PHYSICS

JEE and NEET CRASH COURSE

FLUID MECHANICS

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Introduction

Fluids are the substances that can flow. Therefore liquids and gases both are fluids.

Some Definitions

DENSITY (ρ)

Mass per unit volume is defined as density. So density at a point of a fluid is represented as

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

Dimensions : $[ML^{-3}]$ **S I Unit :** kg/m^3 **C G S Unit :** g/cm^3 or g/cc

$$1\ g/cc = 1000\ kg/m^3 = 1 kg/L\ (\because\ 1L = 10^3\ cm^3 = 10^{-3}\ m^3)$$

The density of water at 4°C (277 K) is $1.0 \times 10^3\ kg/m^3$ and density of mercury is $13.6 \times 10^3\ kg/m^3$.

RELATIVE DENSITY

It is defined as the ratio of the density of the given fluid to the density of pure water at 4°C.

$$\text{Relative density (R.D.)} = \frac{\text{Density of given liquid}}{\text{Density of pure water at } 4^\circ C}$$

Specific Gravity

It is defined as the ratio of the specific weight of the given fluid to the specific weight of pure water at 4°C.

$$\text{Specific gravity} = \frac{\text{Specific weight of given liquid}}{\text{Specific weight of pure water at } 4^\circ\text{C } (9.81\,\text{kN/m}^3)}$$

$$= \frac{\rho_\ell \times g}{\rho_w \times g} = \frac{\rho_\ell}{\rho_w} = \text{R.D. of the liquid}$$

Thus specific gravity of a liquid is numerically equal to the relative density of that liquid and for calculation purposes they are used interchangeably.

NOTE

- Relative density or specific gravity is a unitless and dimensionless positive scalar physical quantity.
- Being a dimensionless/unitless quantity R.D. of a substance is same in SI and CGS system.

Pressure

If a uniform force is exerted normal to an area (A), then pressure (P) is defined as the normal force (F) per unit area i.e.

$$P = \frac{F}{A}$$

$F \perp A$

Dimension of Pressure : $[ML^{-1}T^{-2}]$ ✓

$\frac{MLT^{-2}}{L^2}$

$P = \frac{F\cos\theta}{A}$

Units of Pressure :

SI unit is pascal (Pa) such that 1 Pa = 1 N/m^2

Practical units are atmospheric (atm), bar and torr

1 atm = 1.01325×10^5 Pa = 1.01325 bar = 760 torr $\approx 10^5$ Pa

1 bar = 10^5 Pa ✓

1 torr = Pressure exerted by 1 mm of mercury column

1 mm of Hg = $\rho g h$ =

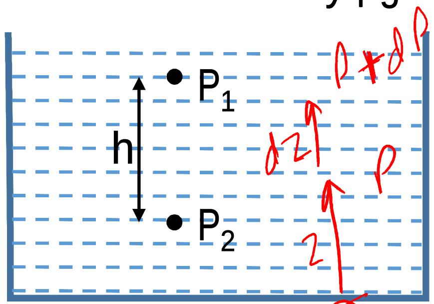
=

$\rho = \rho_0 z^2$

Variation of Pressure

Variation with height

As we go down by ‘h’ in stationary fluid by, pressure increases by ρgh



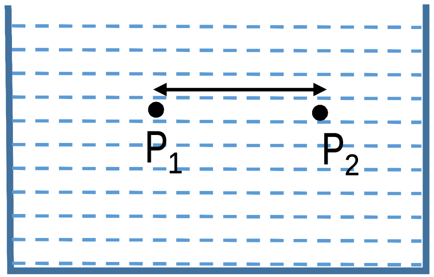
$$P_2 - P_1 = \rho g h$$

$$\boxed{\frac{dP}{dz} = -\rho g}$$

$$\Delta P = -\rho g \Delta z$$

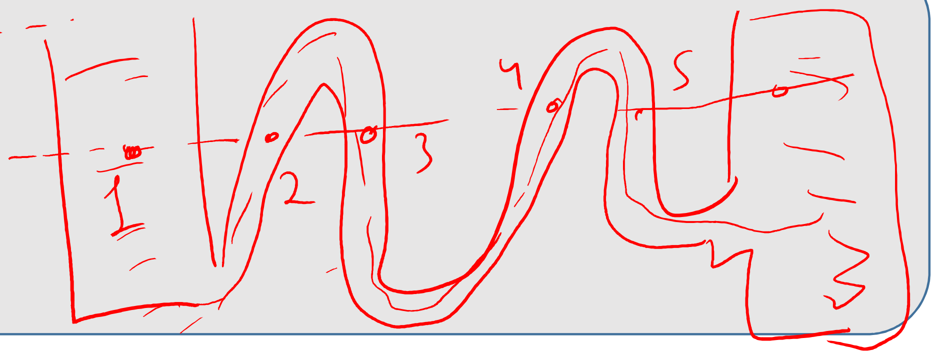
At same horizontal level

In stationary connected fluid, pressure at same horizontal level is always same.



$$P_2 = P_1$$

$$P_1 = P_2 = P_3 = \text{- - - -}$$



Pascal's Law

If the pressure in an enclosed fluid is changed at a particular point, the change is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude.

Applications of pascal's law :- hydraulic jacks, lifts, presses, brakes, etc

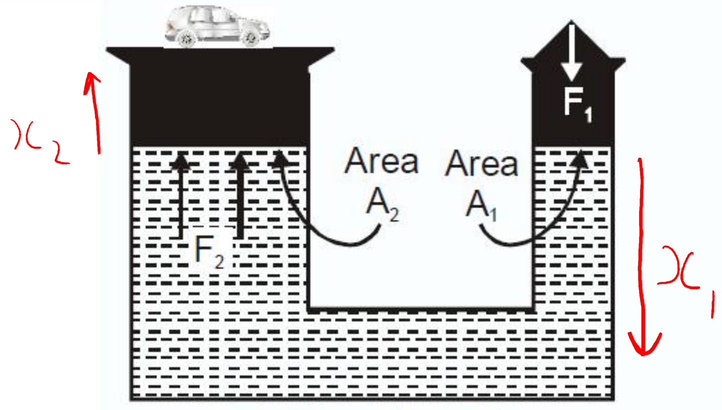
For the hydraulic lift :-

Pressure applied = $\dfrac{F_1}{A_1}$

$\therefore$ Pressure transmitted = $\dfrac{F_2}{A_2}$

$\because$ Pressures $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

$\therefore$ Upward force F_2 on $A_2 = \dfrac{F_1}{A_1} \times A_2$ or $F_2 = \dfrac{A_2}{A_1} \times F_1$



principle of hydraulic lift or press

If $A_2 >> A_1$ then $F_2 >> F_1$

$x_1 >> x_2$

Example

The diameter of the piston P_2 is 50 cm and that of the piston P_1 is 10 cm. What is the force exerted on P_2 when a force of 1 N is applied on P_1 ?

Sol.

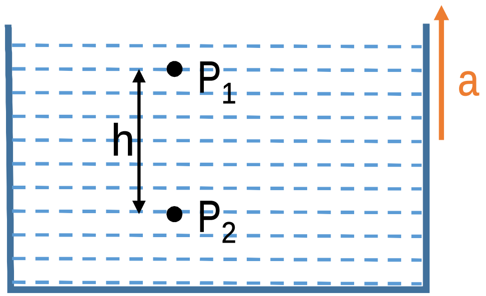
$$\frac{F_2}{F_1} = \frac{A_2}{A_1} = \left(\frac{d_2}{d_1}\right)^2$$

$$= \left(\frac{50}{10}\right)^2 = 25$$

$$F_2 = 25\ N$$

Variation of Pressure in accelerating fluid

Acceleration in vertical direction

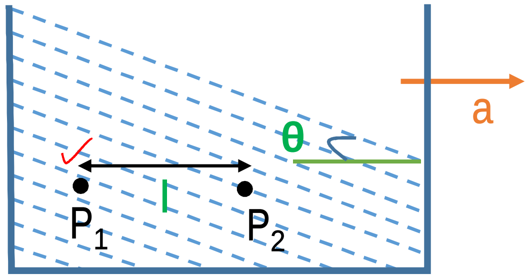


$$P_2 - P_1 = \rho g_{eff} h$$

$g_{eff} = g + a$ if a in upward direction

$g_{eff} = g - a$ if a in downward direction

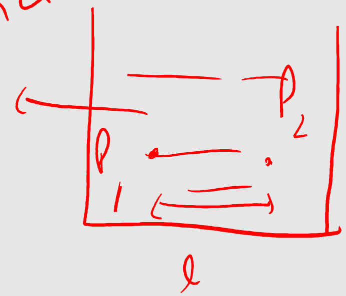
Acceleration in horizontal direction



$$P_1 - P_2 = \rho a l$$

$$\boxed{\tan\theta = \frac{a}{g}}$$

Pseudo = ma



$P_1 - P_2 = \rho a l$

Buoyancy and Archimedes' Principle

Buoyant Force : If a body is partially or wholly immersed in a fluid, it experiences an upward force due to the fluid surrounding it. This phenomenon of force exerted by fluid on the body is called *buoyancy* and force is called *buoyant force* or *upthrust*.

Archimede's Principle : It states that the buoyant force on a body that is partially or totally immersed in a liquid is equal to the weight of the fluid displaced by it.

Up thrust = buoyancy = $V\rho_\ell g$ V = volume submerged ρ_ℓ = density of liquid

Due to upthrust the weight of the body decreases

$W_{App} = W - Th$ (W is the true weight of the body)

Decrease in weight = $W - W_{App} = Th$ = Weight of the fluid displaced

If a body is weighed in air (W_A), in water (W_w) and in a liquid (W_L), then

$$\text{R.D.} = \frac{W_A}{W_A - W_W} = \frac{V\rho_o g}{V\rho_w g} = \rho_{object}/\rho_{water}$$

$$\text{specific gravity of liquid} = \frac{\text{Loss of weight in Liquid}}{\text{Loss of weight in water}} = \frac{W_A - W_L}{W_A - W_W} = \frac{V\rho_{liq}\,g}{V\rho_{water}\,g} = \frac{\rho_{liq}}{\rho_{wat}}$$

Example

A body weighs 160 g in air, 130 g in water and 136 g in oil. What is the specific gravity of oil?

Sol.

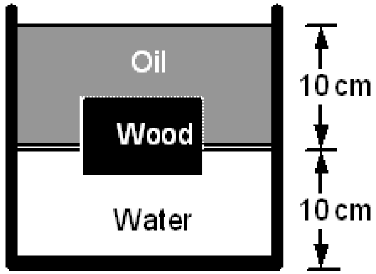
$$\text{Specific gravity of oil} = \frac{\text{Loss of weight in oil}}{\text{Loss of weight in water}}$$

$$= \frac{160-136}{160-130} = \frac{24}{30} = \frac{8}{10} = 0.8$$

Example

A cubical block of wood 10 cm on a side floats at the interface between oil and water, as in Fig.3, with its lower face 2 cm below the interface. The density of the oil is 0.6 g cm^{-3}. The mass of the block is

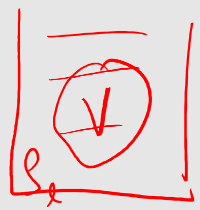
(A) 340 g (B) 680 g
(C) 80 g (D) 10 g



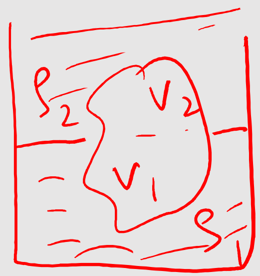
Sol.

$$mg = [100\times2\times1 + 100\times8\times0.6]\ g$$

$$\therefore \quad m = (200 + 480)\ g = 680\ g$$



$Th = V\rho_l g$



$Th = V_1\rho_1 g + V_2\rho_2 g$

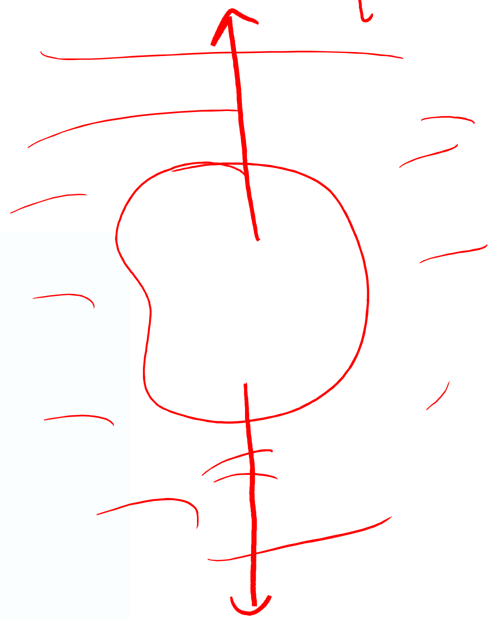
Floatation and it's Laws

$Th = V d_{liq.} g$

When a body of density (ρ_B) and volume (V) is completely immersed in a liquid of density (ρ_L), the forces acting on the body are :

(i) Weight of the body $W = Mg = V\rho_B g$ (ii) Buoyant force or Upthrust $Th = V\rho_L g$

The apparent weight W_{App} is equal to $W - Th$



Case I If density of the body is greater than that of liquid ($\rho_B > \rho_L$)

then $W > Th$

So the body will sink to the bottom of the liquid.

$W_{App} = W - Th = V\rho_B g - V\rho_L g = V\rho_B g\,(1 - \rho_L/\rho_B) = W\,(1 - \rho_L/\rho_B)$.

Case II if density of the body is equal to the density of liquid ($\rho_B = \rho_L$)

then $W = Th$

So the body will float fully submerged in the liquid. It will be in neutral equilibrium.

$$W_{App} = W - Th = 0$$

$W = V d_{body} g$

Case III if density of the body is lesser than that of liquid ($\rho_B < \rho_L$)

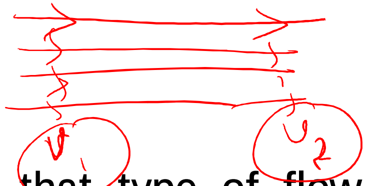
then $W < Th$

So the body will float partially submerged in the liquid. In this case the body will move up and the volume of liquid displaced by the body (V_{in}) will be less than the volume of body (V). So as to make Th equal to W

$$\therefore \quad W_{App} = W - Th = 0$$

Fluid Dynamics

When fluid flows w.r.t. container or vessel



Types of Fluid Flow

1. Steady and Unsteady Flow : Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure and density at a point do not change with time.

In an unsteady flow, the velocity, pressure and density at a point in the flow varies with time.

2. Streamline Flow : In steady flow all the particles passing through a given point follow the same path and hence a unique line of flow. This line or path is called a streamline.
Streamlines do not intersect each other.

Line of flow

3. Laminar and Turbulent Flow : Laminar flow is the flow in which the fluid particles move along well-defined streamlines which are straight and parallel. In laminar flow the velocities at different points in the fluid may have different magnitudes, but there directions are parallel. Thus the particles move in laminar or layers sliding smoothly over the adjacent layer.
Turbulent flow is an irregular flow in which the particles can move in zig-zag way due to which high energy losses take place.

Equation of Continuity

Laminar flow

Based on law of conservation of mass.

In the steady flow the mass of fluid entering into a tube of flow in a particular time interval is equal to the mass of fluid leaving out the tube.

$\rightarrow dt$

$v\,dt$

$dv = A\,v\,dt$

$\text{Rate of flow} = \frac{dV}{dt} = A\,v = \text{constant}$

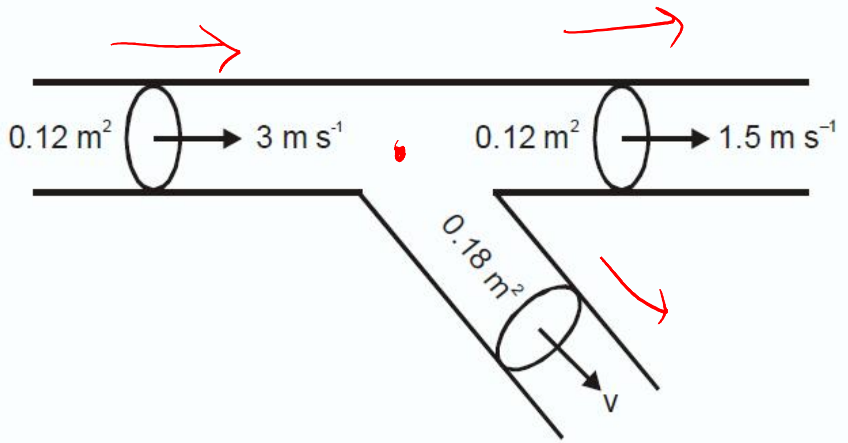
$$A_1 v_1 = A_2 v_2$$

$$Av = \text{constant}$$

Rate of flow = volume of liquid flowing per second = Av

Example

An incompressible liquid travels as shown in figure. Calculate the speed of the fluid in lower branch.



Sol.

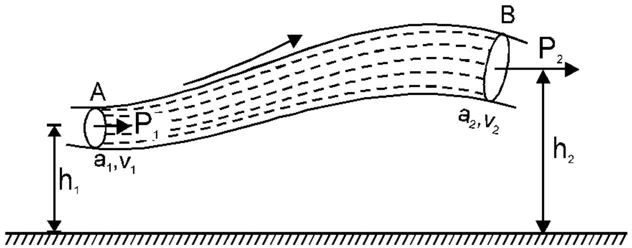
$$0.12 \times 3 = 0.12 \times 1.5 + 0.18 \times v$$

$$\cancel{0.36}^{2} = \cancel{0.18}^{1} + \cancel{0.18}^{1} v$$

$$v = 1$$

Bernoulli's Theorem

Based on law of conservation of energy.



The sum of pressure energy, kinetic energy and potential energy per unit volume remains constant along a streamline in an ideal fluid flow.

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant} \quad \text{(Energy per unit volume)}$$

or $$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant} \qquad \text{(Energy per unit weight)}$$

Here $\frac{P}{\rho g}$ is called ***pressure head***, $\frac{v^2}{2g}$ is called ***velocity/kinetic head*** and

h is called ***gravitational/potential head***

$K.E = \frac{1}{2} m v^2$

$\frac{K.E}{V} = \frac{1}{2} \frac{m}{V} v^2$

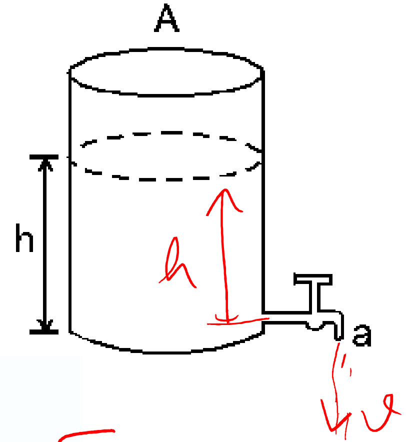
$= \frac{1}{2} \rho v^2$

Torricelli's Theorem (speed of efflux)

$$v = \sqrt{\dfrac{2gh}{1 - \dfrac{a^2}{A^2}}}$$

If A >> a, then

$$v = \sqrt{2gh}$$



Useful Result

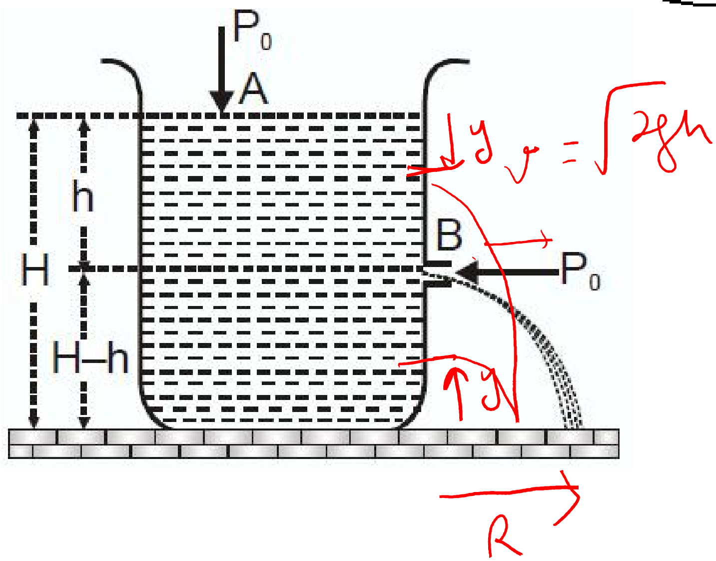
Time $t = \sqrt{2(H-h)/g}$

Range $R = 2\sqrt{h(H-h)}$

Range will be maximum when

$h = H - h$ or $h = H/2$

$$\therefore \quad R_{max.} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)} = H$$

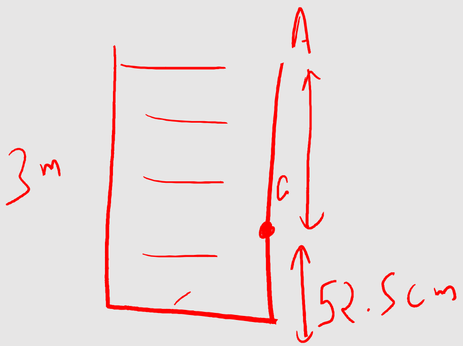


Example

Water is filled in a container upto height 3m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If a/A = 0.1 then v^2 is : (where v is the velocity of water coming out of the hole) (g = 10 m/s^2)

(A) 50 (B) 51 (C) 48 (D) 51.5

Sol.



$$v^2 = \frac{2gh}{1-\left(\frac{a}{A}\right)^2}$$

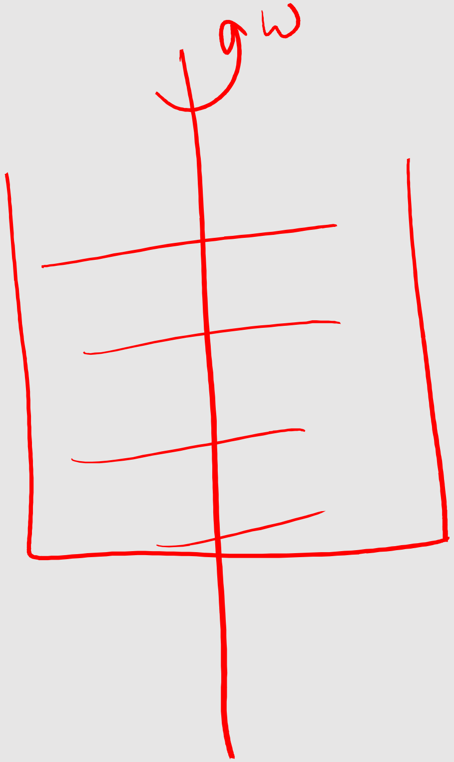
$$= \frac{2\times10\times247.5\text{ cm}}{1-0.01}$$

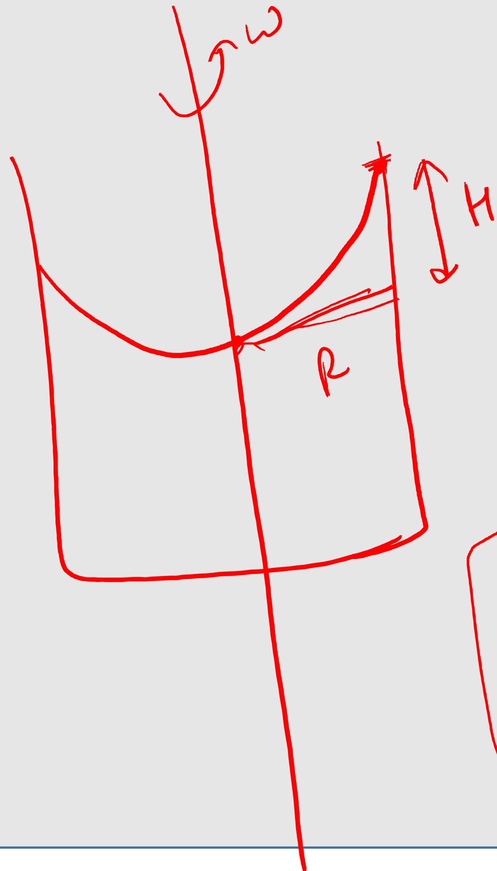
$$= \frac{20\times247.5}{0.99\times100}$$

$$\begin{array}{r} 300 \\ -\ 52.5 \\ \hline 247.5 \end{array}$$

$$= \frac{4950}{99} = 50$$

Extra Concepts





Parabolic

$$y = \frac{\omega^2 x^2}{2g}$$

$$\boxed{H = \frac{\omega^2 R^2}{2g}}$$