Sequence & Series

Problem Solving (JEE Mains)

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JEE Mains Problems

If T_r is the rth term of an AP, for $r = 1, 2, 3, \ldots$. If for some positive integers m and n, we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals

(1998, 2M)

(a) $\frac{1}{mn}$ (b) $\frac{1}{m} + \frac{1}{n}$ (c) 1 (d) 0

$T_m = \frac{1}{n}$, $T_n = \frac{1}{m}$ $T_{mn} = ?$

$a + (m-1)d = \frac{1}{n}$ — (1)

$a + (n-1)d = \frac{1}{m}$ — (2)

$(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$ $d = \frac{1}{mn}$, $a = \frac{1}{mn}$

$T_{mn} = a + (mn-1)d$

$= \frac{1}{mn} + \frac{(mn-1)}{mn}$

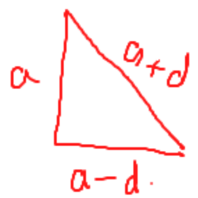
$= \frac{\cancel{1} + mn - \cancel{1}}{mn} = 1$

JEE Mains Problems

Integer Type.

The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side? 6 **(2017 Adv.)**

$a-d,\ a,\ a+d$ (increasing A.P) $(d>0)$



Pythagoras theorem.

$a^2+(a-d)^2=(a+d)^2$ — (1)

$a^2+\cancel{a^2+d^2}-2ad = \cancel{a^2+d^2}+2ad$

$a^2=4ad$

$\boxed{a=4d}$

Area

$\frac{1}{2}\times a\times(a-d)=24$ — (2)

$\frac{1}{\cancel{2}}\times \overset{2}{\cancel{4}}d\times 3d=24$

$d^2=4$

$\boxed{d=2}$

$a=8$

Ans: $a-d=8-2=6$

JEE Mains Problems

If $a_1, a_2, a_3, \ldots$ are in AP such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this AP is

(2019 Main, 12 April II)

(a) 200 (b) 280 (c) 120 (d) 150

First term : a cd : d.

$a_1 + a_7 + a_{16} = 40$

$a + a + 6d + a + 15d = 40$

$3a + 21d = 40$

$a + 7d = \frac{40}{3}$

$S_{15} = \frac{15}{2}(2a + 14d)$

$= 15(a + 7d)$

$= 15 \times \frac{40}{3} = 5 \times 40 = 200$

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JEE Mains Problems

$[3.4] = 3$ $\quad [-3.4] = -4$

For $x \in R$, let $[x]$ denote the greatest integer $\leq x$, then the sum of the series

$$\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \left[-\frac{1}{3} - \frac{2}{100}\right] + \ldots + \left[-\frac{1}{3} - \frac{99}{100}\right] \text{ is}$$

$-1 \quad -1$

(2019 Main, 12 April I)

How many terms: 100

(a) -153 (b) -133

(c) -131 (d) -135

$-\frac{1}{3} - x = -1$

$-x = -\frac{2}{3}$

$x = \frac{2}{3}$

$[-0.33 - 0.99]$

$[-1.32]$

-2

$\frac{2}{3} \times \frac{100}{100} = 66.6$

$$\underbrace{\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] - - - - - - \left[-\frac{1}{3} - \frac{66}{100}\right]}_{67 \text{ terms}} + \underbrace{\left[-\frac{1}{3} - \frac{67}{100}\right] - - - \left[-\frac{1}{3} - \frac{99}{100}\right]}_{33 \text{ terms}}$$

$-1 \quad -1 \quad -1 \qquad -2 \; ---- \; -2$

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Ans: $67(-1) + 33(-2) = -67 - 66 = -133$

JEE Mains Problems

Let the sum of the first n terms of a non-constant AP $a_1, a_2, a_3 \ldots$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this AP, then the ordered pair (d, a_{50}) is equal to **(2019 Main, 9 April I)**

$\boxed{\text{(a) } (A, 50 + 46A)}$ (b) $(50, 50 + 45A)$

(c) $(50, 50 + 46A)$ (d) $(A, 50 + 45A)$

M-1

$S_n = 50n + \frac{n(n-7)}{2}A$

$\frac{n}{2}\{2a + (n-1)d\} = 50n + \frac{n(n-7)}{2}A$

$an + \frac{n^2 d}{2} - \frac{nd}{2} = 50n + \frac{n^2 A}{2} - \frac{7nA}{2}$

$n\left(a - \frac{d}{2}\right) + \frac{n^2 d}{2} = n\left\{50 - \frac{7A}{2}\right\} + n^2\left[\frac{A}{2}\right]$

$a - \frac{d}{2} = 50 - \frac{7A}{2}$ —(1)

$\frac{d}{\cancel{2}} = \frac{A}{\cancel{2}}$ ⟶ (2)

$d = A$

$a = 50 - \frac{7A}{2} + \frac{d}{2}$

$= 50 - \frac{7A}{2} + \frac{A}{2}$

$a = 50 - 3A$

$d = A$

$a_{50} = a + 49d$

$= 50 - 3A + 49A$

$= 50 + 46A$

M-2

$S_n = 50n + \frac{n(n-7)}{2}A$

$S_1 = a_1 = 50 + \frac{1 \times (-6)}{2}A = \boxed{50 - 3A}$

$S_2 = a_1 + a_2 = 100 - 5A$

$\Rightarrow a_1 + a_1 + d = 100 - 5A$

$\Rightarrow \cancel{100} - 6A + d = \cancel{100} - 5A$

$\boxed{d = A}$

JEE Mains Problems

The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is

(2019 Main, 10 Jan I)

(a) 1256 (b) 1465 (c) 1356 (d) 1365

$N_1 = 7K + 2$, $N_2 = 7K + 5$

N_1: $\underbrace{16, 23, 30, 37 \text{ ------- } 93}_{12 \text{ terms}}$ ✓

N_2: $\underbrace{12, 19, 26 \text{ ---- } 96}_{13 \text{ terms}}$ ✓

No common terms

$S_1 = \frac{12}{2}(16+93)$

$S_2 = \frac{13}{2}(12+96)$

JEE Mains Problems

Let $a_1, a_2, \ldots\ldots a_{30}$ be an AP, $S = \sum_{i=1}^{30} a_i$ and

$T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $\boxed{S - 2T = 75,}$

then a_{10} is equal to $\quad a+9d$ **(2019 Main, 9 Jan I)**

(a) 42 (b) 57

(c) 52 ✓ (d) 47

$$a_5 = 27$$

$$a + 4d = 27 \quad \text{—} \ (1)$$

$$a = 7$$

$$a_{10} = a + 9d = 7 + 9 \times 5$$
$$= 52$$

$$S = a_1 + a_2 + a_3 - - a_{30} = \frac{30}{2}(2a + 29d) = 15(2a+29d) = S$$

$$T = a_1 + a_3 + a_5 - - - a_{29} = \frac{15}{2}(2a + 14 \times 2d) = 15(a + 14d) = T.$$

$$a, \quad a+2d, a+4d, \quad , \quad a+28d$$

$$S - 2T = 75$$

$$15(2a+29d) - 2 \times 15(a+14d) = 75$$

$$15[2a + 29d - 2a - 28d] = 75 \Rightarrow d = 5$$

JEE Mains Problems

Let $a_1, a_2, \ldots\ldots a_{30}$ be an AP, $S = \sum_{i=1}^{30} a_i$ and

$T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $\boxed{S - 2T = 75,}$

then a_{10} is equal to $a+9d$ **(2019 Main, 9 Jan I)**

(a) 42 (b) 57

(c) 52 ✓ (d) 47

non constant A.P.

3, 3, 3, 3, 3 — — constant A.P.

$a_5 = 27$

$a + 4d = 27$ — (1)

$a = 7$ ✓

$a_{10} = a + 9d = 7 + 9 \times 5$
$= 52$

$S = a_1 + a_2 + a_3 - - a_{30} = \frac{30}{2}(2a + 29d) = 15(2a + 29d) = S$

$T = a_1 + a_3 + a_5 - - - a_{29} = \frac{15}{2}(2a + 14 \times 2d) = 15(a + 14d) = T.$

$a, \quad a+2d, a+4d, \quad , a+28d$

$S - 2T = 75$

$15(2a + 29d) - 2 \times 15(a + 14d) = 75$

$15[\cancel{2a} + 29d - \cancel{2a} - 28d] = 75 \Rightarrow d = 5$ ✓

JEE Mains Problems

If the sum of first n terms of an AP is cn^2, then the sum of squares of these n terms is **(2009)**

(a) $\frac{n(4n^2-1)c^2}{6}$ (b) $\frac{n(4n^2+1)c^2}{3}$

(c) $\frac{n(4n^2-1)c^2}{3}$ (d) $\frac{n(4n^2+1)c^2}{6}$

$S_n = cn^2$

Put $n=1$

$S_1 = c = a_1$

Put $n=2$

$S_2 = 4c$

$a_1 + a_2 = 4c$

$a + a + d = 4c \Rightarrow d = 4c - 2a$

$= 4c - 2c = 2c$

$a, a+d, a+2d \ldots$

$a = c, \; d = 2c$

$c, 3c, 5c, 7c, 9c \ldots$

$S = c^2 + (3c)^2 + (5c)^2 + (7c)^2 \ldots$ n terms

$= c^2[1^2 + 3^2 + 5^2 + 7^2 \ldots \text{ n terms}]$

$S = c^2 S_1$

$S_1 = 1^2 + 3^2 + 5^2 \ldots$ n terms

$= \Sigma(2n-1)^2$

$= \Sigma(4n^2 + 1 - 4n)$

$S_1 = 4\Sigma n^2 + \Sigma 1 - 4\Sigma n$

$S = \left[\frac{4n(n+1)(2n+1)}{6} + n - \frac{4n(n+1)}{2}\right]c^2$

JEE Mains Problems

ar ar^2

Let a, b and c be in GP with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If $3a$, $7b$ and $15c$ are the first three terms of an AP, then the 4th term of this AP is

(2019 Main, 10 April II)

(a) $5a$ (b) $\frac{2}{3}a$ (c) a (d) $\frac{7}{3}a$

$3a, 7b, 15c$ are in A.P

$2 \times 7b = 3a + 15c$

$14b = 3a + 15c$

$14 \times ar = 3a + 15 \times ar^2$

$14r = 3 + 15r^2$

$15r^2 - 14r + 3 = 0$

$15r^2 - 5r - 9r + 3 = 0$

$5r(3r-1) - 3(3r-1) = 0$

$r = \frac{3}{5}, \frac{1}{3}$

× ✓

$3a, 7 \times a \times \frac{1}{3}, 15 \times a \times \frac{1}{9}$

$3a, \frac{7a}{3}, \frac{5a}{3}$

$\frac{9a}{3}, \frac{7a}{3}, \frac{5a}{3}, \frac{3a}{3}$

$a_4 = a$

JEE Mains Problems

If three distinct numbers a, b and c are in GP and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct? **(2019 Main, 8 April II)**

(a) d, e and f are in GP (b) $\frac{d}{a}, \frac{e}{b}$ and $\frac{f}{c}$ are in AP

(c) d, e and f are in AP (d) $\frac{d}{a}, \frac{e}{b}$ and $\frac{f}{c}$ are in GP

$b^2 = ac.$

equal roots

$ax^2+2bx+c=0 \to \alpha, \alpha$

common root

$dx^2+2ex+f=0$

one ~~root~~ will be $-b/a$.

$D = 4b^2-4ac = 4(b^2-ac)=0$

$\alpha+\alpha = -\frac{2b}{a} \Rightarrow 2\alpha = -\frac{2b}{a} \Rightarrow \boxed{\alpha = -\frac{b}{a}}$

$d\left(-\frac{b}{a}\right)^2 + 2e\left(-\frac{b}{a}\right)+f=0$

$db^2 - 2bea + a^2f = 0$

$d(ac) - 2bea + a^2 f = 0$

$a[cd - 2be + af] = 0$

$2be = cd + af.$

$\frac{2be}{b^2} = \frac{cd}{b^2} + \frac{af}{b^2}$

$\frac{2e}{b} = \frac{cd}{ac} + \frac{af}{ac}$

$\boxed{2\left(\frac{e}{b}\right) = \frac{d}{a} + \frac{f}{c}}$

$2B = A + C$ A.P

$d/a, e/b, f/c$ are in AP

JEE Mains Problems

Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant AP. If these are also the three consecutive terms of a GP, then $\frac{a}{c}$ is equal to

(2019 Main, 9 Jan II)

(a) 2 (b) $\frac{7}{13}$ (c) 4 (d) $\frac{1}{2}$

$a = A + 6D = -8D$

$b = A + 10D = -4D$.

$c = A + 12D = -2D$.

$\frac{a}{c} = \frac{-8\cancel{D}}{-2\cancel{D}} = 4$.

a, b, c are in G.P

$\Rightarrow b^2 = ac$

$\Rightarrow (A+10D)^2 = (A+6D)(A+12D)$

$\Rightarrow \cancel{A^2} + 100D^2 + 20AD = \cancel{A^2} + 72D^2 + 18AD$

$\Rightarrow 28D^{\cancel{2}} = -2A\cancel{D}$.

$\Rightarrow 14D = -A \Rightarrow A = -14D$

JEE Mains Problems

$a \quad ar \quad ar^2$

If a, b and c be three distinct real numbers in GP and $a + b + c = xb$, then x cannot be (2019 Main, 9 Jan I)

(a) 4 (b) 2 (c) −2 (d) −3

$b^2 = ac.$

Concept.

$x + \frac{1}{x} \geq 2$ or $x + \frac{1}{x} \leq -2$

$x > 0$ $x < 0$

$a + b + c = xb$

$a + ar + ar^2 = x(ar)$

$1 + r + r^2 = xr.$

$x = \frac{1}{r} + 1 + r$

$x = r + \frac{1}{r} + 1$

$r + \frac{1}{r} = x - 1$

$r + \frac{1}{r} \geq 2.$ or $r + \frac{1}{r} \leq -2$

For no. to be distinct

$r + \frac{1}{r} > 2$ or $r + \frac{1}{r} < -2$

$x - 1 > 2$ $x - 1 < -2$

$x > 3$ $x < -1$

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JEE Mains Problems

If the 2nd, 5th and 9th terms of a non-constant AP are in GP, then the common ratio of this GP is **(2016 Main)**

(a) $\frac{8}{5}$ (b) $\frac{4}{3}$ (c) 1 (d) $\frac{7}{4}$

$$\left.\begin{aligned} a_2 &= a+d \\ a_5 &= a+4d \\ a_9 &= a+8d \end{aligned}\right\} \text{GP}$$

$$(a+4d)^2 = (a+d)(a+8d)$$

$$\cancel{a^2}+16d^2+8ad = \cancel{a^2}+8d^2+9ad.$$

$$8d^2 = ad$$

$$8d = a.$$

$$a_2 = 9d$$

$$a_5 = 12d$$

$$a_9 = 16d.$$

$$\text{C.r} = \frac{12\cancel{d}}{9\cancel{d}} = \frac{4}{3}$$

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JEE Mains Problems

The sum $\sum_{k=1}^{20} k \frac{1}{2^k}$ is equal to (2019 Main, 8 April II)

(a) $2-\frac{11}{2^{19}}$ (b) $1-\frac{11}{2^{20}}$

(c) $2-\frac{3}{2^{17}}$ (d) $2-\frac{21}{2^{20}}$

$$\sum_{K=1}^{20} K \cdot \frac{1}{2^K} = \frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} - - \quad \frac{20}{2^{20}}$$

$$S = \frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} \text{———} \quad \frac{20}{2^{20}}$$

$$\frac{S}{2} = \quad \frac{1}{2^2} + \frac{2}{2^3} \quad \frac{19}{2^{20}} + \frac{20}{2^{21}}$$

$$\frac{S}{2} = \frac{1}{2} + \left[\frac{1}{2^2} + \frac{1}{2^3} - - \quad \frac{1}{2^{20}}\right] - \frac{20}{2^{21}}$$

$$S = \underbrace{1 + \frac{1}{2} + \frac{1}{2^2} - - \frac{1}{2^{19}}} - \frac{20}{2^{20}}$$

$$= \frac{1\left(1-\left(\frac{1}{2}\right)^{20}\right)}{1-\frac{1}{2}} - \frac{20}{2^{20}}$$

$$2\left(1-\frac{1}{2^{20}}\right) - \frac{20}{2^{20}}$$

$$2 - \frac{2}{2^{20}} - \frac{20}{2^{20}}$$

$c.r = 1/2$

$$= 2 - \frac{22}{2^{20}}$$

$$= 2 - \frac{11}{2^{19}}.$$

JEE Mains Problems

The sum $\sum_{k=1}^{20} k \frac{1}{2^k}$ is equal to (2019 Main, 8 April II)

(a) $2 - \frac{11}{2^{19}}$ (b) $1 - \frac{11}{2^{20}}$

(c) $2 - \frac{3}{2^{17}}$ (d) $2 - \frac{21}{2^{20}}$

$\sum_{K=1}^{20} K \cdot \frac{1}{2^K} = \frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} \cdots \frac{20}{2^{20}}$

$S = \frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} \cdots \frac{20}{2^{20}}$

$\frac{S}{2} = \frac{1}{2^2} + \frac{2}{2^3} \cdots \frac{19}{2^{20}} + \frac{20}{2^{21}}$

$\frac{S}{2} = \frac{1}{2} + \left[\frac{1}{2^2} + \frac{1}{2^3} \cdots \frac{1}{2^{20}}\right] - \frac{20}{2^{21}}$

$S = \underbrace{1 + \frac{1}{2} + \frac{1}{2^2} \cdots \frac{1}{2^{19}}} - \frac{20}{2^{20}}$

$= \frac{1\left(1 - \left(\frac{1}{2}\right)^{20}\right)}{1 - \frac{1}{2}} - \frac{20}{2^{20}}$

$2\left(1 - \frac{1}{2^{20}}\right) - \frac{20}{2^{20}}$

c.r $= 1/2$

$2 - \frac{2}{2^{20}} - \frac{20}{2^{20}}$

$= 2 - \frac{22}{2^{20}}$

$= 2 - \frac{11}{2^{19}}$

JEE Mains Problems

Let $S_n = 1 + q + q^2 + \dots + q^n$ and

$T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \dots + \left(\frac{q+1}{2}\right)^n$, where q is a real number and $q \neq 1$. If

${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + \dots + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$, then α is equal to **(2019 Main, 11 Jan II)**

(a) 2^{100} (b) 202

(c) 200 (d) 2^{99}

$S_n = \frac{1(1-q^n)}{1-q}$

$T_n = \frac{1\left(1-\left(\frac{q+1}{2}\right)^n\right)}{1-\left(\frac{q+1}{2}\right)} = \frac{1-\frac{(q+1)^n}{2^n}}{(1-q)/2}$

$T_{100} = \frac{1-\frac{(q+1)^{100}}{2^{100}}}{(1-q)/2}$

$\alpha = \boxed{\frac{{}^{101}C_1 + {}^{101}C_2 S_1 - - {}^{101}C_{101} S_{100}}{T_{100}}}$

${}^{101}C_1 + {}^{101}C_2 \cdot \frac{(1-q^1)}{1-q} + {}^{101}C_3 \frac{1-q^2}{1-q} + {}^{101}C_4 \left(\frac{1-q^3}{1-q}\right) - - - {}^{101}C_{101}\left(\frac{1-q^{100}}{1-q}\right) = \alpha T_{100}$

${}^{101}C_1 + \left({}^{101}C_2 \times \frac{1}{1-q} - {}^{101}C_2 \frac{q}{1-q}\right) + {}^{101}C_3 \frac{1}{1-q} - {}^{101}C_3 \frac{q^2}{1-q} - - {}^{101}C_{101} \frac{1}{1-q} - {}^{101}C_{101} \frac{q^{100}}{1-q}$

${}^{101}C_1 + \frac{1}{1-q}\left[{}^{101}C_2 + {}^{101}C_3 - - {}^{101}C_{101}\right] - \frac{1}{1-q}\left[{}^{101}C_2 q + {}^{101}C_3 q^2 - - {}^{101}C_{101} q^{100}\right]$

JEE Mains Problems

The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then, the common ratio of this series is

(2019 Main, 11 Jan I)

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$ (c) $\frac{2}{9}$ (d) $\frac{1}{3}$

GP a, ar, ar^2 - ——

$S'_\infty = 3 \Rightarrow \frac{a}{1-r} = 3 \Rightarrow a = 3(1-r)$

New GP a^3, a^3r^3, a^3r^6 - ——

$S''_\infty = \frac{27}{19} \Rightarrow \frac{a^3}{1-r^3} = \frac{27}{19}$

$\Rightarrow \frac{\cancel{27}(1-r)^3}{1-r^3} = \frac{\cancel{27}}{19}$

$19(1-r)^3 = 1-r^3$.

$19\cancel{(1-r)}(1+r^2-2r) = \cancel{(1-r)}(1+r^2+r)$

Solve this & find 'r'

$r = 2/3$.

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JEE Mains Problems

Three positive numbers form an increasing GP. If the middle term in this GP is doubled, then new numbers are in AP. Then, the common ratio of the GP is

(a) $\sqrt{2}+\sqrt{3}$ (b) $3+\sqrt{2}$ **(2014 Main)**

(c) $2-\sqrt{3}$ (d) $2+\sqrt{3}$

$a, ar \quad \frac{a}{r}, a, ar$ G.P

$\frac{a}{r}, 2a, ar$ A.P

$4a = \frac{a}{r} + ar \Rightarrow r + \frac{1}{r} = 4$

$\Rightarrow r^2 - 4r + 1 = 0 \Rightarrow r = 2 \pm \sqrt{3}$

JEE Mains Problems

If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$, $= 10^{11} = \underbrace{K \cdot 10^9}$

then k is equal to **(2014 Main)** $K = 100$

(a) $\frac{121}{10}$ (b) $\frac{441}{100}$ (c) 100 (d) 110

$S = 1 \cdot 10^9 + 2(11)^1(10)^8 + 3 \cdot (11)^2(10)^7 - - - \quad 10 \cdot 11^9 \, 10^0$

A.G.P

$\frac{11}{10} S = \quad 1 \cdot (11)^1 (10)^8 + 2 \cdot (11)^2 (10^7) - - - - \quad 9 \cdot 11^9 \cdot 10^0 + 10 \frac{11^{10}}{10}$

C.r of correspondy GP is $\frac{11}{10}$

$S\left(1 - \frac{11}{10}\right) = \left[10^9 + 11^1 \times 10^8 + 11^2 \times 10^7 + 11^3 \times 10^6 - - \quad 11^9 \times 10^0\right] - 11^{10}$

$-\frac{S}{10} = \dfrac{10^9\left(\left(\frac{11}{10}\right)^{10} - 1\right)}{\frac{11}{10} - 1} - 11^{10}$

$-S = \dfrac{\cancel{10^{10}}\left(\frac{11^{10} - 10^{10}}{\cancel{10^{10}}}\right)}{1/10} - 10 \cdot 11^{10}$

$-S = \cancel{10 \cdot 11^{10}} - 10^{11} - \cancel{10 \cdot 11^{10}} \Rightarrow S = 10^{11}$
