Area Under Curve

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By

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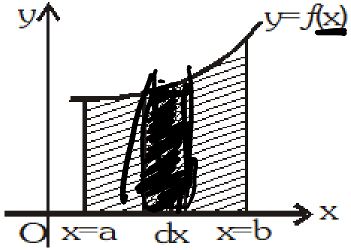
AREA UNDER THE CURVES :

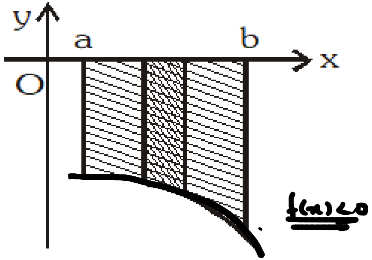
(a) Area bounded by the curve $y = f(x)$, the x-axis and the ordinates at $x = a$ and $x = b$ is given by $\boxed{A = \int_a^b y\,dx}$, where $y = f(x)$ lies above the x-axis and $b > a$. Here vertical strip of thickness dx is considered at distance x.



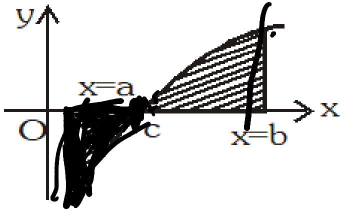
(b) If $y = f(x)$ lies completely below the x-axis then A is negative and we consider the magnitude only, i.e. $\boxed{A = \left|\int_a^b y\,dx\right|}$

$\int_a^b f(x)dx = -ve$

$A = \left|\int_a^b f(x)dx\right|$



(c) If curve crosses the x-axis at $x = c$, then $A = \left|\int_a^c y\,dx\right| + \int_c^b y\,dx$



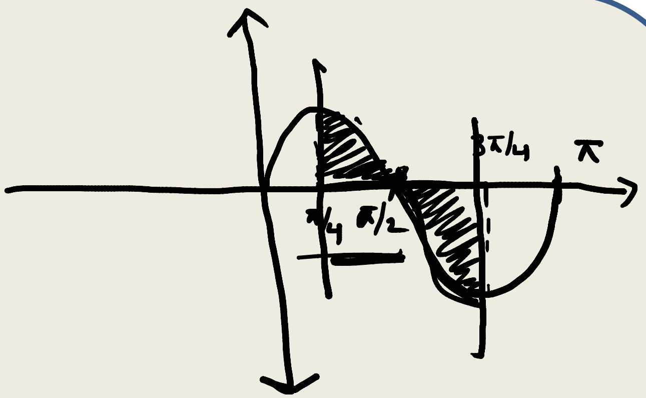
$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

$\left|\int_a^b y\,dx\right|$

Find the area bounded by the curve y = sin2x, x-axis and the lines $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$

$$A = \int_{\pi/4}^{\pi/2} \sin 2x \, dx + \left| \int_{\pi/2}^{3\pi/4} \sin 2x \right|$$

Graph- plotting

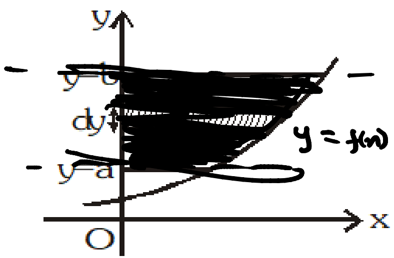


$$= -\frac{\cos 2x}{2}\Big|_{\pi/4}^{\pi/2} + \left| \left[-\frac{\cos 2x}{2} \right]_{\pi/2}^{3\pi/4} \right|$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1 \text{ sq. units}$$

Sometimes integration w.r.t. y is very useful (horizontal strip) :

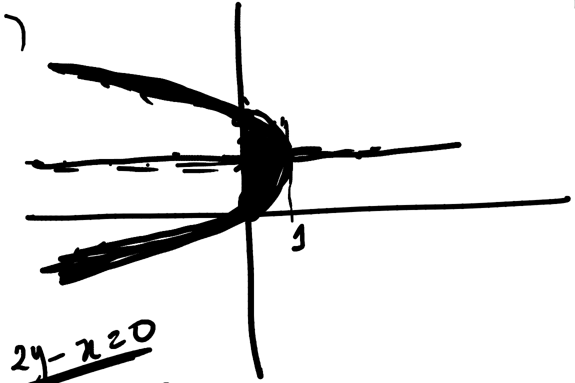


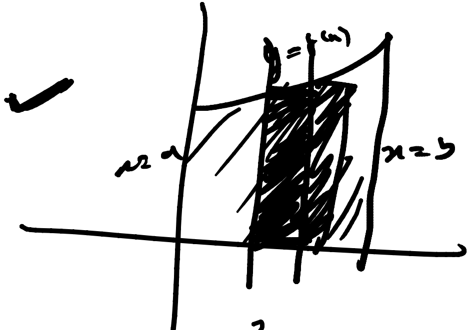
Area bounded by the curve, y-axis and the two abscissae at y = a & y = b is written as $\boxed{A = \int_a^b x\,dy}$.

Q) $(x = 2y - y^2)$ & y- axis

$$\boxed{A = \int_0^2 (2y - y^2)\,dy}$$

$y = (\ \ )$



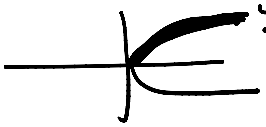


$y^2 = 4ax$

$y^2 - 2y - x = 0$

$y = (\ \ )$

$A = \int y\,dx$

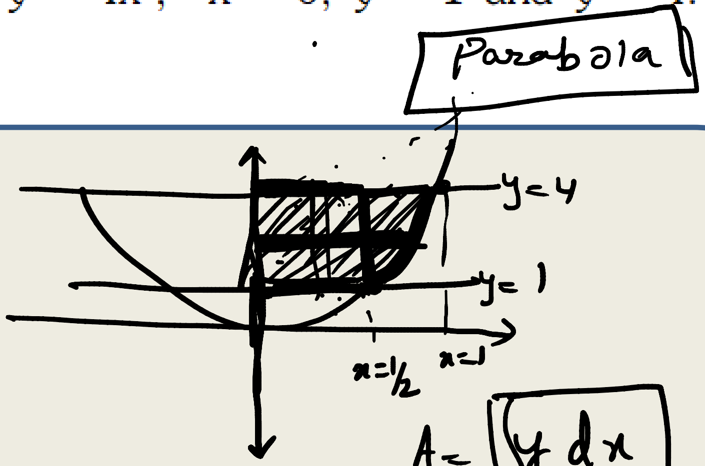


9) Find the area in the first quadrant bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$.

$$A = \int_1^4 x\,dy$$

$$= \int_1^4 \sqrt{\frac{y}{4}}\,dy$$

$$= \frac{1}{2}\int_1^4 \sqrt{y}\,dy$$



$$A = \boxed{\int y\,dx}$$

$$\int_0^{1/2} 3\,dx + \int_{1/2}^{1} (4-4x^2)\,dx$$