficient of Viscosity

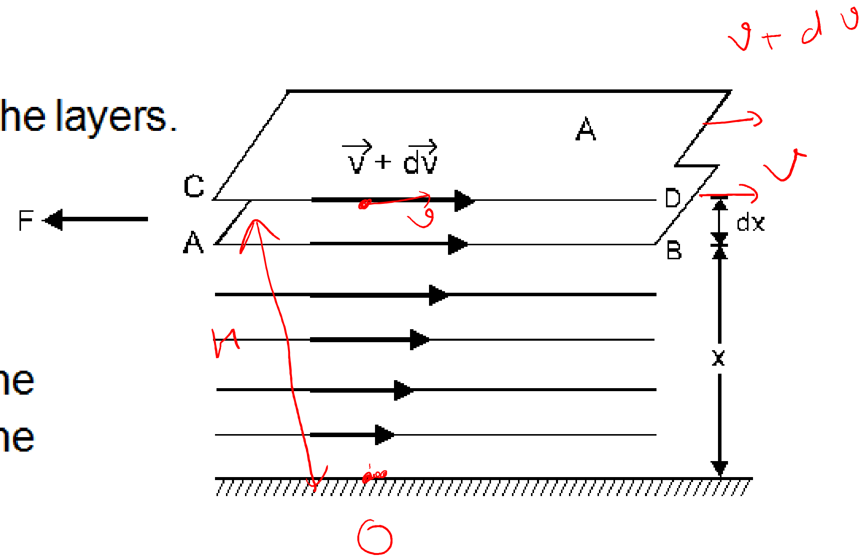
$$\frac{dv}{dx} = \frac{\Delta v}{\Delta x} = \frac{v-0}{H}$$

Consider a flow of a liquid over the horizontal solid surface as shown.

According to Newton, the viscous drag or back ward force (F) between these layers is

(i) directly proportional to the area (A) of the layer and

(ii) directly proportional to the velocity gradient $\left(\frac{dv}{dx}\right)$ between the layers.



i.e. $F \propto A \frac{dv}{dx}$ or $F = -\eta A \frac{dv}{dx}$ ...(1)

η is called Coefficient of viscosity. Negative sign shows that the direction of viscous drag (F) is just opposite to the direction of the motion of the liquid.

Units of Coefficient of Viscosity

$$\eta = \frac{F}{A(\Delta v_x / \Delta z)}$$

$$\therefore \quad \text{dimensions of} \quad \eta = \frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = \frac{[MLT^{-2}]}{[L^2T^{-1}]} = [ML^{-1}T^{-1}]$$

Its unit is kg/(meter-second) or N-s/m^2

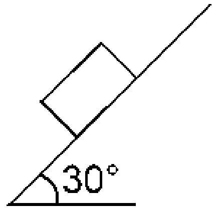
In C.G.S. system, the unit of coefficient of viscosity is dyne-s/cm^2 and is called poise. In SI the unit of coefficient of viscosity is N-s/m^2 and is called decapoise.

1 decapoise = 1 N sm^{-2} = (10^5 dyne) × s × (10^2 cm)$^{-2}$ = 10 dyne s cm^{-2} = 10 poise

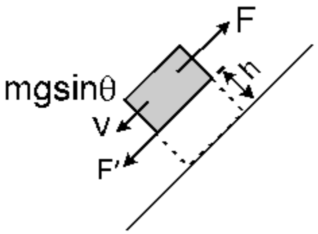
1 poise = 0.1 N-s/m^2

Example

A cubical block (of side 2m) of mass20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m/sec. (g = 10 m/sec^2) find out the thickness of layer of liquid.



Sol.



$$F = F' = \eta A \frac{dv}{dz} = mg \sin\theta \qquad \frac{dv}{dz} = \frac{v}{h}$$

$$20 \times 10 \times \sin 30° = \eta \times 4 \times \frac{10}{h}$$

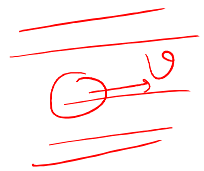
$$h = \frac{40 \times 10^{-2}}{100} \quad - [\eta = 10^{-1} \text{ poise} = 10^{-2} \text{ N-sec-m}^{-2}]$$

$$= 4 \times 10^{-3} \text{ m} = 4 \text{ mm}$$

Stoke's Law and Terminal Velocity

Stoke's Law

Stokes proved that the viscous drag (F) on a spherical body of radius r moving with velocity v in a fluid of viscosity η is given by $F = 6\pi\eta r v$. This is called Stokes' law.



Terminal Velocity

When a body is dropped in a viscous fluid, it is first accelerated and then its acceleration becomes zero and it attains a constant velocity called terminal velocity.

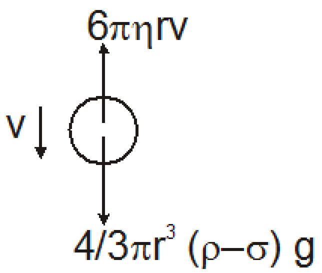
$\rho_{body} > \rho_{liq}$

Calculation of Terminal Velocity

Let us consider a small ball, whose radius is r and density is ρ, falling freely in a liquid (or gas), whose density is σ and coefficient of viscosity η. When it attains a terminal velocity v. It is subjected to two forces :

(i) effective force acting downward

$$= V(\rho-\sigma)g = \frac{4}{3}\pi r^3(\rho-\sigma)g,$$



(ii) viscous force acting upward

$= 6\pi\eta rv.$

Since the ball is moving with a constant velocity v i.e., there is no acceleration in it, the net force acting on it must be zero. That is

$$6\pi\eta rv = \frac{4}{3}\pi r^3(\rho-\sigma)g \qquad \text{or} \qquad v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

Thus, terminal velocity of the ball is directly proportional to the square of its radius

Example

A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

Sol.

Rate of heat loss = power = F × v

$$= 6\,\pi\,\eta\,r\,v \times v \;= 6\,\pi\,\eta\,r\,v^2 \;= 6p\,\eta\,r\left[\frac{2}{9}\frac{gr^2(\rho_0 - \rho_\ell)}{\eta}\right]^2$$

Rate of heat loss $\alpha\ r^5$

Elasticity

The property of a material body by virtue of which it regains its original configuration (i.e. shape and size) when the external deforming force is removed is called elasticity.

Stress

$$\text{stress} = \frac{\textit{restoring force}}{\textit{Area of the body}} = \frac{F}{A}$$

The unit of stress is N/m^2 or Nm^{-2}.

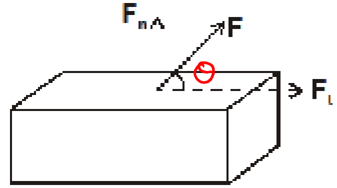
Types of Stress

1. Longitudinal or Normal stress

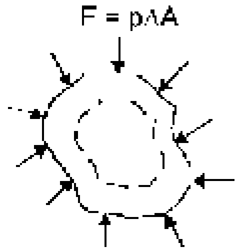
$$\text{Normal (tensile) stress} = \frac{F_n}{A} = \frac{F\sin\theta}{A}$$

2. Tangential or shear stress

$$\textbf{Tangential (shear) stress} = \frac{F_t}{A} = \frac{F\cos\theta}{A}$$

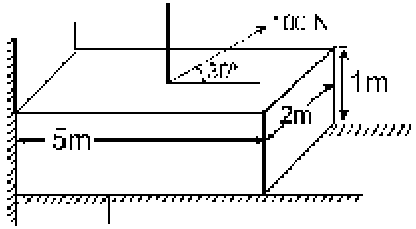


3. Bulk stress or All around stress or Pressure : When force is acting all along the surface normal to the area, then force acting per unit area is known as pressure. The effect of pressure is to produce volume change. The shape of the body may or may not change depending upon the homogeneity of body.



Example

Find out longitudinal stress and tangential stress on a fixed block



Sol.

Longitudinal or normal stress → $\sigma_l = \dfrac{100 \sin 30^\circ}{5 \times 2} = 5\ \text{N/m}^2$

Tangential stress → $\sigma_t = \dfrac{100 \cos 30^\circ}{5 \times 2} = 5\sqrt{3}\ \text{N/m}^2$

Strain

Strain, $\in = \dfrac{\text{change in configuration}}{\text{original configuration}}$

It has no unit

Types of Strain

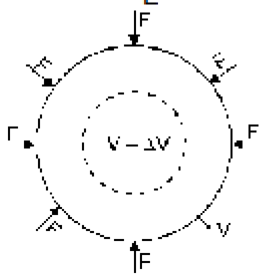
(i) Longitudinal strain :

Longitudinal strain , $\in_\ell = \dfrac{\text{change in length}}{\text{original length}}$ or Longitudinal strain = $\dfrac{\Delta L}{L}$

(ii) Volume strain :

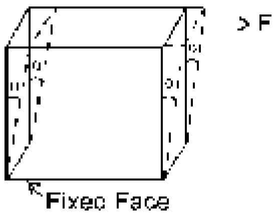
If ΔV = change in volume V = original volume

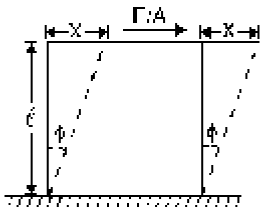
$\in_v$ = volume strain = $\dfrac{\Delta V}{V}$



(iii) Shear Strain :

$\tan \phi$ or $\phi = \dfrac{x}{\ell}$





Hooke's Law and Modulus of Elasticity

According to this law, for small deformation

i.e. stress $\propto$ strain

or stress = constant × strain or $\frac{\text{stress}}{\text{strain}}$ = Modulus of Elasticity.

$\frac{\sigma^2}{2\epsilon_0} = p$

Types of Modulus of Elasticity

Young's modulus of elasticity

$$\text{Young's modulus (Y)} = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

Normal stress = F/A,

Longitudinal strain = $\Delta L/L$

$$Y = \frac{F/A}{\Delta L/L} \quad \frac{FL}{A\Delta L}$$

Bulk modulus :

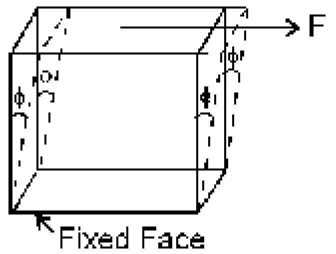
$$B = \frac{\text{Pressure}}{\text{Volume strain}} = \frac{\Delta p}{\frac{\Delta V}{V}} = -\frac{pV}{\Delta V}$$

Negative sign shows that increase in pressure (p) causes decrease in volume (ΔV).

Compressibility : The reciprocal of bulk modulus of elasticity is called compressibility.

Modulus of Rigidity :

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} \text{ or } \eta = \frac{F/A}{\phi} = \frac{F}{A\phi}$$



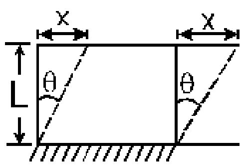
Note:

All modulus of elasticity have same units and dimensional formula i.e. Nm^{-2} or Pascal (Pa) and $[ML^{-1}T^{-2}]$

Example

A rubber cube of side 5 cm has one side fixed while a tangential force equal to 1800 N is applied to opposite face find the shearing strain and the lateral displacement of the strained face. Modulus of rigidity for rubber is 2.4×10^6 N/m^2.

Sol.

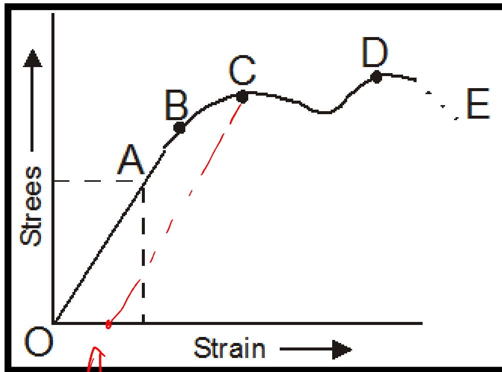


$L = 5 \times 10^{-2}$ m $\Rightarrow$ $\frac{F}{A} = \eta \frac{x}{L}$

$$\text{strain } \theta = \frac{F}{A\eta} = \frac{1800}{25 \times 10^{-4} \times 2.4 \times 10^{6}} = \frac{180}{25 \times 24} = \frac{3}{10} = 0.3 \text{ radian}$$

$\frac{x}{L} = 0.3 \Rightarrow$ $x = 0.3 \times 5 \times 10^{-2} = 1.5 \times 10^{-2}$ m $= 1.5$ mm

Variation of Strain with Stress



OA → Limit of Proportionality
OB → Elastic limit
C → Yield Point
CD → Plastic behaviour
D → Ultimate point
DE → Fracture

A material for which the plastic range CD is relatively high is called ductile material. These materials get permanently deformed before breaking. The materials for which plastic range is relatively small are called brittle materials. These materials break as soon as elastic limit is crossed.

permanent set

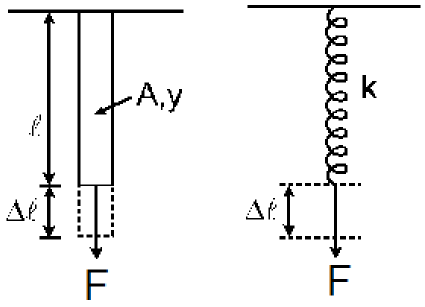
Analogy of Rod as a spring

For rod, $F = \frac{Ay}{\ell}\Delta\ell$

For spring, $F = k\Delta\ell$

Comparing, we get

$k = \frac{Ay}{\ell}$ = equivalent spring constant.



$T = 2\pi\sqrt{\frac{M}{K}}$

$= 2\pi\sqrt{\frac{ML}{YA}}$

$U = \frac{1}{2}kx^2$

$U = \frac{1}{2}\frac{YA}{L}\cdot\Delta L^2$

Elastic potential energy stored in a stretched wire or in a rod

Strain energy stored in equivalent spring

$Y = \frac{F}{A} \frac{\ell}{x}$

$$U = \frac{1}{2}kx^2$$

where $x = \frac{F\ell}{Ay}$, $k = \frac{Ay}{\ell}$ $\quad U = \frac{1}{2}\frac{Ay}{\ell}\frac{F^2\ell^2}{A^2y^2} = \frac{1}{2}\frac{F^2\ell}{Ay}$.

$$U = \frac{1}{2}\frac{F}{A} \times \frac{F}{Ay} \times A\ell \qquad \left[\text{ Strain} = \frac{F}{Ay} \right]$$

$$U = \frac{1}{2}\text{ stress} \times \text{strain} \times \text{volume}$$

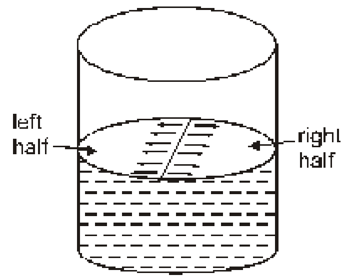
$$\text{strain energy density} = \frac{\text{strain energy}}{\text{volume}} = \frac{1}{2}\frac{(\text{stress})^2}{y} = \frac{1}{2}y(\text{strain})^2 = \frac{1}{2}\text{ stress} \times \text{strain}$$

Surface Tension

The tension generated in the surface is called surface tension force. Due to surface tension, the liquid surface behaves like a stretched membrane (rubber sheet) and try to minimize its area.

> The surface tension of a liquid can be measured as the force per unit length on an imaginary line drawn on the liquid surface, which acts perpendicular to the line on its either side at every point and tangentially to the liquid surface.

$$T = \frac{F}{\ell}$$



Note:

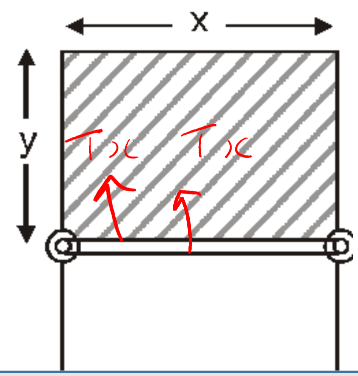
If we increase the temperature, surface tension constant (T) decreases.

If we add highly soluble substances like NaCl, $ZnSO_4$ etc. then surface tension constant (T) increases.

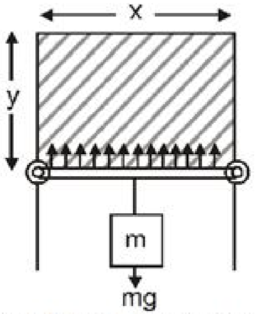
If we add sparingly soluble substances like soap, phenol, then surface tension (T) decreases.

Example

Between a frame and a light slider, a thin film of soap solution is made. Whose length is x and width is y. Find surface tension force on the slider. To keep the slider in equilibrium, how much weight should be suspended ?



Sol.



The surface will act like a tight membrane and pull the slider with a force

$$F = (T)(\ell)$$

Since this a film, it will have two surfaces: the front surface and the back surface. On the front surface, contact length is x, and also on the back surface contact length is x. So total contact length will be

$$\ell = x + x = 2x$$

So surface tension force on slider. $F = (T)(2x)$

For equilibrium, this force will be balanced by weight of suspended block.

$$(T)(2x) = mg \quad \Rightarrow \quad m = \frac{2Tx}{g}$$

$F = T \times \ell$

Surface energy

Potential energy stored due to surface tension force is called surface energy.

Surface energy U = (T)(A) = (T) (surface area)

So, Surface tension is surface energy per unit surface area.
Numerically surface energy density is same as surface tension.

Example

1000 small water drops, each of radius r, combine and form a big drop. In this process, find decrease in surface energy.

Sol.

Suppose radius of big drop is R. During this process, mass will be conserved, so volume will also be conserved.

$$(\text{Volume})_{\text{initial}} = (\text{Volume})_{\text{final}}$$

$$\left(\frac{4}{3}\pi r^3\right) \times 1000 = \left(\frac{4}{3}\pi R^3\right)$$

$\Rightarrow$ $R = 10r$

Loss in surface energy

$$\Delta U_{\text{loss}} = T\Delta A_{\text{loss}} = T(\,(4\pi r^2) \times 1000 - 4\pi(10r)^2\,)$$

$$\Delta U_{\text{loss}} = (T)\,(900 \times 4\pi r^2)$$

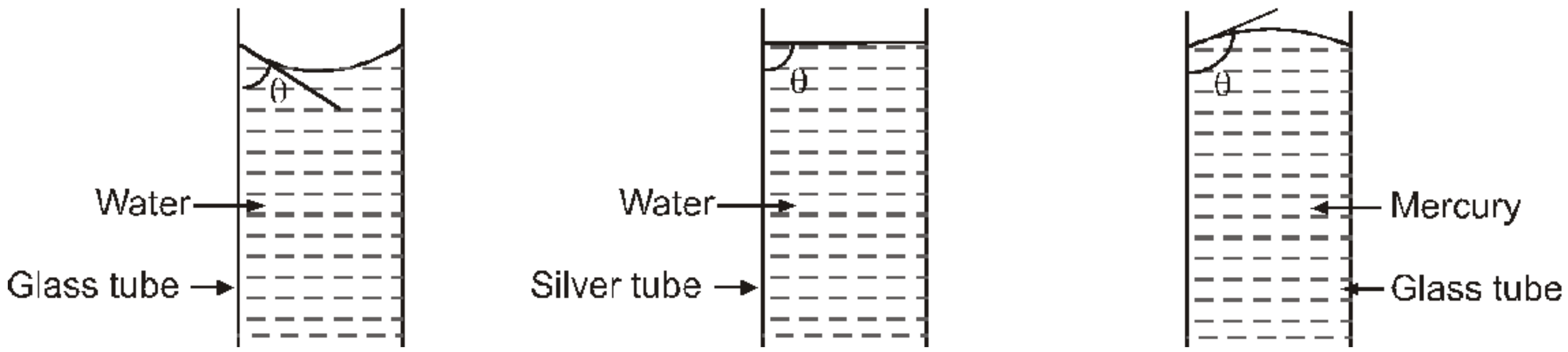
Angle of Contact, Cohesive and Adhesive force

Cohesive force : The force of attraction between the molecules of the same substance is called cohesive force.

Adhesive force : The force of attraction between different substances is called adhesive force.

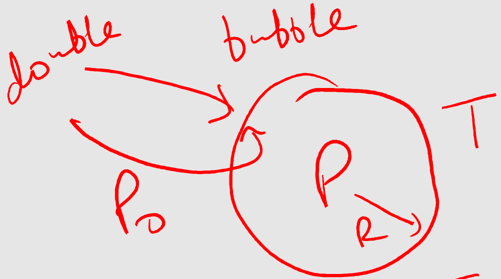
Angle of Contact : At point of contact, angle between the tangent to the liquid surface and solid surface submerged in liquid is called angle of contact (θ).

Shape of liquid surface is called meniscus.

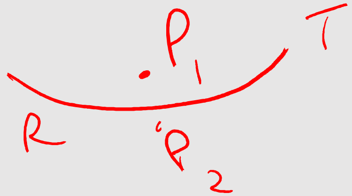


Excess Pressure inside Bubble and Drop

$$P - P_0 = \frac{2T}{R}$$



$$P - P_0 = \frac{4T}{R}$$



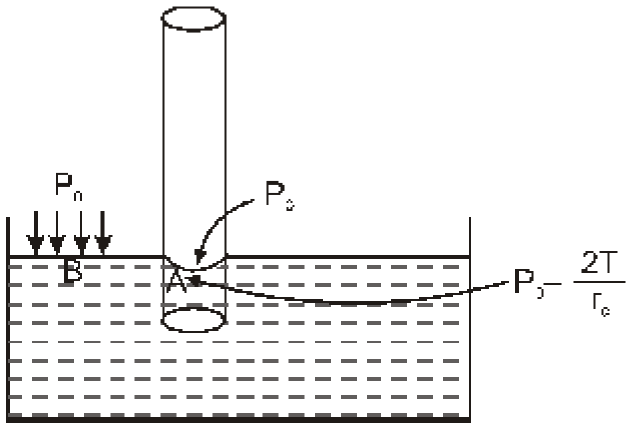
$$P_1 - P_2 = \frac{2T}{R}$$

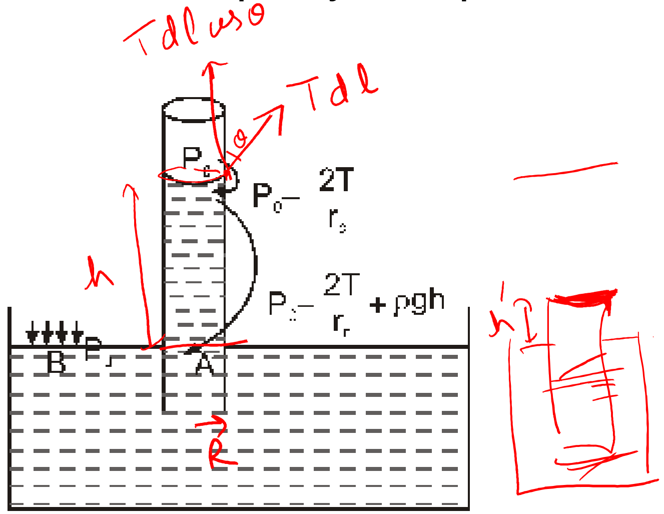
$$P - P_0 = \frac{2T}{R}$$

Always concave side has higher pressure.

Capillary Action

A glass tube of very small diameter is called capillary. The phenomenon of rise or fall of liquid in a capillary tube is called capillarity.





$$P_0 - \frac{2T}{r_c} + \rho g h = P_0$$

$$\Rightarrow \quad h = \frac{2T}{\rho g r_c}$$

$$\Rightarrow \quad h = \frac{2T\cos\theta}{\rho g R}$$

r_c = radius of curvature of the water surface

R = radius of capillary tube

$r_c = R\sec\theta$