PHYSICS

JEE and NEET Crash Course

Problem Solving Class
(Fluid Mechanics)

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P-Q1302

Density of ice is $\mathbf{900 kg/m^3}$. A piece of ice is floating in water of density $\mathbf{1000 kg/m^3}$. Find the fraction of the piece of ice outside the water

(A) 0.9 (B) 0.5 (C) 0 (D) 0.1

$W = F_B$

$V \times 900 \times g = V' \times 1000 \times g$

$V' = \frac{9}{10} V$

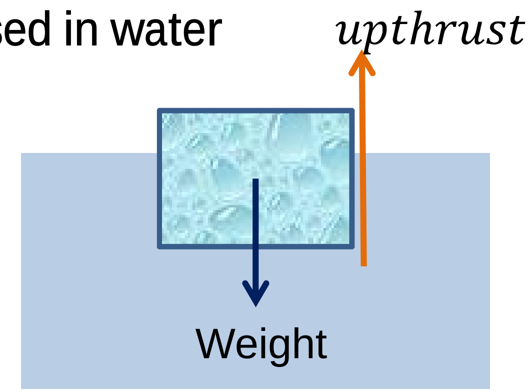
$\frac{1}{10}$

P-Q1302-Solution

Ans [D]

let V be the total volume and V_i the volume of ice piece immersed in water

for equilibrium of ice piece



$$weight = upthrust$$

Force balance at equilibrium

$$V\rho_i g = V_i \rho_w g$$

Upthurst due to water

$$\rho_i = 900 kg/m^3$$

$$\rho_w = 1000 kg/m^3$$

$$\frac{V_i}{V} = \frac{900}{1000} = 0.9$$

$$f = 1 - 0.9 = 0.1$$

Fraction of piece outside

P-Q1329

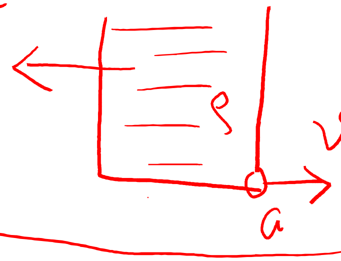
A light cylindrical vessel is kept on a horizontal surface. Its base area is A. A hole of cross sectional area a is made just at its bottom side. The ~~minimum~~ maximum coefficient of friction necessary for sliding of the vessel due to the impact force of the emerging liquid is (a<< A)

(A) varying *(B) a/A*

(C) 2a/A *(D) None of these*

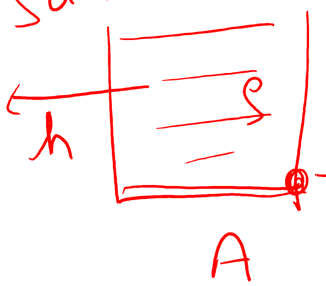
$F = \rho a v^2$

thrust



$v = \sqrt{2gh}$

$F = \rho a v^2$



$v = \sqrt{2gh}$

$F \geq \mu m g$

$\cancel{\rho} a v^2 \geq \mu . \cancel{\rho} A h g$

$a . 2g\cancel{h} \geq \mu A \cancel{h} g$

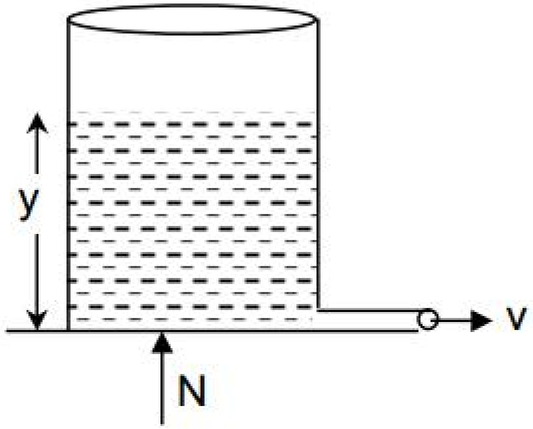
$$\boxed{\mu \leq \frac{2a}{A}}$$

P-Q1329-Solution

Ans [C]

The velocity of efflux of the liquid is given by v

$$v = \sqrt{2gy}$$



The impact force on the emerging liquid on the vessel +liquid content is equal to

$$F = v\frac{dm}{dt} = va\rho v = a\rho v^2$$

$$\Rightarrow F = a\rho\,(\sqrt{2gy})^2 = 2a\rho gy$$

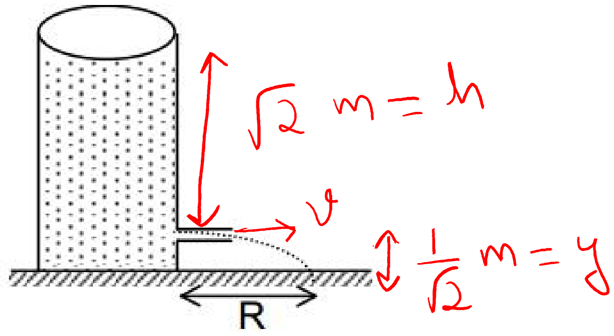
The force of friction $= f = F = 2a\rho gy$

$$\Rightarrow \mu N = 2a\rho gy \quad \Rightarrow \mu\,(A\rho gy) = 2a\,\rho gy$$

$$\Rightarrow \mu = \frac{2a}{A}.$$

P-Q1333

A small hole is made at a height of $h' = (1/\sqrt{2})$ m from the bottom of a cylindrical water tank and at a depth of $h = \sqrt{2}$ m from the upper level of water in the tank. The distance, where the water emerging from the hole strikes the ground is:



(A) $2\sqrt{2}$m

(B) 1 m

✓ (C) 2 m

(D) None of these.

$$v = \sqrt{2gh}$$

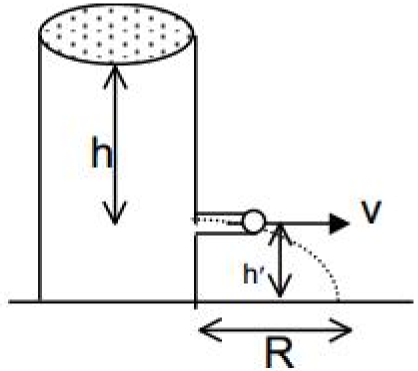
$$R = vT = \sqrt{2\cancel{g}h}\sqrt{\frac{2y}{\cancel{g}}} = 2\sqrt{hy}$$

$$= 2\sqrt{\sqrt{2} \times \frac{1}{\sqrt{2}}}$$

$$= 2\ m$$

P-Q1333-Solution

Ans [C]



$v = \sqrt{2gh}$. The range $R = v_2 \times t$,

where t = time of fall can be given by

$h' = 1/2\, gt^2 \quad \Rightarrow \quad t = \sqrt{\dfrac{2h'}{g}}$

Using 2nd eq. of motion and putting $u = 0$

$\Rightarrow \; R = v\sqrt{\dfrac{2h'}{g}}$; putting $v = \sqrt{2gh}$, we obtain $R = 2\sqrt{hh'}$

Putting $h' = \dfrac{1}{\sqrt{2}}$ m and $h = \sqrt{2}$ m, we obtain R = 2 m.

P-Q13105

The velocity of kerosene oil in a horizontal pipe is 5 m/s. If $g = 10 m/s^2$ then the velocity head of oil will be

(A) 1.25 m (B) 12.5 m

(C) 0.125 m (D) 125 m

velocity head $= \frac{K.E}{wt} = \frac{\frac{1}{2}mv^2}{mg} = \frac{v^2}{2g}$

$= \frac{25}{20} = 1.25 m$

P-Q13105-Solution

Ans [A]

$$\text{Velocity head } h = \frac{v^2}{2g} = \frac{(5)^2}{2 \times 10} = 1.25\ m$$

$\because v = \sqrt{2gh}$

P-Q13104

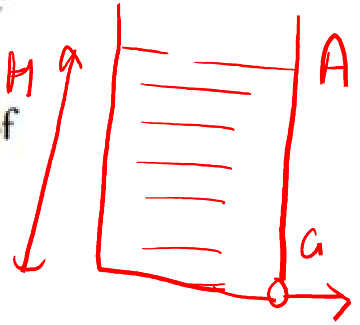
A large tank is filled with water to a height H. A small hole is made at the base of the tank. It takes T_1 time to decrease the height of water to $\frac{H}{\eta}(\eta > 1)$; and it takes T_2 time to take out the rest of water. If $T_1 = T_2$, then the value of η is

(A) 2

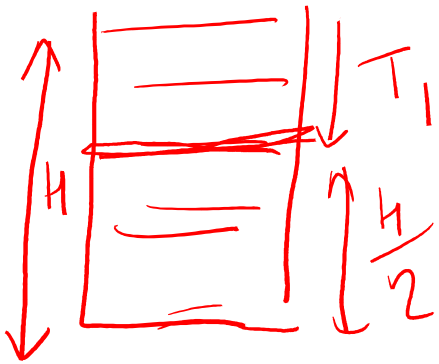
(B) 3

(C) 4

(D) $2\sqrt{2}$



Time for vessel to become empty $t = \frac{A}{a}\sqrt{\frac{2H}{g}}$



$$T_1 = \frac{A}{a}\sqrt{\frac{2H}{g}} - \frac{A}{a}\sqrt{\frac{2H/2}{g}}$$

$$T_2 = \frac{A}{a}\sqrt{\frac{2H/2}{g}}$$

$$T_1 = T_2 \Rightarrow 1 - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow 1 = \frac{2}{\sqrt{2}}$$

$$\eta = 4$$

P-Q13104-Solution

Ans [C]

$$t = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H_1} - \sqrt{H_2}\right]$$

Using efflux speed relation

The velocity gained after fall from *h*

height in air will be ,$v = \sqrt{2gh}$

$$\text{Now, } T_1 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{\frac{H}{\eta}}\right] \quad \text{1}$$

$$\text{and } T_2 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{H}{\eta}} - \sqrt{0}\right] \quad \text{2}$$

According to problem $T_1 = T_2$

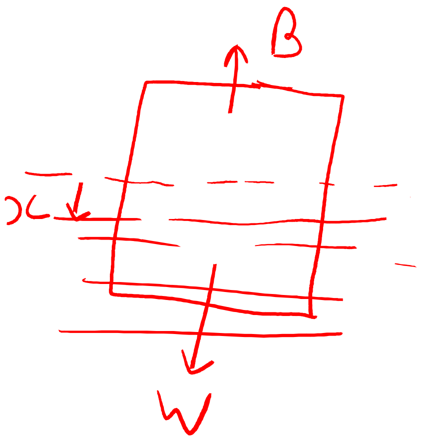
Use equation 1 and 2

$$\therefore \sqrt{H} - \sqrt{\frac{H}{\eta}} = \sqrt{\frac{H}{\eta}} - 0 \Rightarrow \sqrt{H} = 2\sqrt{\frac{H}{\eta}} \Rightarrow \eta = 4$$

P-Question

A cylindrical block of wood (density =650 kg m^{-3}), base area $30cm^2$ and height 54cm, floats in a liquid of density 900kgm^{-3}. The block is depressed slightly and then released. The time period of the resulting oscillations of the block would be equal to that of a simple pendulum of length (nearly)…..

1) 65cm   2) 52cm   3) $39cm$ 4) $26cm$



$$F = Kx$$

$$F_{extra} = V_{extra}\rho_l g = Ax\rho_L g$$

$$= (A\rho_L g)x$$

$$T = \cancel{2\pi}\sqrt{\frac{m}{K}} = \cancel{2\pi}\sqrt{\frac{l}{g}}$$

$$\frac{m}{K} = \frac{l}{g} \Rightarrow l = \frac{\cancel{A}hd_w \times \cancel{g}}{\cancel{A}\rho_L \cancel{g}} = \frac{\overset{6}{\cancel{54}} \times 65\cancel{0}}{\underset{10}{\cancel{90\cancel{0}}}}$$

$$= \frac{390}{10} = 39cm$$

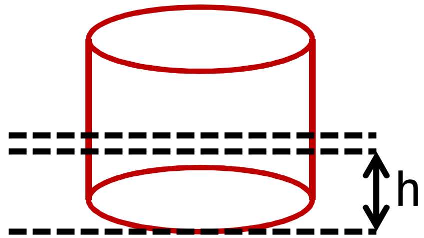
P-Solution

Ans [3]

Let block is floating with dissolve depth 'h' then about equilibrium.

$$M_{block}g = F_{up}$$

$$AH\rho_B g = Ah\rho_L g \dots .(1)$$



when block is depressed by distance x then

$F_{extra\ buoyancy\ force} = A\, x\rho_L g$ acts as restoring force on block mass m.

Hence , ma= $Ax\rho_2 g$

$m\omega^2 x = Ax\rho_L g$

$$\omega^2 = \frac{A\rho_L g}{V\rho_B} \Rightarrow \omega^2 = \frac{A\rho_L g}{AH\rho_B} \Rightarrow \omega^2 = \frac{\rho_L g}{H\rho_B}$$

But for simple pendulum, $\omega^2 = \frac{g}{L}$

Equating the above equations , we have

Length of pendulum $L = \frac{H\rho_B}{\rho_L} = 39\text{cm}$

P-Question

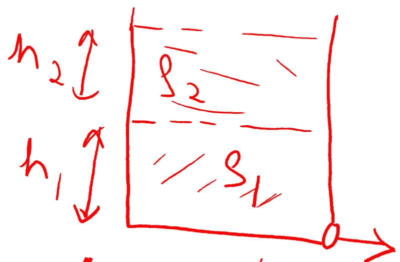
A tank with a small hole at the bottom has been filled with water and kerosene (specific gravity 0.8). The height of water is 3m and that of kerosene 2m. When the hole is opened the velocity of fluid coming out from it is nearly : (take g = 10ms^{-2} and density of water = 103 kg m^{-3})

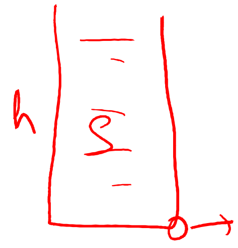
1) 10.7 ms^{-1}

2) 9.6 ms^{-1} ✓

3) 8.5 ms^{-1}

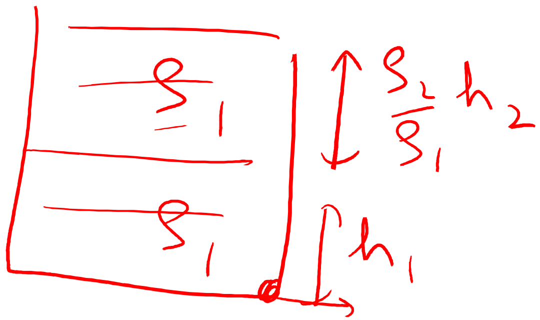
4) 7.6 ms^{-1}





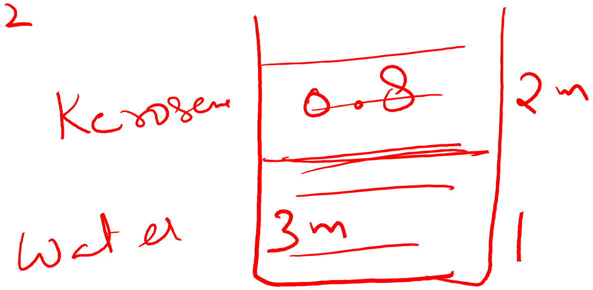
$\rho_2 h_2 = \rho_1 h' \Rightarrow h' = \frac{\rho_2}{\rho_1} h_2$

$v = \sqrt{2gh}$



$0.8 \times 2 = 1 \times h'$

$h' = 1.6$



$v = \sqrt{2gh''}$ 4.6

$= \sqrt{2 \times 10 \times 4.6}$

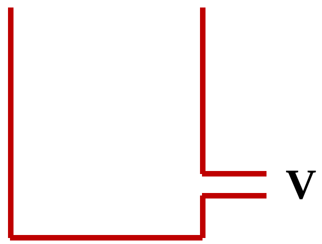
$= \sqrt{92}$

P-Solution

Ans [2]

$$P_1 + \frac{1}{2}\rho {v_1}^2 fgh \ = P_2 + fgh + \frac{1}{2}\rho {v_2}^2$$

$$= p_a\ (h\rho g)_k + (h\rho g)_w = \frac{1}{2}\ \rho.0 = p_a + \frac{1}{2}\rho_w .\ {v_2}^2$$



$${v_2}^2 = \frac{2g\ (h_k\rho_k + h_w\rho_w)}{\rho_w} \qquad v_2 = v_{\ out}$$

$$v_{out} = \sqrt{\frac{2 \times 10(2 \times 0.8 \times 10^3 + 0.3 \ \times 10^3)}{10^3}}$$

$$\sqrt{20 \times 4.6} = \sqrt{96} \quad = 9.6\ ms^{-1}$$

P-Question

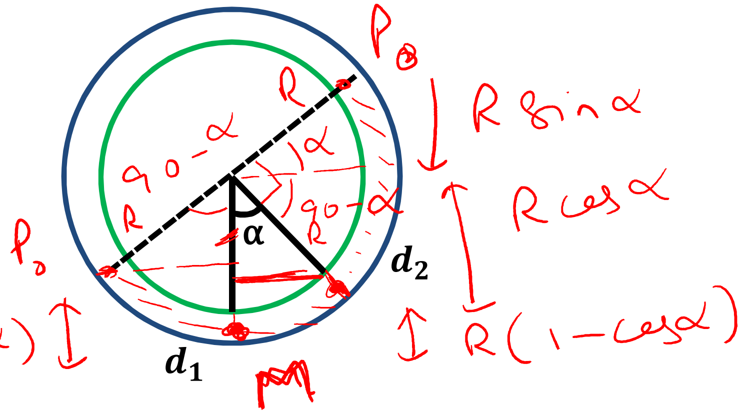
There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90^0 angle at center. Radius joining their interface makes angle α with vertical. Ratio $\frac{d_1}{d_2}$ is : [JEEMAINS - 2014]

1) $\frac{1+sin\alpha}{1-sin\alpha}$

2) $\frac{1+cos\alpha}{1-cos\alpha}$

3) $\frac{1+tan\alpha}{1-tan\alpha}$ ✓

4) $\frac{1+sin\alpha}{1-cos\alpha}$



$$\cancel{P_0} + d_1 g R(1-\sin\alpha) = \cancel{P_0} + d_2 g R(\sin\alpha + \cos\alpha) + d_1 g R(1-\cos\alpha)$$

$$d_1 \cancel{g} \cancel{R} (\cos\alpha - \sin\alpha) = d_2 \cancel{g} \cancel{R} (\sin\alpha + \cos\alpha)$$

$$\Rightarrow \frac{d_1}{d_2} = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha} = \frac{\tan\alpha + 1}{1-\tan\alpha}$$

P-Solution

Ans [~~1~~] 3

$$\frac{d_1}{d_2} = \frac{1 + tan \propto}{1 - tan \propto}$$

P-Question

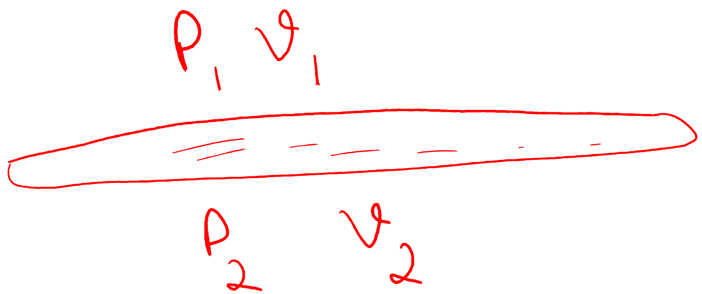
Air of density 1.2 kgm^{-3} is blowing across the horizontal wings of an aeroplane in such a way that its speeds above and below the wings are 150ms^{-1} and 100ms^{-1}, respectively.The pressure difference between the upper and lower sides of the wings, is :

1) 60 Nm^{-2}

2) 180 Nm^{-2}

3) 12500 Nm^{-2}

4) 7500 Nm^{-2} ✓



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$= \frac{1}{2} \times 1.2\,(22500 - 10000)$$

$$= 0.6 \times 12500$$

$$= 7500$$

P-Solution

Ans [4]

$$\Delta P = \frac{1}{2}\rho({v_2}^2 - {v_1}^2)$$

P-Question

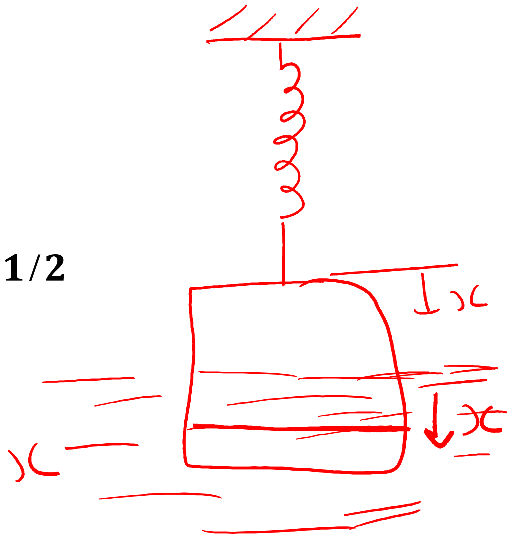
A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a mass less spring of spring constant k such that it is half submerged in a liquid of density σ at equilibrium position. When the cylinder is given a downward push and released, it starts oscillating vertically with a small amplitude. The time period T of the oscillations of the cylinder will be:

1) smaller than $2\pi\left[\frac{M}{(k+A\sigma g)}\right]^{1/2}$

2) $2\pi\sqrt{\frac{M}{k}}$

3) Larger than $2\pi\left[\frac{M}{(k+A\sigma g)}\right]^{1/2}$

4) $2\pi\left[\frac{M}{(k+A\sigma g)}\right]^{1/2}$



Extra force

$F = Kx + A x d_e g$

$= (K + A d_e g) x = (K + A d_e g) x$

$T = 2\pi\sqrt{\frac{M}{K + A\sigma g}}$

P-Solution

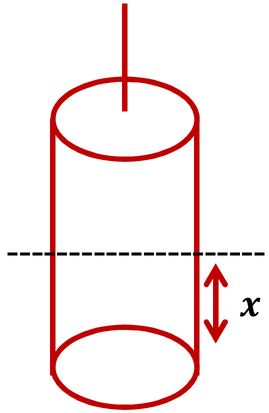
Ans [~~1~~] 4

$$F = \boldsymbol{\sigma} A\, x\, g + k\, x = [(\sigma A g + k)x =]x|-m w^2 x|$$

$$\boldsymbol{\sigma} A\, x\, g + k\, x = [(\sigma A g + k)x] =|-m w^2 x|$$

$\because$ it is in SHM $\quad \boldsymbol{\omega} = \frac{2\pi}{T}$; $\qquad T = 2\boldsymbol{\pi}\sqrt{\frac{M}{k + AFx}}$

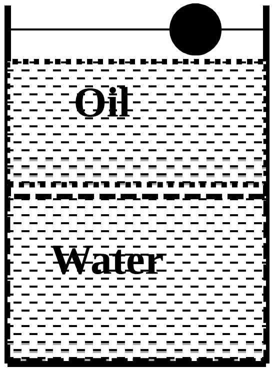
$$\boldsymbol{\omega} = \sqrt{\sigma A g + {}^{k}/_{m}}$$



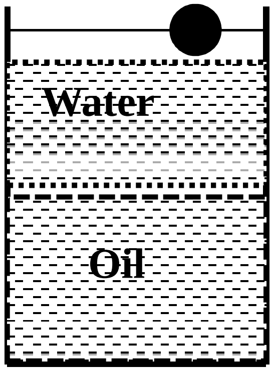
P-Question

A ball is made of a material of density $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ?

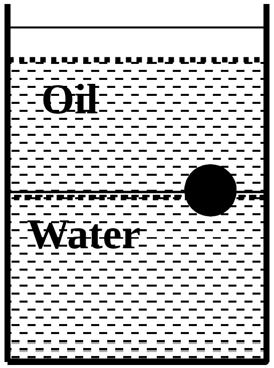
1)



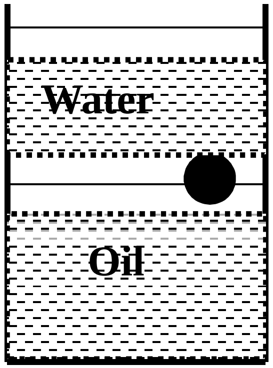
2)



3)



4)



P-Solution

Ans [3]

i. $\boldsymbol{\rho_{oil} < \rho_{water}}$Þ **oil will be above water**

ii. $\boldsymbol{\rho_{oil} < \rho < \rho_{water}}$Þ **ball will be in equilibrium at oil-water interface**

P-Question

A U- tube is partially filled with water. Oil, which does not mix with water, is next poured into one side, until water rises by 25cm on the other side. If the density of the oil is 0.8g/cc, the oil level will stand higher than the water level by

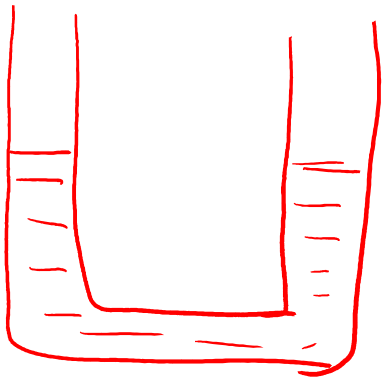
1) 6.25cm **2) 18.75 cm**

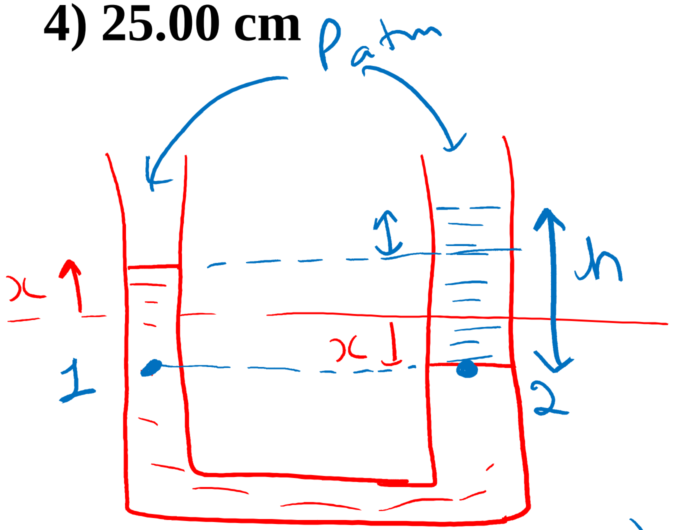
3) 12.50 cm **4) 25.00 cm**

$x = 25\,cm$

$P_1 = P_2$

$\cancel{P_{atm}} + \rho_w g(2x) = \cancel{P_{atm}} + \rho_{oil} g h$





$1 \times g\,(50\,cm) = 0.8\,g\,h$

$h = \frac{50}{\cancel{8}\,4} \times \cancel{10}\,5 = \frac{250}{4} = 62.5c$

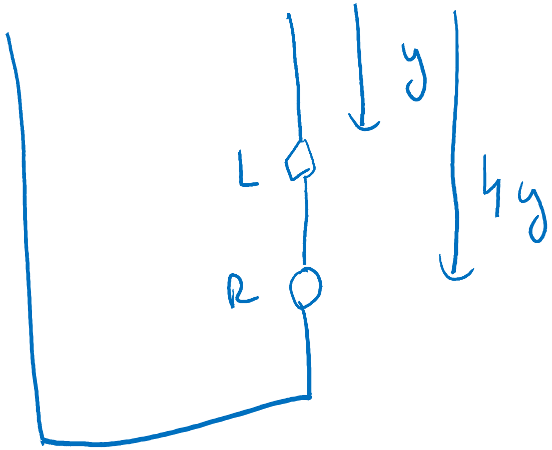
P-Solution

Ans [~~2~~] 3

P-Question

A large open tank has two holes in the wall. One is a square hole of side 'L' at a depth 'y' from the top and the other is a circular hole of radius R at a depth '4y' from the top. When the tank is completely filled with water, the quantities of water flowing out per second from the two holes are the same. Then value of R is....

1) $\frac{L}{\sqrt{2\pi}}$ ✓ **2)** $2\pi L$ **3)** $L\sqrt{\frac{2}{\pi}}$ **4)** $\frac{L}{2\pi}$



$$\frac{dV}{dt} = Av$$

$$L^2\sqrt{\cancel{2g}y} = \pi R^2\sqrt{\cancel{2g}\cdot 4y}$$

$$L^2 = 2\pi R^2$$

$$\boxed{R = \frac{L}{\sqrt{2\pi}}}$$

P-Solution

Ans [1]

$$Q = A_1 v_1 = A_2 v_2$$

$$A_1 = L^2$$
$$A_2 = \pi r^2$$

$$v_1 = \sqrt{2gy}$$
$$v_2 = \sqrt{2g.4y}$$

P-Question

A horizontal pipe of non-uniform cross-section allows water to flow through it with a velocity 1 ms^{-1} when pressure is 50 kPa at a point. If the velocity of flow has to be 2 ms^{-1} at some other point, the pressure at that point should be....

1) 50 kpa

2) 100 kpa

3) 48.5 kpa

4) 24.25 kpa

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$50\times10^3 + \frac{1}{2}\times10^3\times1 = P_2 + \frac{1}{\cancel{2}}\times10^3\times\cancel{4}^{\,2}$$

$$P_2 = 50\times10^3 + \frac{1}{2}\times10^3 - 2\times10^3$$

$$= 48.5\times10^3\ Pa = 48.5\ KPa$$

P-Solution

Ans [3]

$$P_1 + \frac{1}{2}\rho {v_1}^2 = P_2 + \frac{1}{2}\rho {v_2}^2$$