# PHYSICS

## JEE and NEET Crash Course

# Problem Solving Class

# (SHM)

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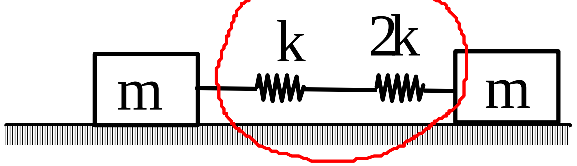
# P-Q1001

The time period for small oscillations of  the two blocks will be




A) $2\pi\sqrt{\frac{3m}{k}}$  B) $2\pi\sqrt{\frac{3m}{2k}}$

C) $2\pi\sqrt{\frac{3m}{4k}}$  D) $2\pi\sqrt{\frac{3m}{8k}}$

$T = 2\pi\sqrt{\frac{m}{K}}$

$T = 2\pi\sqrt{\frac{\mu}{K}}$ where, $\mu = \frac{m_1 m_2}{m_1+m_2}$

Reduced mass

$K_{eq.} = \frac{K_1 K_2}{K_1+K_2} = \frac{K \cdot 2K}{3K}$

$= \frac{2K}{3}$

$\mu = \frac{m \times m}{2m} = \frac{m}{2}$

$T = 2\pi\sqrt{\frac{m/2}{\frac{2K}{3}}} = 2\pi\sqrt{\frac{3m}{4K}}$

## P-Q1001-Solution

### Ans [C]

Springs are connected in series

$$K_{eq} = \frac{k(2k)}{k+2k} = \frac{2k}{3}$$

$$\text{Time period } T = 2\pi\sqrt{\frac{\mu}{K_{eq}}}$$

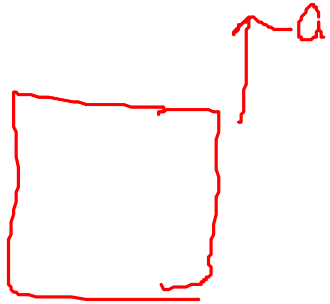
$$\text{Where } \mu = \frac{m_1.m_2}{m_1+m_2}$$

$$\text{Here } \mu = \frac{m}{2}$$

$$\therefore T = 2\pi\sqrt{\frac{m}{2}.\frac{3}{2k}} = 2\pi\sqrt{\frac{3m}{4k}}$$

## P-Q1002

A simple pendulum has a period T. It is taken inside a lift moving up with uniform acceleration of g/3. Now its time period will be




$A)\ \frac{\sqrt{2}}{3}T$ B) $\frac{1}{3}T$ C) $\frac{\sqrt{3}}{2}T$ D) $\frac{2}{\sqrt{3}}T$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T' = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{3l}{4g}} = \frac{\sqrt{3}}{2}\cdot 2\pi\sqrt{\frac{l}{g}}$$

$$T' = \frac{\sqrt{3}}{2}T$$

$$g_{eff} = g + a = g + \frac{g}{3} = \frac{4g}{3}$$

# P-Q1002-Solution

## Ans [C]

Initial Time Period is $T$

When pendulum present in accelerated system them

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

When object move with acceleration $\vec{a}$ then $\overrightarrow{g_{eff}} = \vec{g} - \vec{a}$

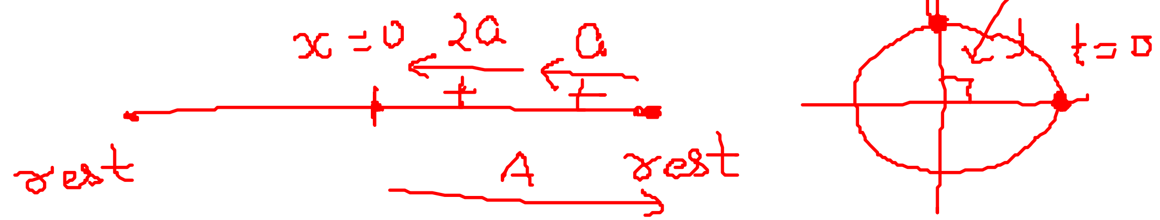
Lift moving up so $g_{eff}$ = g + g/3 = 4g/3

When moving up **a** is opposite sign so **g + a**

Putting the value of $g_{eff}$ we get $T' = \frac{\sqrt{3}}{2}T$

## P-Question

A particle moves with simple harmonic motion in a straight line. In first $t$ $s$, after starting from rest it travels a distance and in next $t$ $s$ it travels 2a, in same direction, then … [JEE MAINS–2014]



1) **Amplitude of motion is 3a**

2) **Time period of oscillations is 8 $t$**

3) **Amplitude of motion is 4a**

4) **Time period of oscillations is 6 $t$**

## P-Solution

**KEY: 4**

$a=A(1-\cos\omega\tau)$

$3a=A(1-\cos 2\omega\tau)$

$a=A(2\sin^2\omega\tau)$

$a=A(2)(4\sin^2\omega\tau\,\cos^2\omega\tau)$

$a=8A.\frac{a}{2A}\cos^2\omega\tau$

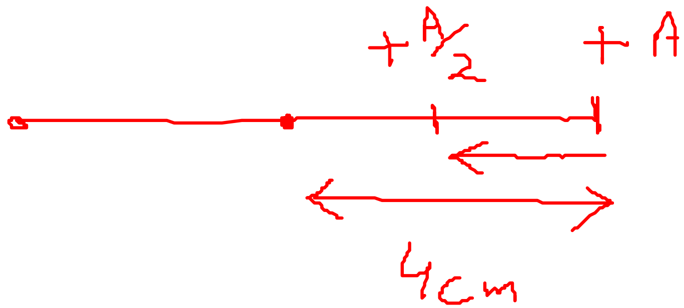
$\frac{1}{4}=\cos^2\omega\tau$

$\cos\omega\tau=\frac{1}{2}\text{Þ}\ \omega\tau=\frac{\pi}{3}$

$\left(\frac{2\text{p}}{T}\right)\tau=\frac{\pi}{3}\text{Þ}\ T=6\tau$

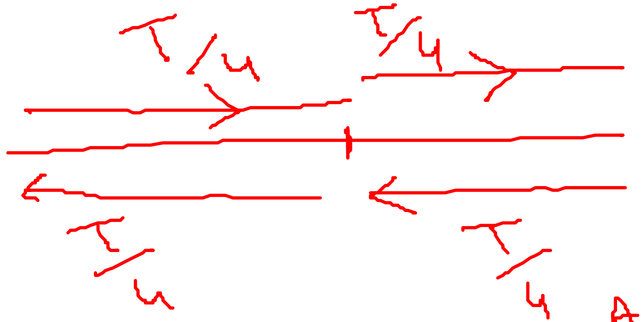
## P-Q1031

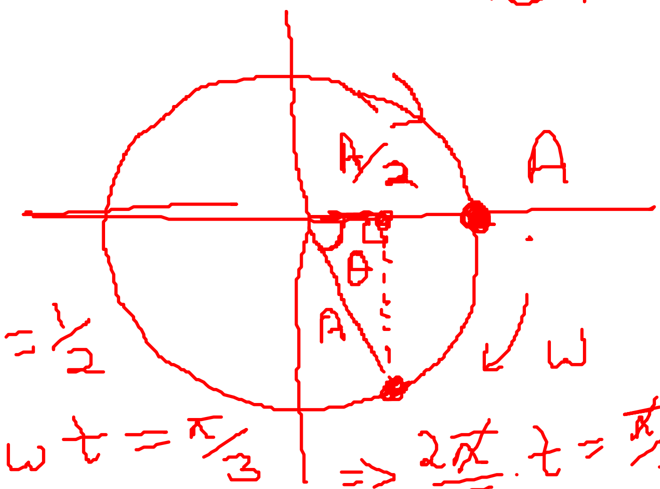
A particle executing S.H.M. of amplitude 4 *cm* and $T = 4$ *sec*. The time taken by it to move from positive extreme position to half the amplitude is




(A) 1 *sec*  (B) 1/3 *sec*

(C) 2/3 *sec*  (D) $\sqrt{3/2}$ *sec*




$\cos\theta = \frac{A/2}{A} = \frac{1}{2}$

$\theta = \frac{\pi}{3} \Rightarrow \omega t = \pi/3 \Rightarrow \frac{2\pi}{T} \cdot t = \frac{\pi}{3} \Rightarrow t = \frac{T}{6}$

## P-Q1031-Solution

### Ans [C]

Equation of motion $y = a.cos\omega t$

$$\Rightarrow \frac{a}{2} = a\cos\omega t \Rightarrow \cos\omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{3}$$

$$\Rightarrow \frac{2\pi t}{T} = \frac{\pi}{3} \Rightarrow t = \frac{\frac{\pi}{3}\times T}{2\pi} = \frac{4}{3\times 2} = \frac{2}{3}\sec$$

## P-Question

The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10s it will decrease to $\boldsymbol{\alpha}$ times its original magnitude where $\boldsymbol{\alpha}$ equals. [ JEE MAINS– 2013 ]

**1)** $\mathbf{0.81}$ **2)** $\mathbf{0.729}$ **3)** $\mathbf{0.6}$ **4)** **0.7**

$A = A_0 e^{-Kt}$

$0.9 A_0 = A_0 e^{-5K} \Rightarrow 0.9 = e^{-5K}$

$\alpha A_0 = A_0 e^{-15K}$

$= e^{-5K \times 3} = \left(e^{-5K}\right)^3$

$\alpha = e$

$= (0.9)^3 = 0.729$

$A = A_0 e^{-\frac{bt}{2m}}$

$= A_0 e^{-Kt}$

$E = \frac{1}{2} K A^2$

$(a^b)^c = a^{b \times c}$

## P-Solution

**KEY: 2**

**Amplitude decreases exponentially. In 5s it remains 0.9 times original magnitude therefore in total 15s it will remains (0.9), (0.9) ,(0.9) times of original magnitude so it is 0.728 times its original value.**

**A=$A_0e^{-kt}$ Þ 0.9$A_0$=$A_0e^{-5k}$**

**and $\propto A_0 = A_0e^{-15k}$**

**Solving Þ α =0.729**

## P-Q1033

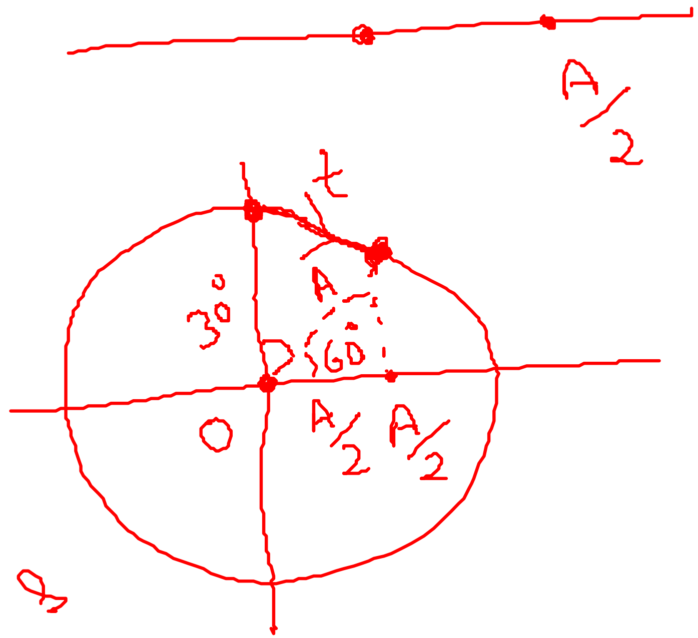
The periodic time of a body executing simple harmonic motion is 3 *sec*. After how much interval from time $t = 0$, its displacement will be half of its amplitude

(A) $\frac{1}{8}$ *sec*

(B) $\frac{1}{6}$ *sec*

(C) $\frac{1}{4}$ *sec*

(D) $\frac{1}{3}$ *sec*



$\theta = \omega t$

$\frac{\pi}{6} = \frac{2\pi}{T} \cdot t$

$t = \frac{T}{12}$

$= \frac{1}{4}$

# P-Q1033-Solution

## Ans [C]

Equation of motion y=a sin ωt

$$y = a\sin\frac{2\pi}{T}t \Rightarrow \frac{a}{2} = a\sin\frac{2\pi t}{3} \Rightarrow \frac{1}{2} = \sin\frac{2\pi t}{3}$$

$$\Rightarrow \sin\frac{2\pi t}{3} = \sin\frac{\pi}{6} \Rightarrow \frac{2\pi t}{3} = \frac{\pi}{6} \Rightarrow t = \frac{1}{4}\sec$$

## P-Question

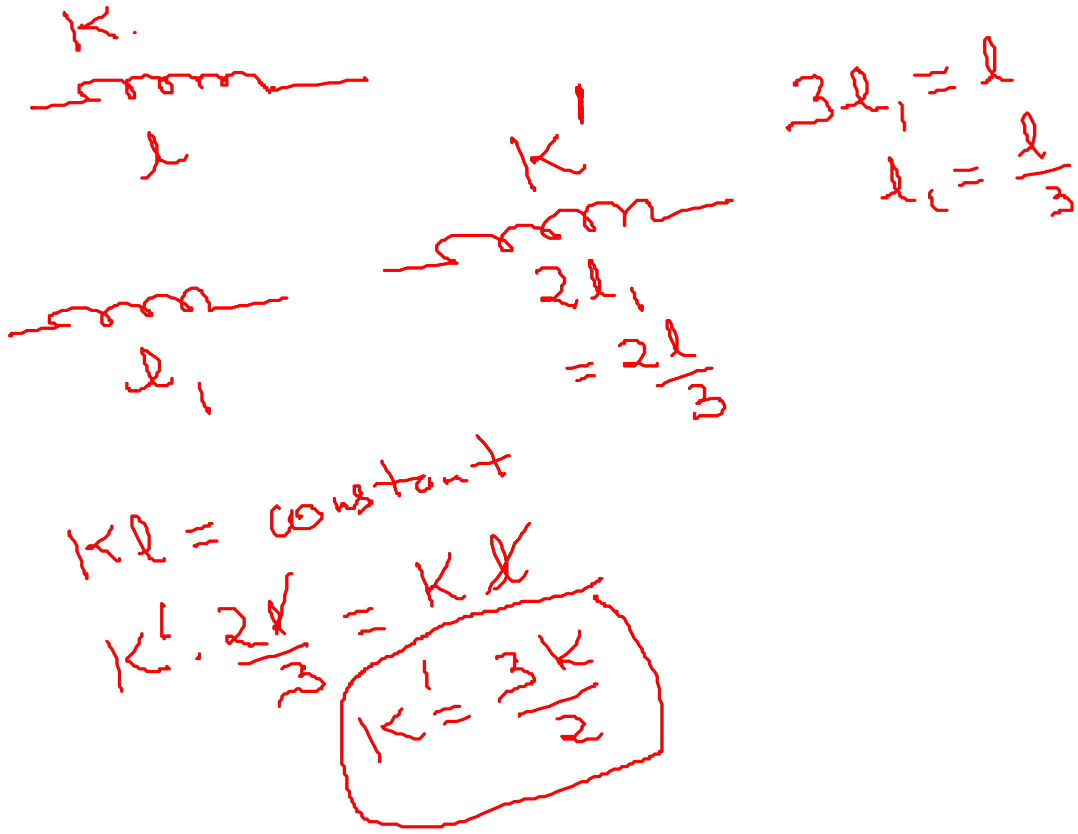
A spring of force constant ‘k’ is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of ….. [ 2000, IIT 99 ]

1) $\frac{2k}{3}$

2) $\frac{3k}{2}$

3) 3K

4) 6K



## P-Solution

**KEY: 2**

$kl = k_1 l_1 = k_2 l_2$

$$k_2 = \frac{kl}{l_2} = \frac{k(1+2)}{2} = \frac{3k}{2}$$

## P-Q1034

A simple pendulum performs simple harmonic motion about X = 0 with an amplitude A and time period T. The speed of the pendulum at $X$ =A/2 will be

(A) $\frac{\pi A\sqrt{3}}{T}$ ✓

(B) $\frac{\pi A}{T}$

(C) $\frac{\pi A\sqrt{3}}{2T}$

(D) $\frac{3\pi^2 A}{T}$

$$v = \omega\sqrt{A^2 - x^2}$$

$$= \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}}$$

$$= \frac{2\pi}{T}\frac{\sqrt{3}}{2}A$$

$$= \frac{\pi A\sqrt{3}}{T}$$

# P-Q1034-Solution

## Ans [A]

Velocity of a particle executing S.H.M. is given by

$$v = \omega\sqrt{a^2 - x^2} = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{2\pi}{T}\sqrt{\frac{3A^2}{4}} = \frac{\pi A\sqrt{3}}{T}.$$

At any point x(distance from centre) the velocity will be this

**P-Q1035**

A body of mass 5 *gm* is executing S.H.M. about a point with amplitude 10 *cm*. Its maximum velocity is 100 *cm/sec*. Its velocity will be 50 *cm/sec* at a distance

(A) 5

(B) $5\sqrt{2}$

(C) $5\sqrt{3}$ ✓

(D) $10\sqrt{2}$

$v_{max} = \omega A \Rightarrow 100 = \omega \times 10 \Rightarrow \omega = 10$ rad/s

$v = \omega\sqrt{A^2 - x^2}$

$50 = 10\sqrt{10^2 - x^2}$

$25 = 100 - x^2$

$x^2 = 75$

$x = \sqrt{75} = 5\sqrt{3}$ cm

# P-Q1035-Solution

## Ans [C]

It is given $v_{max} = 100\ cm/sec$ , $a = 10\ cm.$

$$\Rightarrow v_{max} = a\omega \Rightarrow \omega = \frac{100}{10} = 10\ rad/sec$$

For max velocity y will be zero

Hence $v = \omega\sqrt{a^2 - y^2} \Rightarrow 50 = 10\sqrt{(10)^2 - y^2}$

$$\Rightarrow y = 5\sqrt{3}\ cm$$

## P-Question

A simple pendulum has a time period $T_1$ when on the earth's surface, and $T_2$ when taken to height R above the earth's surface, where R is the radius of the earth. The value of $T_2/T_1$ is ……

[IIT SCREENING 2001]

1) **1**

2) $\sqrt{2}$

3) **4**

4) **2** ✓

$$T_1 = 2\pi\sqrt{\frac{l}{g}}$$

$$g_h = \frac{GM}{(R+h)^2} = \frac{GM}{4R^2} = g/4$$

$$T_2 = 2\pi\sqrt{\frac{l}{g/4}} = 2\pi\sqrt{\frac{4l}{g}} = 2\left(2\pi\sqrt{\frac{l}{g}}\right)$$

$$\frac{T_2}{T_1} = \frac{2}{1}$$

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}}$$

## P-Solution

**KEY: ~~2~~ 4**

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\frac{T_h}{T_e} = \sqrt{\frac{g_e}{g_h}} = \frac{R + h}{R} = \frac{2R}{R} = 2$$

## P-Q1036

A simple harmonic oscillator has a period of 0.01 *sec* and an amplitude of 0.2 *m*. The magnitude of the velocity in $m\ sec^{-1}$ at the centre of oscillation is

(A) $20\pi$

(B) 100

(C) $40\pi$

(D) $100\pi$

$$v_{max} = \omega A = \frac{2\pi}{T} \cdot A$$

$$= \frac{2\pi}{0.01} \times 0.2 = 2\pi \times 20$$

$$= 40\pi$$

## P-Q1036-Solution

### Ans [C]

At centre $v_{max} = a\omega = a.\frac{2\pi}{T} = \frac{0.2 \times 2\pi}{0.01} = 40\pi$ ← For max velocity y will be zero

## P-Q1038

A body is executing S.H.M. When its displacement from the mean position is 4 *cm* and 5 *cm*, the corresponding velocity of the body is 10 *cm/sec* and 8 *cm/sec*. Then the time period of the body is

(A) $2\pi\ sec$

(B) $\pi / 2\ sec$

(C) $\pi\ sec$ ✓

(D) $3\pi / 2\ sec$

$$v = \omega\sqrt{A^2 - x^2}$$

$$10 = \omega\sqrt{A^2 - 4^2} \Rightarrow 100 = \omega^2 A^2 - 16\omega^2 \Rightarrow 100 + 16\omega^2 = \omega^2 A^2$$

$$8 = \omega\sqrt{A^2 - 5^2} \Rightarrow 64 = \omega^2 A^2 - 25\omega^2 \Rightarrow 64 + 25\omega^2 = \omega^2 A^2$$

$$100 + 16\omega^2 = 64 + 25\omega^2$$

$$36 = 9\omega^2 \;\;(4 = \omega^2) \Rightarrow \omega = 2$$

$$\Rightarrow \frac{2\pi}{T} = 2 \Rightarrow T = \pi\ s$$

## P-Q1038-Solution

### Ans [C]

Using the eq. of velocity we can calculate ω

$$v = \omega\sqrt{a^2 - y^2} \Rightarrow 10 = \omega\sqrt{a^2 - (4)^2} \text{ and } 8 = \omega\sqrt{a^2 - (5)^2}$$

On solving $\omega = 2 \Rightarrow \omega = \dfrac{2\pi}{T} = 2 \Rightarrow T = \pi\ sec$

## P-Q1040

If the displacement of a particle executing SHM is given by $y = 0.30\sin(220t + 0.64)$ in *metre*, then the frequency and maximum velocity of the particle is

(A) 35 *Hz*, 66 *m* / *s*  (B) 45 *Hz*, 66 *m* / *s*

(C) 58 *Hz*, 113 *m* / *s*  (D) 35 *Hz*, 132 *m* / *s*

$y = A\sin(\omega t + \phi)$

$\omega = 220$

$2\pi f = 220$

$2 \times \frac{22}{7} f = 220$

$f = 35$ Hz

$v_{max} = \omega A$

$= 220 \times 0.3$

$= 22 \times 3$

$= 66$ m/s

## P-Q1040-Solution

### Ans [A]

$$n = \frac{\omega}{2\pi} = \frac{220}{2\pi} = 35\ Hz$$

$$v_{max} = \omega a = 220 \times 0.30\ m/s = 66\ m/s$$

Maximum velocity will be at centre where y=0

## P-Q1072

A particle in SHM is described by the displacement equation $x(t) = A\cos(\omega t + \theta)$. If the initial $(t = 0)$ position of the particle is $1\ cm$ and its initial velocity is $\pi\ cm/s$, what is its amplitude? The angular frequency of the particle is $\pi s^{-1}$

(A) $1\ cm$

(B) $\sqrt{2}\ cm$

(C) $2\ cm$

(D) $2.5\ cm$

$x = A\cos(\omega t + \theta)$ —(1)

$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \theta)$ —(2)

$1 = A\cos\theta$

$\pi = -\omega A \sin\theta$

$\pi = -\pi A \sin\theta$

$1 = A\cos\theta$ —(3)

$-1 = A\sin\theta$ —(4)

$(3)^2 + (4)^2$

$1 = A^2\cos^2\theta$

$+\ 1 = A^2\sin^2\theta$

$2 = A^2$

$A = \sqrt{2}$

## P-Q1072-Solution

## Ans [B]

Given, $v = \pi\, cm/\sec$, $x = 1\, cm$ and $\omega = \pi s^{-1}$

using $v = \omega\sqrt{a^2 - x^2} \Rightarrow \pi = \pi\sqrt{a^2 - 1}$

$\Rightarrow 1 = a^2 - 1 \Rightarrow a = \sqrt{2}\, cm.$

Given that $\pi$ is the velocity and angular frequency

## P-Q1074

The period of a simple pendulum is doubled, when

(A) Its length is doubled

(B) The mass of the bob is doubled

(C) Its length is made four times

(D) The mass of the bob and the length of the pendulum are doubled

$T = 2\pi\sqrt{\frac{l}{g}}$

## P-Q1074-Solution

### Ans [C]

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$$

For double time period its length is has to be made four times

Remember time period doesn't depend on the mass suspended.

## P-Q1075

The period of oscillation of a simple pendulum of constant length at earth surface is T. Its period inside a mine is

(A) Greater than T (B) Less than T

(C) Equal to T (D) Cannot be compared

$T = 2\pi\sqrt{\frac{l}{g}}$ $g \downarrow$ $T \uparrow$

$g_d = g\left(1 - \frac{d}{R}\right)$

## P-Q1075-Solution

### Ans [A]

Inside the mine g will decrease

hence from $T = 2\pi\sqrt{\frac{l}{g}}$ ; $T$ increase

## P-Q1076

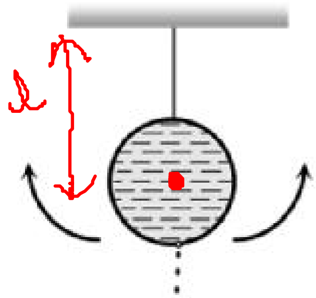
A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will

(A) Remains unchanged

(B) Increase

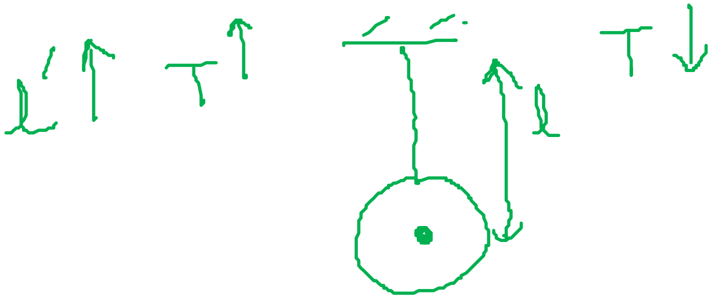
(C) Decrease

(D) Become erratic




$T = 2\pi\sqrt{\frac{l}{g}}$

l = distance b/w pt. of suspension & C.O.M of bob

## P-Q1076-Solution

### Ans [B]

When a little mercury is drained off, the position of center of gravity of ball falls (wrt fixed) so that effective length so pendulum increases hence T increases.

## P-Q1077

The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (If it is a second's pendulum on earth)

(A) $\frac{1}{\sqrt{2}}$ *sec*

(B) $2\sqrt{2}$ *sec*

(C) 2 *sec*

(D) $\frac{1}{2}$ *sec*

## P-Q1077-Solution

### Ans [B]

As we know $g = \frac{GM}{R^2}$

$$\Rightarrow \frac{g_{\text{earth}}}{g_{\text{planet}}} = \frac{M_e}{M_p} \times \frac{R_\rho^2}{R_e^2} \Rightarrow \frac{g_e}{g_p} = \frac{2}{1}$$

$$\text{Also } T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}} \Rightarrow \frac{2}{T_p} = \sqrt{\frac{1}{2}}$$

$$\Rightarrow T_p = 2\sqrt{2} \text{ sec.}$$

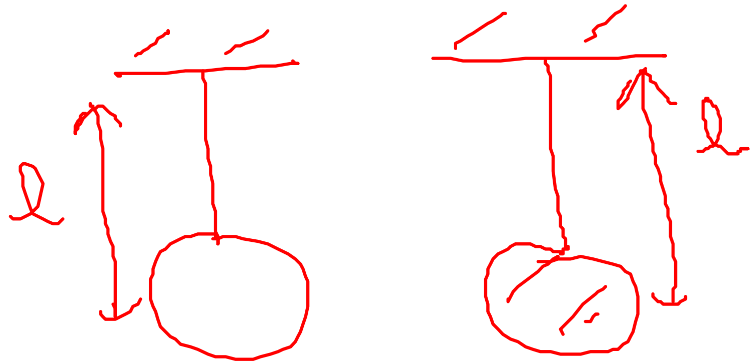
A seconds pendulum is a pendulum whose period is precisely two seconds.

## P-Q1078

The time period of a second's pendulum is 2 *sec*. The spherical bob which is empty from inside has a mass of 50 *gm*. This is now replaced by another solid bob of same radius but having different mass of 100 *gm*. The new time period will be

(A) 4 *sec*   (B) 1 *sec*

(C) 2 *sec*   (D) 8 *sec*




$$T = 2\pi\sqrt{\frac{l}{g}}$$

## P-Q1078-Solution

### Ans [C]

$$T = 2\pi\sqrt{\frac{l}{g}}$$

It is independent of mass

## P-Q1099

A simple pendulum is executing simple harmonic motion with a time period *T*. If the length of the pendulum is increased by 21%, the percentage increase in the time period of the pendulum of increased length is

(A) 10% (B) 21%

(C) 30% (D) 50%

$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$

$l' = 1.21\, l$

$\sqrt{l'} = 1.1\sqrt{l}$

10%