Problem Solving on Area Under Curve

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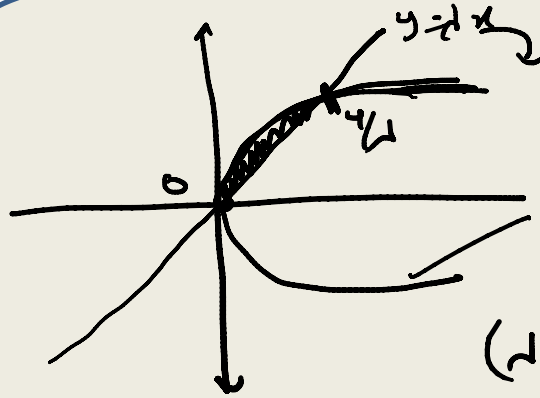
(1) If the area (in sq units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to

(2019 Main, 12 April II)

(a) $2\sqrt{6}$ (b) 48 (c) 24 (d) $4\sqrt{3}$

+4



$A = 1/9$ $\lambda = ?$

$\frac{8}{3} \times \frac{a^2}{\lambda^3} = \frac{1}{9}3$

$(\lambda x)^2 = 4\lambda x$

$\lambda^2 x^2 = 4\lambda x$

$x = \frac{4}{\lambda}$

$\lambda = 24$

$y = mx$

$y^2 = 4ax$

$A = \left(\frac{8}{3}\frac{a^2}{m^3}\right)$

$A = \int_0^{4/\lambda} \left(\sqrt{4\lambda x} - \lambda x\right) dx$

=

9)

If the area (in sq units) of the region $\{(x, y): y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2} + b$, then $a - b$ is equal to

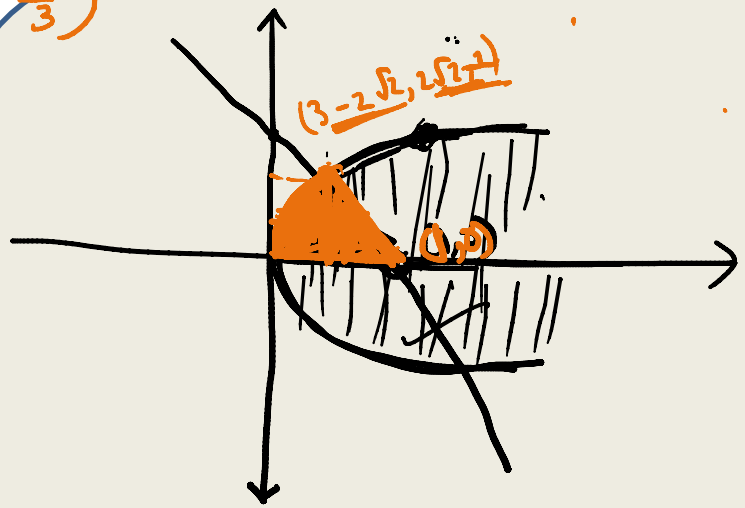
(2019 Main, 12 April I)

(a) $\frac{10}{3}$ (b) 6 (c) $\frac{8}{3}$ (d) $-\frac{2}{3}$

$A = \int y\,dx$

$= \int x\,dy$

$\frac{8}{3} - \left(-\frac{2}{3}\right)$



$x + y = 1$, $y^2 = 4x$

$y^2 = 4(1-y)$

$x = 3 - 2\sqrt{2}$

$y^2 + 4y = 4$

$(y+2)^2 = 8$

$y = 2\sqrt{2} - 2$

$$\int_0^{3-2\sqrt{2}} 2\sqrt{x}\,dx + \int_{3-2\sqrt{2}}^{1} (1-x)\,dx$$

$A = \int_0^{2\sqrt{2}-2} (x_2 - x_1)\,dy$

$= \int_0^{2\sqrt{2}-2} \left[(1-y) - \left(\frac{y^2}{4}\right)\right] dy$

$= y - \frac{y^2}{2} - \frac{y^3}{12} \Big|_0^{2\sqrt{2}-2}$

$= y\left(1 - \frac{y}{2} - \frac{y^2}{12}\right)$

$= (2\sqrt{2}-2)\left[1 - (\sqrt{2}-1) - \left(1 - \frac{2\sqrt{2}}{3}\right)\right]$

$1 - \sqrt{2} + \frac{2\sqrt{2}}{3}$

$(2\sqrt{2}-2)\left(\frac{3-\sqrt{2}}{3}\right)$

$\frac{6\sqrt{2} - 10 + 2\sqrt{2}}{3}$

$\left(\frac{8\sqrt{2}-10}{3}\right)$

$8 + 4 - 8\sqrt{2}$

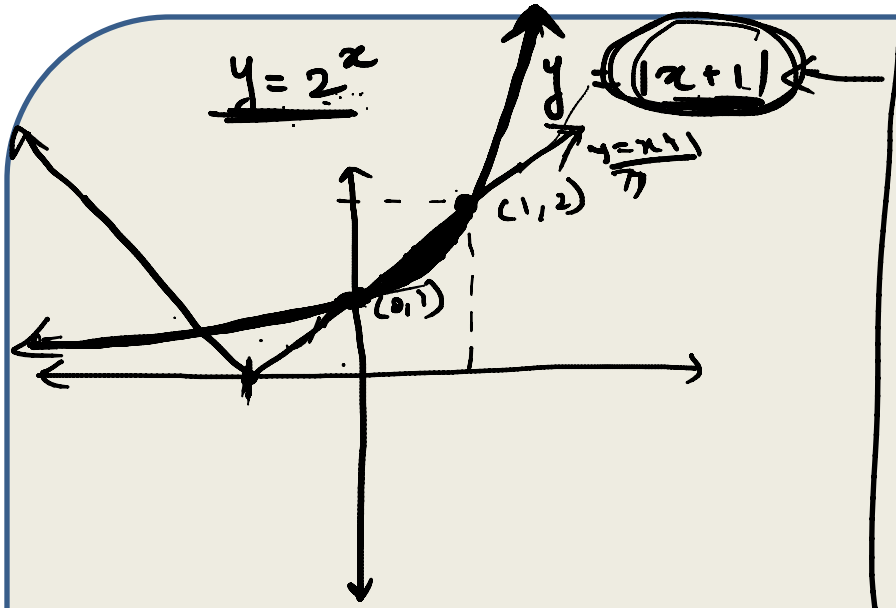
$\frac{12 - 8\sqrt{2}}{12} = 1 - \frac{2\sqrt{2}}{3}$

9)

The area (in sq units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is

(2019 Main, 10 April II)

(a) $\frac{3}{2}$ (b) $\log_e 2 + \frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2} - \frac{1}{\log_e 2}$



$$A = \int_0^1 [(x+1) - (2^x)]\,dx$$

$$= \left(\frac{x^2}{2} + x - \frac{2^x}{\log 2}\right)\Big|_0^1$$

$$= \frac{3}{2} - \frac{2}{\log 2} + \frac{1}{\log 2}$$

$$= \left(\frac{3}{2} - \frac{1}{\log 2}\right)$$

9) The area (in sq units) of the region $A = \left\{(x, y) : \frac{y^2}{2} \le x \le y + 4\right\}$ is

(2019 Main, 9 April II)

(a) 30 (b) $\frac{53}{3}$ (c) 16 (d) 18

$\int |f(x) - g(x)|\, dx$

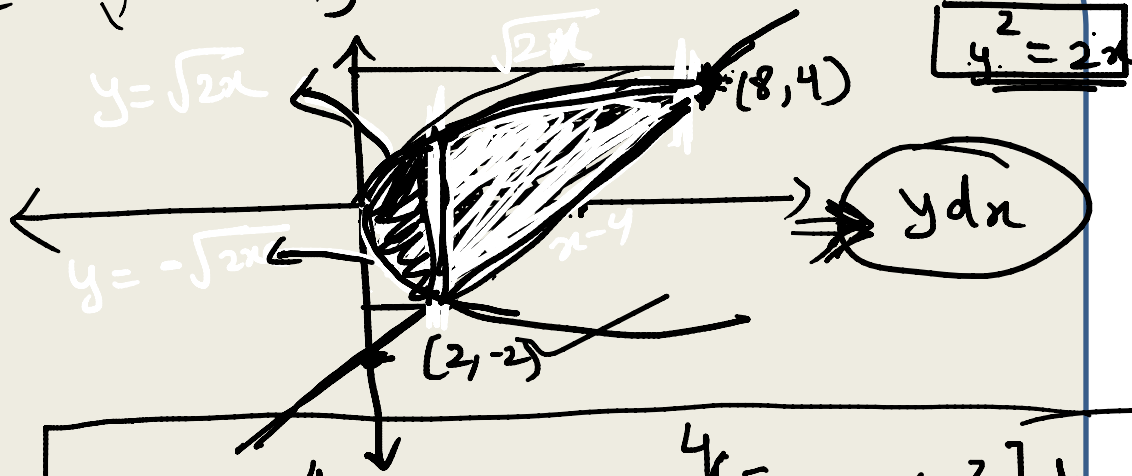
$y^2 = 2x$

$y = \pm\sqrt{2x}$

$A = \left\{(x,y) : \frac{y^2}{2} \le x \le y+4\right\}$

$x = y + 4$

$y^2 = 2x$



$x \le y + 4$

$y \ge x - 4$

$y^2 = 2(y+4)$

$y^2 - 2y - 8 = 0$

$(y-1)^2 = 9$

$y = 4, -2$

$A = \int_{-2}^{4} x\,dy = \int_{-2}^{4} \left[(y+4) - \left(\frac{y^2}{2}\right)\right] dy$

$\left(24 - \frac{32}{3}\right) - \left(-6 + \frac{1}{3}\right)$

$30 - \frac{36}{2}$ 12

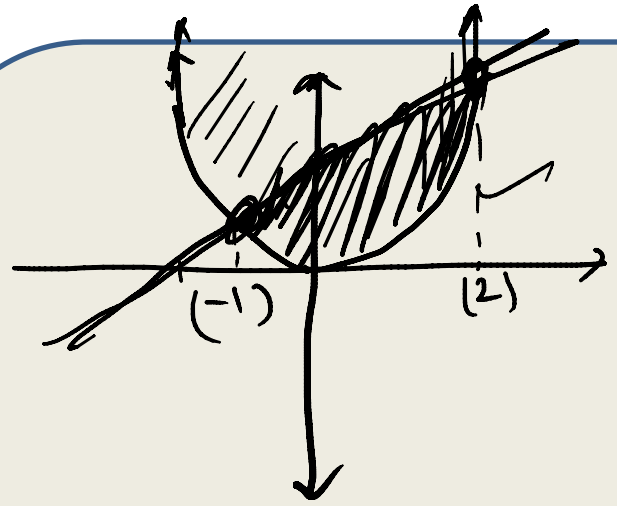
$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{6}\right]_{-2}^{4}$

$= \left(8 + 16 - \frac{64}{6}\right) - \left(2 - 8 + \frac{8}{6}\right)$

9) Eun

The area (in sq units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is **(2019 Main, 9 April I)**

(a) $\frac{13}{6}$ (b) $\frac{9}{2}$ (c) $\frac{31}{6}$ (d) $\frac{10}{3}$



$y = x^2$, $y = x + 2$

$x^2 = x + 2$

$x = -1, 2$

$$A = \int_{-1}^{2} [(x+2) - (x^2)]\, dx$$

$$= \left(\frac{x^2}{2} + 2x - \frac{x^3}{3} \right) \Big|_{-1}^{2}$$

$$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

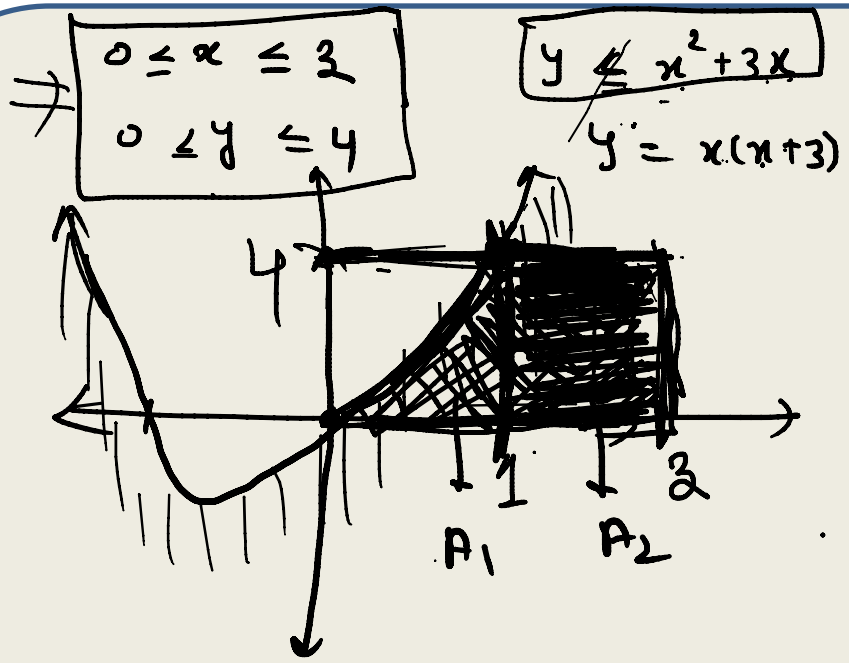
$$= \left(\frac{10}{3}\right) - \left(\frac{-7}{6}\right)$$

$$= \frac{27}{6} \quad \left(\frac{9}{2}\right)$$

The area (in sq units) of the region
$A = \{(x, y) \in R \times R | 0 \le x \le 3,$
$0 \le y \le 4, y \le x^2 + 3x\}$ is

(2019 Main, 8 April I)

(a) $\frac{53}{6}$ (b) 8 (c) $\frac{59}{6}$ (d) $\frac{26}{3}$



$2 \times 4 = 8 = A_2$

$\int_0^1 (x^2+3x)\,dx$

$= \frac{x^3}{3} + \frac{3x^2}{2} \Big|_0^1$

$A_1 = \frac{11}{6}$

$8 + \frac{11}{6}$

$= \frac{59}{6}$