PHYSICS

JEE and NEET CRASH COURSE

Waves

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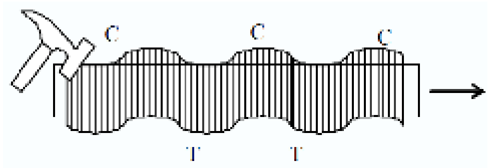
Wave motion

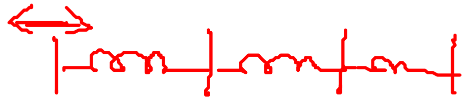
In wave motion energy or momentum is transferred from one place to another without the actual transfer of matter.

Transverse Waves

The particles of medium vibrate in a direction perpendicular to the direction of propagation of wave.

Crest (C) & Trough (T) are formed

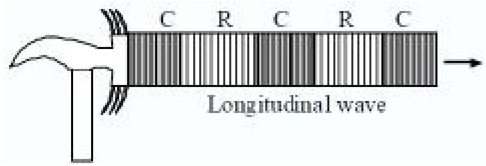




Longitudinal Waves

The particles of medium vibrate in a direction of wave propagation

Wave proceeds in form of compression (C) & rarefaction (R)



Equation of plane progressive wave

dirn of oscill. of particles

(i) Equation of progressive wave in positive x direction

$y = A \sin \omega (t - x/v) = A \sin\left(\omega t - \frac{\omega}{v} x\right) = A \sin(\omega t - Kx)$

$\frac{\omega}{v} = \frac{2\pi}{\lambda}$ propagation constant or ang. wave no.

(ii) Equation of progressive wave in negative x direction

$y = A \sin \omega (t + x/v) = A \sin(\omega t + Kx)$

Wave velocity and particle velocity

Wave velocity ; $v = \frac{\omega}{K}$

particle velocity

$v_p = \frac{\partial y}{\partial t} = \omega A \cos(\omega t - Kx)$

particle's acc. $a_p = \frac{\partial}{\partial t} v_p = -\omega^2 A \sin(\omega t - Kx)$

$= -\omega^2 y$

( ) x ( ) t

Same sign

(−ve x-dirn)

( ) x ( ) t

opp. sign

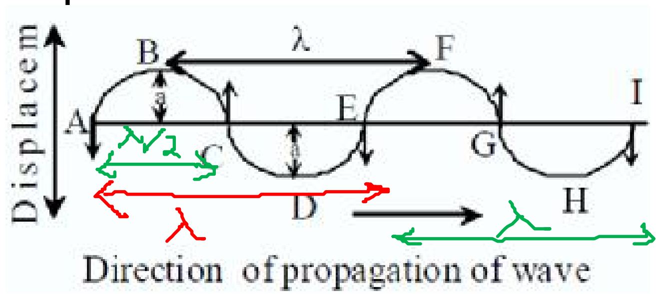
(+ve x-dirn)

→ distance travelled by wave in one T.P.

Wave Length (λ) and Intensity

The distance between two consecutive particle in the same phase or the distance travelled by the wave in one periodic time and denoted by λ

$\boxed{v = f\lambda}$ or $\boxed{\lambda = vT}$



Intensity of Wave

Opp. phase = A & C

In medium, propagation energy perpendicular to per unit area per second is called intensity of wave.

Avg. Intensity $I = 2\pi^2 f^2 a^2 \rho v$

a = amplitude
ρ = density of medium

Same phase, $\Delta x = n\lambda = \lambda, 2\lambda, \ldots$

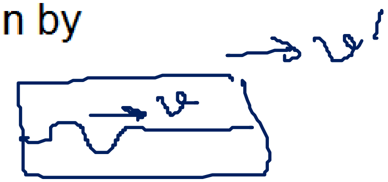
Opp. phase; $\Delta x = (2n-1)\frac{\lambda}{2} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$

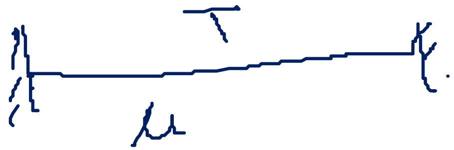
$n = 1, 2, 3, 4, \ldots$

The Speed of Transverse Waves on Strings

The speed of a wave on a string is given by

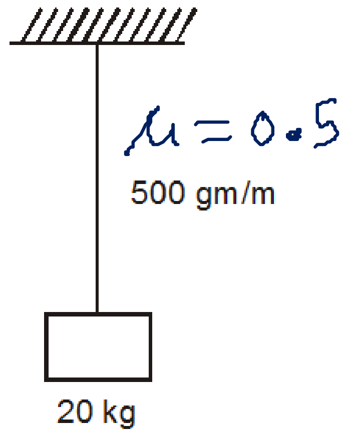
$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$





where T is tension in the string (in Newtons) and μ is mass per unit length of the string (kg/m).
It should be noted that v is speed of the wave w.r.t. the medium (string).
In case the tension is not uniform in the string or string has nonuniform linear mass density then v is speed at a given point and T and μ are corresponding values at that point.

Example. Find speed of the wave generated in the string as in the situation shown. Assume that the tension is not affected by the mass of the cord.



Solution : T = 20 × 10 = 200 N

$$v = \sqrt{\frac{200}{0.5}} = 20\,m/s$$

Speed of Sound Waves

Velocity of sound waves in a linear solid medium is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

where Y = young's modulus of elasticity and ρ = density.

Velocity of sound waves in a fluid medium (liquid or gas) is given by

$$v = \sqrt{\frac{B}{\rho}}$$

where, ρ = density of the medium and B = Bulk modulus of the medium

Newton's formula : Newton assumed isothermal process.

PV = constant

and hence B = P

$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{RT}{M}}$$ where M = molar mass of air

29 gm

Laplace's correction : Laplace established that propagation of sound in a gas is an adiabatic process and hence PV^{γ} = constant

where, $B = -V\frac{dP}{dV} = \gamma P$

and hence speed of sound in a gas, $\boxed{v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}}$

$\gamma = C_p/C_v$

Interference

When waves of equal frequency & nearly equal amplitude is superimposed, interference occurs.

Mathematical Interference -

At a time t, at point x two waves of equal frequency $y_1 = a_1\sin(\omega t - kx_1 + \phi_1)$ & $y_2 = a_2\sin(\omega t - kx_2 + \phi_2)$ is super-imposed, then Amplitude A & Intensity I of Resultant wave :

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \Delta\phi$$

$$\& \; I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \Delta\phi$$

$$\Delta\phi = k(x_1 - x_2) + \phi_2 - \phi_1$$

Intensity of resultant wave changes periodically from minimum to maximum & maximum to minimum from one point to another point

$I \propto a^2$

$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\Delta\phi}$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$

$\Delta\phi = K\Delta x + \Delta\phi_0$

path diff.

Constructive Interference :	**Destructive Interference :**
Where, phase difference $\Delta\phi = 2\pi n$	Where phase difference $\Delta\phi = (2n - 1)\pi$
path difference $\Delta x = n\lambda$,	path difference $\Delta x = \left(n - \frac{1}{2}\right)\lambda$
$I_{max} \propto (a_1 + a_2)^2$ $\quad A_{max} = a_1 + a_2$	$I_{min} \propto (a_1 - a_2)^2$ $\quad A_{min} = \lvert A_1 - A_2 \rvert$

If phase diff is only due to path diff

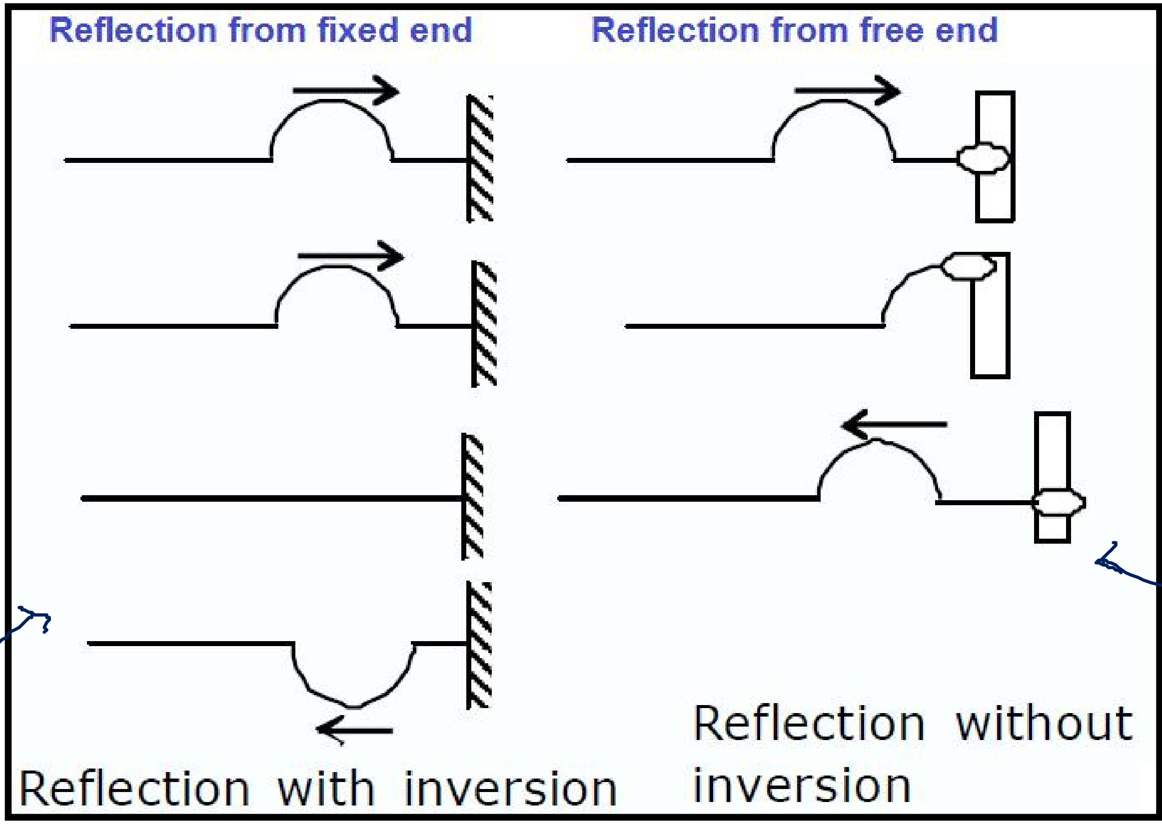
$\Delta\phi = K\Delta x$

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

Reflection & Transmission of Wave

There is a phase change of π



There is no phase change

Standing Waves

Suppose two sine waves of equal amplitude and frequency propagate on a long string in opposite directions. The equations of the two waves are given by

$$y_1 = A \sin(\omega t - kx) \quad \text{and} \quad y_2 = A \sin(\omega t + kx).$$

These waves interfere to produce what we call standing waves.

The resultant displacements of the particles of the string are given by the principle of superposition as

$$y = y_1 + y_2$$

$$= A[\sin(\omega t - kx) + \sin(\omega t + kx)] = 2A \sin \omega t \cos kx$$

or, $$y = (2A \cos kx) \sin \omega t.$$

Amplitude of standing wave

The amplitude of the wave

$A_s = |2A \cos kx|$ $\qquad$ $A_{max} = 2A$

is not constant but varies periodically with position (and not with time as in beats).

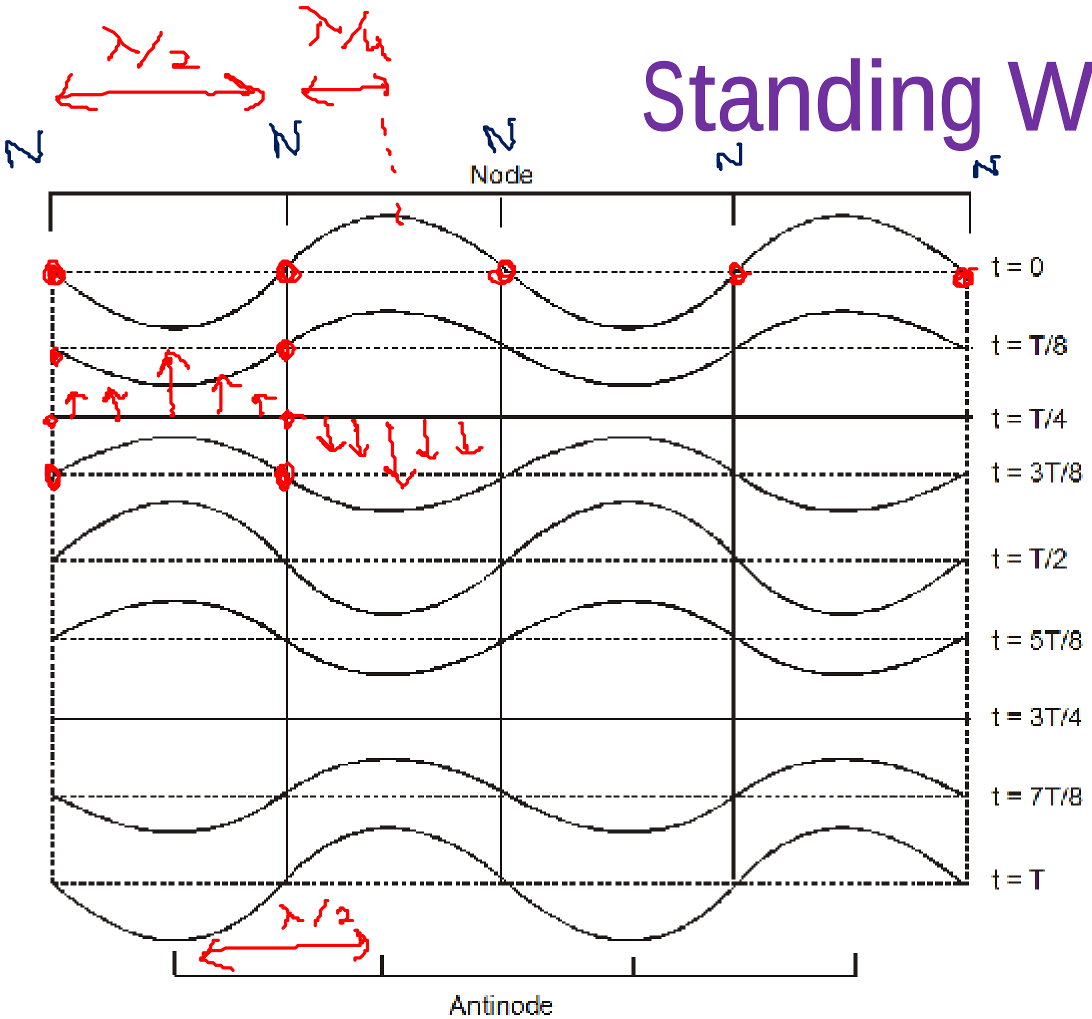
The points for which amplitude is minimum are called nodes ($A_s = 0$)

The points for which amplitude is maximum are called antinodes ($A_s = 2A$)

in a stationary wave, nodes and antinodes are also equally spaced with spacing ($\lambda/2$)

nodes and antinodes are alternate with spacing ($\lambda/4$).

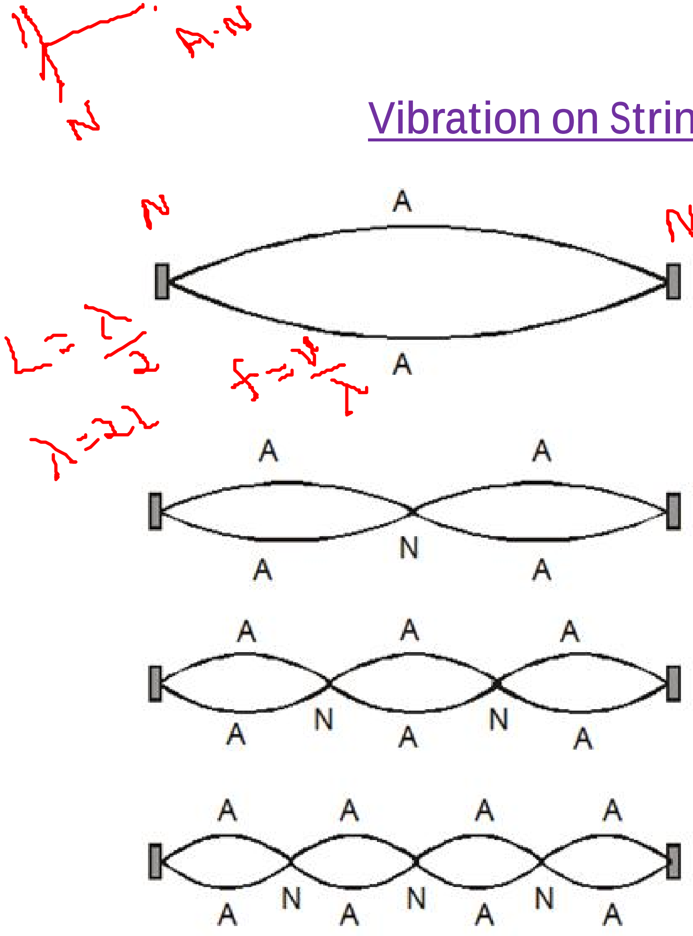
Standing Waves



Vibration of String fixed at both ends or organ pipe open at both ends

A·N

Vibration on String



$L = \frac{\lambda}{2}$

$\lambda = 2L$

$f = \frac{v}{\lambda}$

(a)
Fundamental
or first harmonic
$f_1 = v/2L$

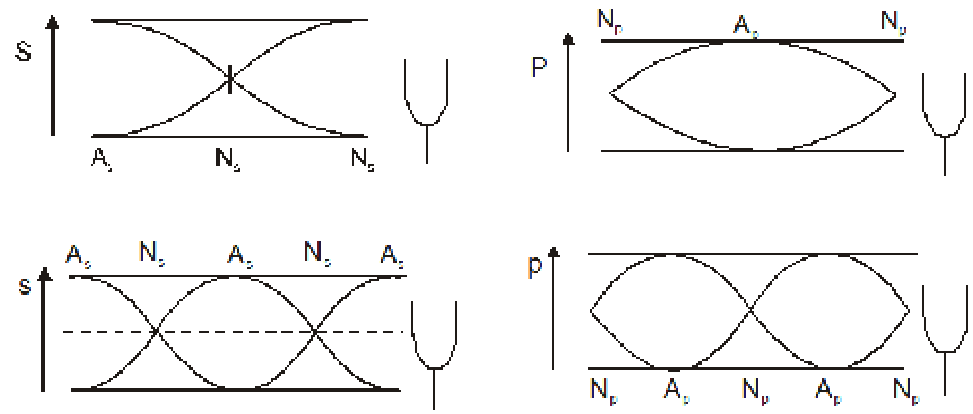
min. freq. or max λ

(b)
second harmonic
or first overtone =
$f_2 = 2f_1 = 2v/2L$

(c)
third harmonic
or 2nd
overtone =
$f_3 = 3f_1 = 3v/2L$

(d)
fourth harmonic
or 3rd
overtone =
$f_4 = 4f_1 = 4v/2L$

Vibration in Organ Pipe



$$f_n = nf_1$$

(n−1)th overtone or nth harmonic

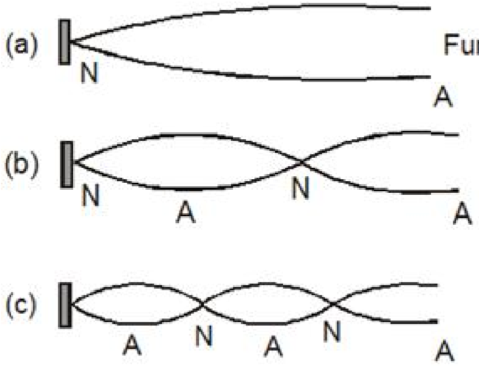
fundamental freq $(v/2L)$

$L = \frac{\lambda}{4}$

$\lambda = 4L$

Vibration of String fixed at one end

Vibration on String

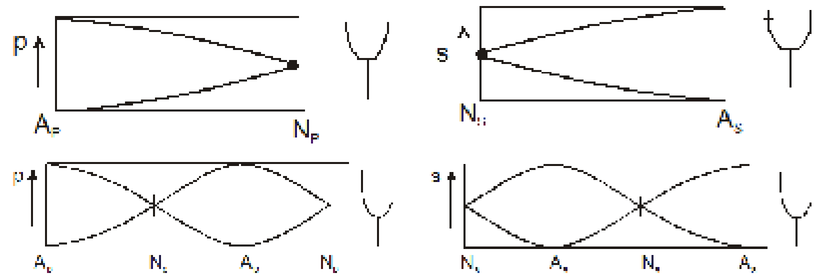


(a) Fundamental $f_1 = v/4L$

(b) First Overtone $f_2 = \frac{3v}{4L} = 3f_1$

(c) Second Overtone $f_3 = \frac{5v}{4L} = 5f_1$

Vibration in Organ Pipe



$f_n = (2n-1)f_1$

Here only odd harmonics are present

$(n-1)^{th}$ overtone or $(2n-1)^{th}$ harmonic

fundemental $\left(\frac{v}{4L}\right)$

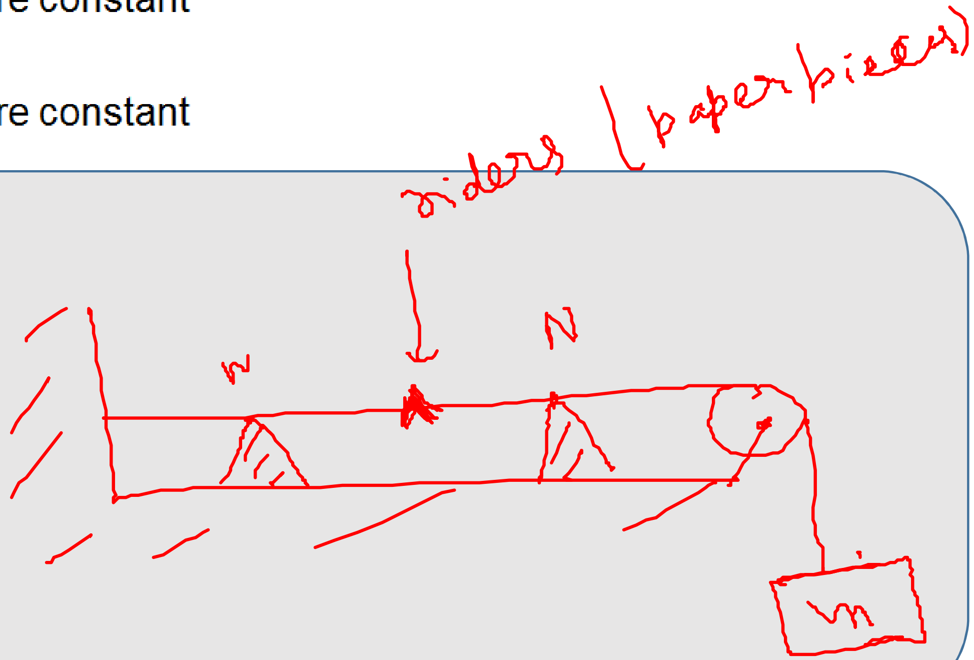
Laws of transverse vibrations of a string - Sonometer Wire

(a) Law of length $f \propto \frac{1}{L}$ so $\frac{f_1}{f_2} = \frac{L_2}{L_1}$; if T & μ are constant

(b) Law of tension $f \propto \sqrt{T}$ so $\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}$; if L & μ are constant

(c) Law of mass $f \propto \frac{1}{\sqrt{\mu}}$ so $\frac{f_1}{f_2} = \sqrt{\frac{\mu_2}{\mu_1}}$; if T & L are constant

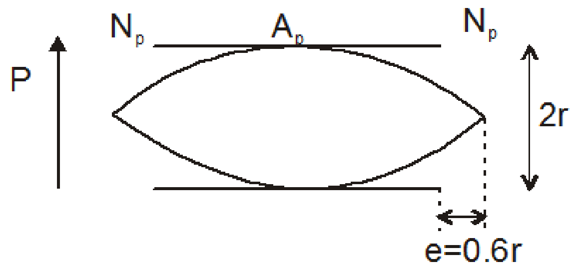
$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$



End Correction

As mentioned earlier the displacement antinode at an open end of an organ pipe lies slightly outside the open end. The distance of the antinode from the open end is called end correction and its value is given by

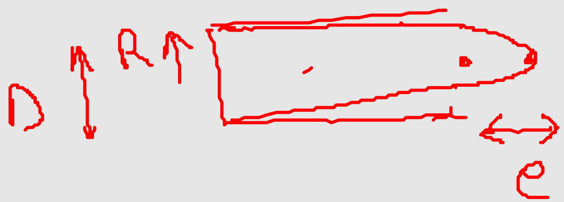
$$e = 0.6\ r \quad = 0.3\,D$$



where r = radius of the organ pipe.

with end correction, the fundamental frequency of a closed pipe (f_c) and an open organ pipe (f_0) will be given by

$$f_c = \frac{v}{4(\ell + 0.6r)} \qquad \text{and} \qquad f_0 = \frac{v}{2(\ell + 1.2r)}$$



Loudness

DECIBEL SCALE :

The logarithmic scale which is used for comparing two sound intensity is called **decibel scale**.

The intensity level β described in terms of decibels is defined as

$$L = 10 \log \left(\frac{I}{I_0} \right) \text{ (dB)}$$

Here I_0 is the threshold intensity of hearing for human ear

i.e. $I = 10^{-12}$ watt/m^2.

$$L_2 - L_1 = 10 \log \left(\frac{I_2}{I_0} \right) - 10 \log \left(\frac{I_1}{I_0} \right)$$

$$= 10 \log \left(\frac{I_2}{\cancel{I_0}} \times \frac{\cancel{I_0}}{I_1} \right) = 10 \log \frac{I_2}{I_1}$$

Interference in time : Beats

When two sound waves of same amplitude and different frequency superimpose, then intensity at any point in space varies periodically with time. This effect is called beats.

Beat frequency,	$f_B = f_1 - f_2$
Time Period,	$T_B = \dfrac{1}{f_1 - f_2}$

IMPORTANT POINTS :

(i) Beat phenomenon can be used for determining an unknown frequency by sounding it together with a source of known frequency.

(ii) If the arm of a tuning fork is waxed or loaded, then its frequency decreases.

(iii) If arm of tuning fork is filed, then its frequency increases.

$$\boxed{f_B = |f_1 - f_2|}$$

Doppler's Effect

The apparent change in frequency or wavelength observed by the observer when there is a relative motion between the source and the observer, is known as Doppler Effect.

If source and observer both are moving with velocities v_s and v_0 along the line joining them

Speed of sound

The observed frequency, $$f' = f\left(\frac{v \pm v_0}{v \mp v_s}\right)$$

observer

source

O → officer

S → servant

Approach → freq ↑

Separate → freq ↓

S

O

+ / − → approach

sep. → − / +

$f' = f\left(\frac{v - v_s\cos\theta}{v}\right)$

v_s

$v_s\cos\theta$

S

O