Problem Solving on Differential Equations

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If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ is equal to **(2017 Main)**

(a) $\frac{1}{3}$ (b) $-\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) $\frac{4}{3}$

The general solution of the differential equation $(y^2 - x^3)dx - xydy = 0 \ (x \neq 0)$ is (where, C is a constant of integration) **(2019 Main, 12 April II)**

(a) $y^2 - 2x^2 + Cx^3 = 0$
(b) $y^2 + 2x^3 + Cx^2 = 0$
(c) $y^2 + 2x^2 + Cx^3 = 0$
(d) $y^2 - 2x^3 + Cx^2 = 0$

Consider the differential equation, $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$.

If value of y is 1 when $x = 1$, then the value of x for which $y = 2$, is **(2019 Main, 12 April I)**

(a) $\frac{5}{2} + \frac{1}{\sqrt{e}}$ (b) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (c) $\frac{1}{2} + \frac{1}{\sqrt{e}}$ (d) $\frac{3}{2} - \sqrt{e}$

Let $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 1$. Then **(2019 Main, 10 April II)**

(a) $y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$ (b) $y'\left(\frac{\pi}{4}\right) + y'\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

(c) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$ (d) $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$

The solution of the differential equation $x\frac{dy}{dx}+2y=x^2(x\neq 0)$ with $y(1)=1$, is **(2019 Main, 9 April I)**

(a) $y=\frac{x^2}{4}+\frac{3}{4x^2}$

(b) $y=\frac{x^3}{5}+\frac{1}{5x^2}$

(c) $y=\frac{3}{4}x^2+\frac{1}{4x^2}$

(d) $y=\frac{4}{5}x^3+\frac{1}{5x^2}$

If a curve passes through the point $(1, -2)$ and has slope of the tangent at any point (x, y) on it as $\frac{x^2 - 2y}{x}$, then the curve also passes through the point

(2019 Main, 12 Jan II)

(a) $(\sqrt{3}, 0)$ (b) $(-1, 2)$

(c) $(-\sqrt{2}, 1)$ (d) $(3, 0)$

Let f be a differentiable function such that $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}$, $(x > 0)$ and $f(1) \neq 4$. Then, $\lim_{x \to 0^+} x f\left(\frac{1}{x}\right)$

(2019 Main, 10 Jan II)

(a) does not exist

(b) exists and equals $\frac{4}{7}$

(c) exists and equals 0

(d) exists and equals 4

Let $y = y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x$, $x \in (0, \pi)$.

If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(2018 Main)

(a) $\frac{4}{9\sqrt{3}}\pi^2$ (b) $\frac{-8}{9\sqrt{3}}\pi^2$ (c) $-\frac{8}{9}\pi^2$ (d) $-\frac{4}{9}\pi^2$

The curve amongst the family of curves represented by the differential equation, $(x^2 - y^2)dx + 2xydy = 0$, which passes through $(1, 1)$, is **(2019 Main, 10 Jan II)**

(a) a circle with centre on the Y-axis

(b) a circle with centre on the X-axis

(c) an ellipse with major axis along the Y-axis

(d) a hyperbola with transverse axis along the X-axis.

Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t)$ is equal to

(2014 Main)

(a) $400 - 300e^{\frac{t}{2}}$

(b) $300 - 200e^{-\frac{t}{2}}$

(c) $600 - 500e^{\frac{t}{2}}$

(d) $400 - 300e^{-\frac{t}{2}}$

At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P with respect to additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employees 25 more workers, then the new level of production of items is

(2013 Main)

(a) 2500 (b) 3000 (c) 3500 (d) 4500

Given that the slope of the tangent to a curve $y = y(x)$ at any point (x, y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is **(2019 Main, 8 April II)**

(a) $x^2 \log_e |y| = -2(x - 1)$ (b) $x \log_e |y| = x - 1$

(c) $x \log_e |y| = 2(x - 1)$ (d) $x \log_e |y| = -2(x - 1)$

Let f be a differentiable function such that $f(1) = 2$ and $f'(x) = f(x)$ for all $x \in R$. If $h(x) = f(f(x))$, then $h'(1)$ is equal to **(2019 Main, 12 Jan II)**

(a) $4e^2$ (b) $4e$ (c) $2e$ (d) $2e^2$

The solution of the differential equation, $\frac{dy}{dx} = (x - y)^2$, when $y(1) = 1$, is **(2019 Main, 11 Jan II)**

(a) $\log_e \left| \frac{2 - y}{2 - x} \right| = 2(y - 1)$

(b) $-\log_e \left| \frac{1 + x - y}{1 - x + y} \right| = x + y - 2$

(c) $\log_e \left| \frac{2 - x}{2 - y} \right| = x - y$

(d) $-\log_e \left| \frac{1 - x + y}{1 + x - y} \right| = 2(x - 1)$

A solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$$ is

(a) $y = 2$

(b) $y = 2x$

(c) $y = 2x - 4$

(d) $y = 2x^2 - 4$