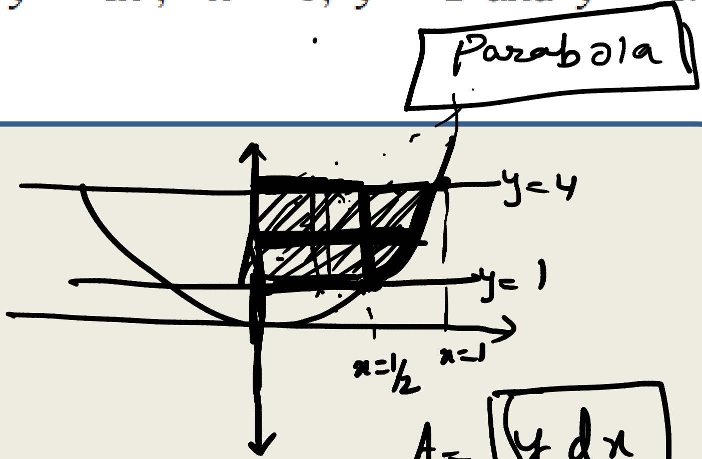
9) Find the area in the first quadrant bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$.

$$A = \int_1^4 x\,dy$$

$$= \int_1^4 \sqrt{\frac{y}{4}}\,dy$$

$$= \frac{1}{2}\int_1^4 \sqrt{y}\,dy$$



$$A = \boxed{\int y\,dx}$$

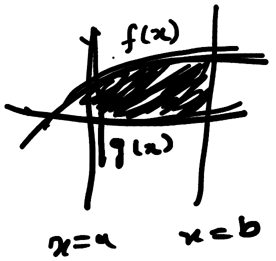
$$\int_0^{1/2} 3\,dx + \int_{1/2}^{1} (4-4x^2)\,dx$$

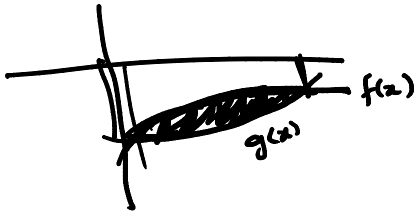
AREA ENCLOSED BETWEEN TWO CURVES :

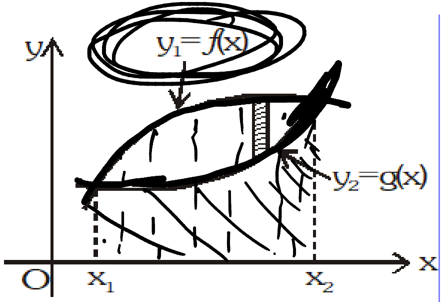
(a) Area bounded by two curves $y = f(x)$ & $y = g(x)$ such that $f(x) > g(x)$ is

$$A = \int_{x_1}^{x_2} (y_1 - y_2)dy$$

$$\boxed{A = \int_{x_1}^{x_2} [f(x) - g(x)]dx}$$







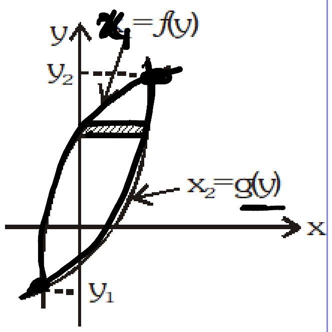
(b) In case horizontal strip is taken we have

$$A = \int_{y_1}^{y_2} (x_1 - x_2)dy$$

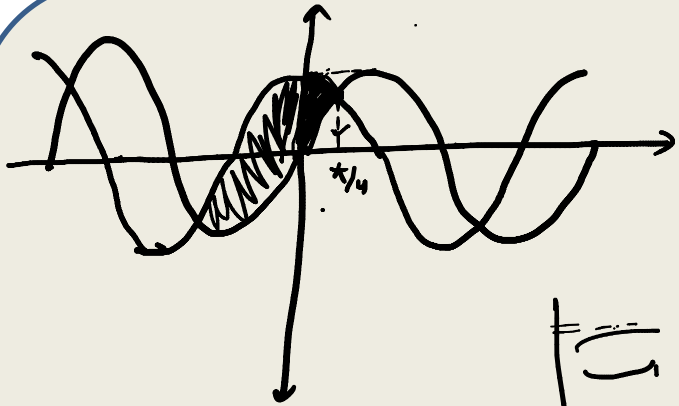
$$A = \int_{y_1}^{y_2} [f(y) - g(y)]dy$$

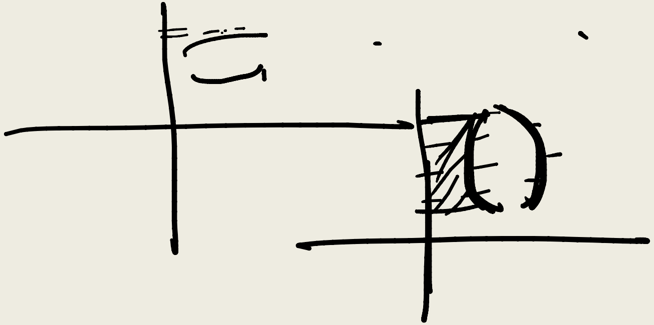
$$A = \int x\,dy$$

$$A = \int_{y_1}^{y_2} [f(y) - g(y)]\,dy$$



9) Find the area enclosed between y = sin x ; y = cos x and y-axis in the 1st quadrant





$0^\circ < \theta < 90^\circ$

$$A = \int_0^{\pi/4} (\cos x - \sin x)\, dx$$

$$= (\sin x + \cos x)\Big|_0^{\pi/4}$$

$$= (\sqrt{2} - 1) \text{ sq. units}$$

IMPORTANT POINTS :

(a) Since area remains invariant even if the co-ordinate axes are shifted, hence shifting of origin in many cases proves to be very convenient in computing the area.

Q2) Find the area of the region common to the circle $x^2 + y^2 + 4x + 6y - 3 = 0$ and the parabola $x^2 + 4x = 6y + 14$.

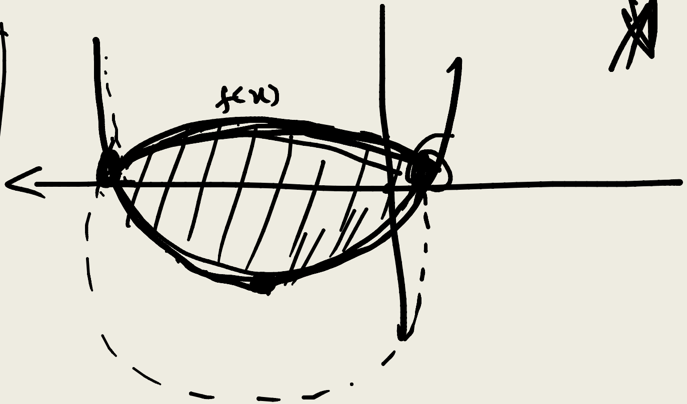
$(x+2)^2 + (y+3)^2 = 16$

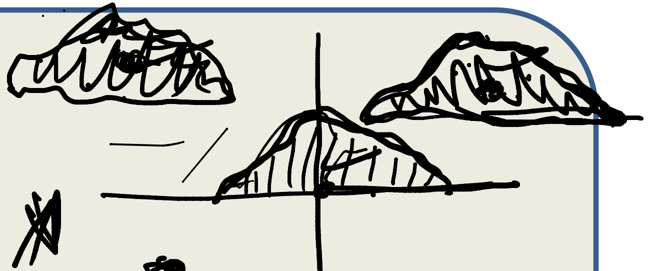
$(x+2)^2 = 6(y+3)$

$$\boxed{\begin{aligned} X^2 + Y^2 &= 16 \\ X^2 &= 6Y \end{aligned}}$$

$y^2 + 6y - 16 = 0$

$(y+8)(y-2) = 0$





$$\int_{-8}^{2} \left(\sqrt{16 - x^2} - \frac{x^2}{6}\right) dx$$

The area bounded by a curve & an axis is equal to the area bounded by the inverse of that curve & the other axis, i.e., the area bounded by $y = f(x)$ and x-axis (say) is equal to the area bounded by $y = f^{-1}(x)$ and y-axis.

Q) If $y = g(x)$ is the inverse of a bijective mapping $f : R \to R$ $\boxed{f(x) = 6x^5 + 4x^3 + 2x}$, find the area bounded by $g(x)$, the x-axis and the ordinate at $x = 12$.

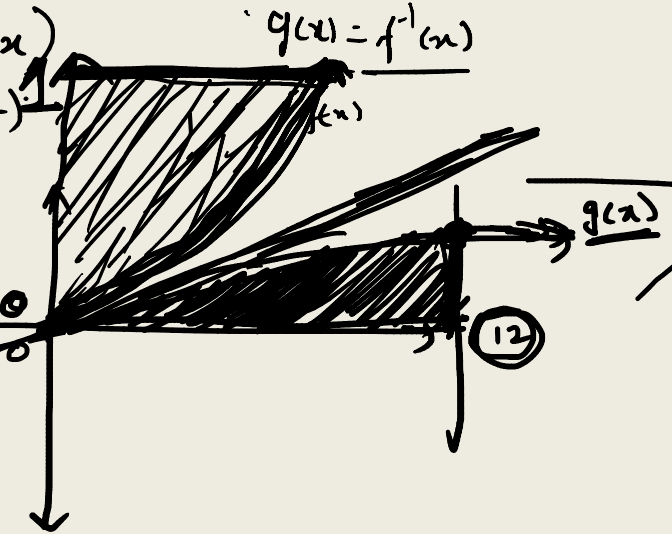
$$f(x) = (6x^5 + 4x^3 + 2x)$$

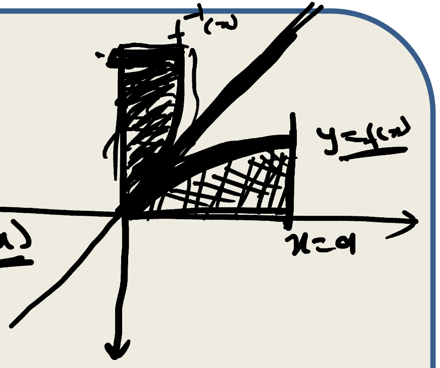
$$= x(6x^4 + 4x^2 + 2)$$

$$f'(x) = (30x^4 + 12x^2 + 2)$$

$$g(x) = f^{-1}(x)$$

$$\int_0 \left[1 - (6x^5 + 4x^3 + 2x)\right] dx$$





$$f^{-1}(x) = y$$

$$f^{-1}(12) = y$$

$$\boxed{12 = f(y)}$$

USEFUL RESULTS :

(a) Whole area of the ellipse, $x^2/a^2 + y^2/b^2 = 1$ is πab sq.units.

(b) Area enclosed between the parabolas $y^2 = 4ax$ & $x^2 = 4by$ is $16ab/3$ sq.units.

(c) Area included between the parabola $y^2 = 4ax$ & the line $y = mx$ is $8a^2/3m^3$ sq.units.