Complex Numbers

Problem Solving (JEE Mains)

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JEE Mains Problems

$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number n, then **(2019 Main, 12 April II)**

(a) $n = 20$ and $\text{Re}(z) = -10$ (b) $n = 40$ and $\text{Re}(z) = 10$

✓(c) $n = 40$ and $\text{Re}(z) = -10$ (d) $n = 20$ and $\text{Re}(z) = 10$

$\frac{2z-n}{2z+n} = 2i-1$

$\frac{2z-n}{2z+n} + 1 = 2i - 1 + 1$

$\frac{2z - \cancel{n} + 2z + \cancel{n}}{2z+n} = 2i$

$\frac{4z}{2z+n} = 2i$

$2z = 2zi + ni$

$2z(1-i) = ni$

$2z = \frac{ni}{1-i} \times \frac{1+i}{1+i}$

$2z = \frac{ni-n}{2}$

$\boxed{z = -\frac{n}{4} + \frac{n}{4}i}$

$\frac{n}{4} = 10 \Rightarrow n = 40$

$\text{Re}(z) = -\frac{n}{4} = -10$

JEE Mains Problems

Let $\left(-2-\frac{1}{3}i\right)^3 = \frac{x+iy}{27}$ $(i=\sqrt{-1})$, where x and y are real numbers, then $y-x$ equals (2019 Main, 11 Jan I)

(a) 91 (b) 85 (c) -85 (d) -91

$$\frac{(-6-i)^3}{\cancel{27}} = \frac{x+iy}{\cancel{27}}$$

$$-(6+i)^3 = x+iy$$

$$(x+iy) = -(6+i)^3$$

$$= -[216+i^3+3\times36i+3\times6i^2]$$

$$= -[216-i+108i-18]$$

$$x+iy = -198-107i$$

$x=-198$

$y=-107$

$y-x = -107+198$

$= 91$

JEE Mains Problems

All the points in the set $S = \left\{ \dfrac{\alpha + i}{\alpha - i} ; \alpha \in \mathbf{R} \right\} (i = \sqrt{-1})$ lie on a

(2019 Main, 9 April I)

(a) circle whose radius is $\sqrt{2}$.
(b) straight line whose slope is -1.
(c) circle whose radius is 1.
(d) straight line whose slope is 1.

M-2. $\alpha = \tan\theta$

$x = \dfrac{\tan^2\theta - 1}{\tan^2\theta + 1}$, $y = \dfrac{2\tan\theta}{\tan^2\theta + 1} = \sin 2\theta$

$x = -\cos 2\theta$, $y = \sin 2\theta$

$\boxed{x^2 + y^2 = 1}$

$z = \dfrac{\alpha + i}{\alpha - i}$

$x + iy = \dfrac{\alpha + i}{\alpha - i} \times \dfrac{\alpha + i}{\alpha + i}$

$x + iy = \dfrac{\alpha^2 + i^2 + 2\alpha i}{\alpha^2 - i^2} = \dfrac{\alpha^2 - 1 + 2\alpha i}{\alpha^2 + 1}$

$x = \dfrac{\alpha^2 - 1}{\alpha^2 + 1}$, $y = \dfrac{2\alpha}{\alpha^2 + 1}$

M-1

$x^2 + y^2 = \dfrac{(\alpha^2 - 1)^2}{(\alpha^2 + 1)^2} + \dfrac{4\alpha^2}{(\alpha^2 + 1)^2}$

$= \dfrac{\alpha^4 + 1 + 2\alpha^2}{(\alpha^2 + 1)^2}$

$x^2 + y^2 = \dfrac{(\alpha^2 + 1)^2}{(\alpha^2 + 1)^2}$

$\Rightarrow \boxed{x^2 + y^2 = 1}$

JEE Mains Problems

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5+3z}{5(1-z)}$, then

(2019 Main, 9 April II)

(a) $4\,\mathrm{Im}(\omega) > 5$ ✓(b) $5\mathrm{Re}(\omega) > 1$

(c) $5\,\mathrm{Im}(\omega) < 1$ (d) $5\mathrm{Re}(\omega) > 4$

$w = x + iy$

$5w - 5zw = 5 + 3z$

$3z + 5zw = 5(w-1)$

$z(3+5w) = 5(w-1)$

$z = \frac{5(w-1)}{(3+5w)}$

$|z| < 1$

$\left|\frac{5w-5}{3+5w}\right| < 1$

$\Rightarrow \frac{|5w-5|}{|3+5w|} < 1$

$\Rightarrow |5w-5| < |3+5w|$

$\Rightarrow |5x+5iy-5|^2 < |3+5x+5iy|^2$

$\Rightarrow (5x-5)^2 + \cancel{(5y)^2} < (3+5x)^2 + \cancel{(5y)^2}$

$\Rightarrow \cancel{25x^2} + 25 - 50x < 9 + \cancel{25x^2} + 30x$

$\Rightarrow 80x > 16$

$\Rightarrow x > \frac{1}{5} \Rightarrow 5x > 1$

$\Rightarrow 5\mathrm{Re}(w) > 1$

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JEE Mains Problems

Let $A = \left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary}\right\}$

$\rightarrow Re(z) = 0$

Then, the sum of the elements in A is (2019 Main, 9 Jan I)

(a) $\frac{3\pi}{4}$ (b) $\frac{5\pi}{6}$ (c) π (d) $\frac{2\pi}{3}$ ✓

$\sin^2\theta = \sin^2\alpha$.

$\theta = n\pi \pm \alpha$.

$$z = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$$

$$z = \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$$

$$Re(z) = \frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$$

$\Rightarrow 3 - 4\sin^2\theta = 0$

$\sin^2\theta = 3/4$

$\sin^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2$

$\sin^2\theta = \sin^2\frac{\pi}{3}$

$\theta = n\pi \pm \frac{\pi}{3}, \ n \in I$

$n = -2$ ✗

$n = -1$ ✗

$n = 0$ $\boxed{\frac{\pi}{3}}, \boxed{-\frac{\pi}{3}}$

$n = 1$ $\pi - \frac{\pi}{3} = 2\pi/3$

$n = 2$ ✗

Solution: $\frac{\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}$

Sum $= \frac{2\pi}{3}$

JEE Mains Problems

A value of θ for which $\dfrac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is

(2016 Main)

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{6}$ (c) $\sin^{-1}\left(\dfrac{\sqrt{3}}{4}\right)$ (d) $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

$$z = \frac{2+3i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$$

$$\mathrm{Re}(z) = \frac{2-6\sin^2\theta}{1+4\sin^2\theta} = 0$$

$$2 = 6\sin^2\theta$$

$$\Rightarrow \sin^2\theta = 1/3 \qquad \Rightarrow \sin\theta = \pm\frac{1}{\sqrt{3}}$$

$$\theta = \pm\sin^{-1}\frac{1}{\sqrt{3}}$$

JEE Mains Problems

$i^{4m} + i^{4m+1} + i^{4m+2} + i^{4m+3} = 0$

The value of sum $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals (1998, 2M)

(a) i (b) $i - 1$ (c) $-i$ (d) 0

$$\sum_{n=1}^{13} i^n + i^{n+1} = (i + i^2 + i^3 + i^4 - - - i^{13}) + (i^2 + i^3 - - - i^{14})$$

$$= i^{13} + i^{14}$$

$$= i + i^2$$

$$= i - 1$$

JEE Mains Problems

The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$, is

(a) 8 (b) 16 **(1980, 2M)**

(c) 12 (d) None of these

$\frac{1+i}{1-i} = \left(\frac{1+i}{1-i}\right) \times \frac{(1+i)}{1+i}$

$= \frac{\cancel{1}+\cancel{i^2}+2i}{1-i^2} = \frac{2i}{2} = i$

$i^n = 1$

Smallest n is 4.

JEE Mains Problems

The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$, is $\rightarrow n=4$

(a) 8 (b) 16 **(1980, 2M)**

(c) 12 (d) None of these

$$\frac{1+i}{1-i} = \left(\frac{1+i}{1-i}\right) \times \frac{(1+i)}{1+i}$$

$$= \frac{\cancel{1}+\cancel{i^2}+2i}{1-i^2} = \frac{2i}{2} = \cancel{2}.i$$

$i^n = 1$

Smallest n is 4.

JEE Mains Problems

The equation $|z - i| = |z - 1|$, $i = \sqrt{-1}$, represents

(2019 Main, 12 April I)

(a) a circle of radius $\frac{1}{2}$

(b) the line passing through the origin with slope 1

(c) a circle of radius 1

(d) the line passing through the origin with slope -1

$\rightarrow \quad z = x + iy$

$|x + iy - i| = |x + iy - 1|$

$|x + i(y-1)|^2 = |x - 1 + iy|^2$

$x^2 + (y-1)^2 = (x-1)^2 + y^2$

$\cancel{x^2} + \cancel{y^2} + \cancel{1} - 2y = \cancel{x^2} + \cancel{1} - 2x + \cancel{y^2}$

$-2x = -2y$

$\Rightarrow \boxed{y = x}$ *

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JEE Mains Problems

If $a > 0$ and $z = \dfrac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\dfrac{2}{5}}$, then $\bar{z}$ is equal to **(2019 Main, 10 April I)**

(a) $\frac{1}{5} - \frac{3}{5}i$ (b) $-\frac{1}{5} - \frac{3}{5}i$

(c) $-\frac{1}{5} + \frac{3}{5}i$ (d) $-\frac{3}{5} - \frac{1}{5}i$

$$z = \frac{\cancel{1} + \cancel{i^2} + 2i}{\alpha - i} = \frac{2i}{\alpha - i} \times \frac{\alpha + i}{\alpha + i}$$

$$z = \boxed{\frac{-2 + 2\alpha i}{\alpha^2 + 1}}$$

$$|z| = \sqrt{\frac{4}{(\alpha^2+1)^2} + \frac{4\alpha^2}{(\alpha^2+1)^2}} = \sqrt{\frac{2}{5}}$$

$$\frac{4 + 4\alpha^2}{(\alpha^2+1)^2} = \frac{2}{5}$$

$$\frac{\cancel{4}\,2\,\cancel{(\alpha^2+1)}}{(\alpha^2+1)^2} = \frac{\cancel{2}}{5}$$

$$10 = \alpha^2 + 1$$

$$\alpha^2 = 9$$

$$\alpha = \pm 3$$

$$\Rightarrow \alpha = 3.$$

$$z = \frac{2i}{\alpha - i} \quad \frac{-2 + 2\alpha i}{\alpha^2 + 1}$$

$$z = \frac{-2 + 6i}{10} = \frac{-1 + 3i}{5}$$

$$\bar{z} = -\frac{1}{5} - \frac{3}{5}i$$

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JEE Mains Problems

Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then, the minimum value of $|z_1 - z_2|$ is **(2019 Main, 12 Jan II)**

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) 0 ✓

$|z_1| = 9$

$z_1 = x_1 + iy_1$

$\sqrt{x^2+y^2} = 9$

$\boxed{x^2+y^2 = 81}$

$\uparrow$

z_1

$C_1(0,0)$

$r_1 = 9$

$|z_2 - 3 - 4i| = 4.$

$z_2 = x_2 + iy_2.$

$\boxed{(x_2-3)^2 + (y_2-4)^2 = 16}$

$(x-3)^2 + (y-4)^2 = 16$

$\uparrow$

z_2

$C_2(3,4)$

$r_2 = 4.$

C_1C_2 $\quad r_1 + r_2 \quad |r_1 - r_2|$

$C_1C_2 = 5.$ $\quad 5.$

$C_1C_2 = |r_1 - r_2|$

JEE Mains Problems

If $\frac{z-\alpha}{z+\alpha}(\alpha \in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is (2019 Main, 12 Jan I)

(a) $\sqrt{2}$ (b) $\frac{1}{2}$ (c) 1 (d) 2 ✓

$z + \bar{z} = 2\mathrm{Re}(z)$

$z + \bar{z} = 0$

$$\frac{z-\alpha}{z+\alpha} + \left(\overline{\frac{z-\alpha}{z+\alpha}}\right) = 0$$

$$\frac{z-\alpha}{z+\alpha} + \frac{\overline{z-\alpha}}{\overline{z+\alpha}} = 0$$

$$\frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\bar{\alpha}}{\bar{z}+\bar{\alpha}} = 0$$

$$\frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\alpha}{\bar{z}+\alpha} = 0$$

$$(z-\alpha)(\bar{z}+\alpha) + (z+\alpha)(\bar{z}-\alpha) = 0$$

$$z\bar{z} + \cancel{\alpha z} - \cancel{\alpha\bar{z}} - \alpha^2 + z\bar{z} + \cancel{\alpha\bar{z}} - \cancel{\alpha z} - \alpha^2 = 0$$

$$\cancel{2}z\bar{z} = \cancel{2}\alpha^2$$

$$|z|^2 = \alpha^2 \Rightarrow \alpha^2 = 4 \Rightarrow \alpha = \pm 2.$$

JEE Mains Problems

Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$).

Then, $|z|$ is equal to (2019 Main, 11 Jan II)

(a) $\frac{\sqrt{34}}{3}$ (b) $\frac{5}{3}$ (c) $\frac{\sqrt{41}}{4}$ (d) $\frac{5}{4}$

$z = x + iy$

$\sqrt{x^2+y^2} + x + iy = 3 + i$

$\Rightarrow \sqrt{x^2+y^2} = 3 - x + i(1-y)$

$\Rightarrow \sqrt{x^2+y^2} = 3 - x$

$\cancel{x^2} + y^2 = 9 + \cancel{x^2} - 6x$

$6x = 8 \Rightarrow x = 4/3$

$0 = 1 - y$

$y = 1$

$z = \frac{4}{3} + i$

$|z| = \sqrt{\frac{16}{9} + 1} = \sqrt{\frac{25}{9}} = \frac{5}{3}$

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JEE Mains Problems

$z\bar{z} = |z|^2$

A complex number z is said to be unimodular, if $|z| \neq 1$.
If z_1 and z_2 are complex numbers such that $\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. $|z_2| \neq 1$
Then, the point z_1 lies on a **(2015 Main)**

(a) straight line parallel to X-axis
(b) straight line parallel to Y-axis
(c) circle of radius 2
(d) circle of radius $\sqrt{2}$

$$\left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$$

$$|z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$$

$$(z_1 - 2z_2)(\overline{z_1 - 2z_2}) = (2 - z_1\bar{z}_2)(\overline{2 - z_1\bar{z}_2})$$

$$(z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$$

$$z_1\bar{z}_1 - \cancel{2z_1\bar{z}_2} - \cancel{2\bar{z}_1 z_2} + 4z_2\bar{z}_2 = 4 - \cancel{2\bar{z}_1 z_2} - \cancel{2z_1\bar{z}_2} + z_1 z_2\bar{z}_1\bar{z}_2$$

$$|z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2|z_2|^2$$

$$|z_1|^2 - |z_1|^2|z_2|^2 + 4|z_2|^2 - 4 = 0$$

$$|z_1|^2(1 - |z_2|^2) - 4(1 - |z_2|^2) = 0$$

$$(|z_1|^2 - 4)(1 - |z_2|^2) = 0$$

$$|z_1|^2 = 4 \quad \text{or} \quad |z_2|^2 = 1 \; \times$$

$$|z_1|^2 = 4$$

$$|z_1| = 2$$

JEE Mains Problems

If z is a complex number of unit modulus and argument θ, then $\arg\left(\dfrac{1+z}{1+\bar{z}}\right)$ is equal to **(2013 Main)**

(a) $-\theta$ (b) $\dfrac{\pi}{2}-\theta$ (c) θ (d) $\pi-\theta$

$|z|=1 \qquad \arg(z)=\theta$

$z = re^{i\theta} = e^{i\theta}$

$\bar{z} = e^{-i\theta} = \dfrac{1}{e^{i\theta}} = \dfrac{1}{z}.$

$\arg\left(\dfrac{1+z}{1+\bar{z}}\right) = ? = \arg(z)$

$\dfrac{1+z}{1+\bar{z}} = \dfrac{1+z}{1+\frac{1}{z}} = \dfrac{\cancel{1+z}}{\cancel{(z+1)}/z} = z,$

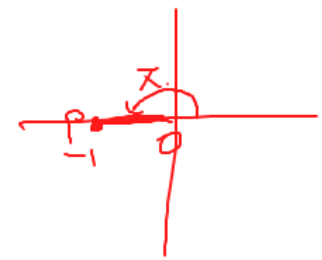
$\boxed{\dfrac{1+z}{1+\bar{z}} = z}$

JEE Mains Problems

If $\arg(z) < 0$, then $\arg(-z) - \arg(z)$ equals (2000, 2M)

(a) π
(b) $-\pi$
(c) $-\pi/2$
(d) $\pi/2$

$$\arg \frac{z_1}{z_2} = \arg z_1 - \arg z_2$$



$$\arg(-z) - \arg(z) = \arg\left(\frac{-z}{z}\right) = \arg(-1)$$

$$= \pi$$

JEE Mains Problems

$\cos\theta + i\sin\theta$

$-\pi < \theta \leq \pi$.

Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of z then (2019 Main, 10 Jan II)

(a) $R(z) > 0$ and $I(z) > 0$ (b) $I(z) = 0$

(c) $R(z) < 0$ and $I(z) > 0$ (d) $R(z) = -3$

$z = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^5 + \left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right]^5$

$= \cos\frac{5\pi}{6} + \cancel{i\sin\frac{5\pi}{6}} + \cos\left(-\frac{5\pi}{6}\right) + \cancel{i\sin\left(-\frac{5\pi}{6}\right)}$

$= 2\cos\frac{5\pi}{6}$

$= 2\cos\left(\pi - \frac{\pi}{6}\right) = -\sqrt{3}$

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JEE Mains Problems

If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then (2019 Main, 10 April II)

(a) $\bar{z}w = -i$ (b) $z\overline{w} = \frac{1-i}{\sqrt{2}}$

(c) $\bar{z}w = i$ (d) $z\overline{w} = \frac{-1+i}{\sqrt{2}}$

$\bar{z}w = -i$

$\overline{\bar{z}w} = \overline{-i}$

$z\bar{w} = i$

$\rightarrow$ $|zw| = 1 \Rightarrow |z||w| = 1$

$\arg z = \frac{\pi}{2} + \arg(w)$

$|z| = r, \ |w| = \frac{1}{r}$

$\arg w = \theta$,

$\arg z = \frac{\pi}{2} + \theta$

$z = r e^{i(\frac{\pi}{2}+\theta)}, \quad w = \frac{1}{r} e^{i\theta}$

$\bar{z} = r e^{-i(\frac{\pi}{2}+\theta)}, \quad w = \frac{1}{r} e^{i\theta}$

$\bar{z}w = e^{-i(\frac{\pi}{2}+\theta)} \times e^{i\theta}$

$\bar{z}w = e^{i(\theta - \frac{\pi}{2} - \theta)} = e^{-\frac{\pi}{2}i}$

$= \cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2}) = -i$

JEE Mains Problems

$z = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

If $z = \frac{\sqrt{3}}{2} + \frac{i}{2} (i = \sqrt{-1})$, then $(1 + iz + z^5 + iz^8)^9$ is equal to **(2019 Main, 8 April II)**

$-i\omega$

(a) 1 (b) $(-1+2i)^9$ (c) -1 (d) 0

$\omega = -\frac{1}{2} + \frac{\sqrt{3}i}{2}$

$\boxed{\omega^2 = -\frac{1}{2} - \frac{\sqrt{3}i}{2}}$

$-\omega^2 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$-\omega = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

$-i\omega = \frac{1}{2}i - \frac{\sqrt{3}}{2}i^2$

$-i\omega = \frac{1}{2}i + \frac{\sqrt{3}}{2}$

$z = -i\omega$

$$(1+iz+z^5+iz^8)^9 = [1 + i(-i\omega) + (-i\omega)^5 + i(-i\omega)^8]^9$$

$$= [1 - i^2\omega - i^5\omega^5 + i^9\omega^8]^9$$

$$= [1 + \omega - \cancel{i\omega^2} + \cancel{i\omega^2}]^9$$

$$= [1+\omega]^9 = (-\omega^2)^9$$

$$= -\omega^{18} = -1$$

JEE Mains Problems

Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$, $\to \omega$, $\to \omega^2$

If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then $\arg z$ is equal to

(2019 Main, 9 Jan II)

(a) $\frac{\pi}{4}$ ✓ (b) $\frac{\pi}{6}$ (c) 0 (d) $\frac{\pi}{3}$

$z = 3 + 6i\,\omega^{81} - 3i\,\omega^{93}$

$z = 3 + 6i - 3i$

$z = 3 + 3i$

$\alpha = \tan^{-1}\frac{3}{3} = \frac{\pi}{4}$

$\theta = \frac{\pi}{4}$

$\theta = \pi - \alpha$	$\theta = \alpha$
$\theta = \alpha - \pi$	$\theta = -\alpha$

JEE Mains Problems

Let z_0 be a root of the quadratic equation, $x^2+x+1=0$, $\rightarrow \omega$ $\rightarrow \omega^2$
If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to

(2019 Main, 9 Jan II)

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) 0 (d) $\frac{\pi}{3}$

$z = 3 + 6i\omega^{81} - 3i\omega^{93}$

$z = 3 + 6i - 3i$

$z = 3 + 3i$

$\alpha = \tan^{-1}\frac{3}{3} = \frac{\pi}{4}$

$\theta = \frac{\pi}{4}$

$\theta = \pi - \alpha$	$\theta = \alpha$
$\theta = \alpha - \pi$	$\theta = -\alpha$

JEE Mains Problems

Let $z = \cos\theta + i\sin\theta$. Then, the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is **(2009)**

(a) $\frac{1}{\sin 2°}$ (b) $\frac{1}{3\sin 2°}$

(c) $\frac{1}{2\sin 2°}$ (d) $\frac{1}{4\sin 2°}$ ✓

$$z^{2m-1} = (\cos\theta + i\sin\theta)^{2m-1} = \cos(2m-1)\theta + i\sin(2m-1)\theta$$

$$\text{Im}(z^{2m-1}) = \sin(2m-1)\theta$$

$$\sum_{m=1}^{15} \sin(2m-1)\theta = \sin\theta + \sin3\theta + \sin5\theta \cdots \sin 29\theta$$

Angles in A.P

$\alpha = \theta$, $\beta = 2\theta$, $n = 15$

$\beta/2 = \theta$

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) \cdots \sin(\alpha+(n-1)\beta)$$

$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cdot \sin\left(\alpha + (n-1)\frac{\beta}{2}\right)$$

$$S = \frac{\sin 15\theta}{\sin\theta} \cdot \sin(\theta + 14\theta)$$

$$= \frac{\sin 15\theta \cdot \sin 15\theta}{\sin\theta}$$

$$= \frac{\sin 30° \cdot \sin 30°}{\sin\theta} = \frac{\sin^2 30°}{\sin\theta}$$

$$= \frac{1}{4\sin\theta} = \frac{1}{4\sin 2°}$$