to Find r.m.s. value we calculate ~~power~~ Heat in full cycle.

learn.

IIT-JEE/NEET-PHYSICS

TOPIC - AC

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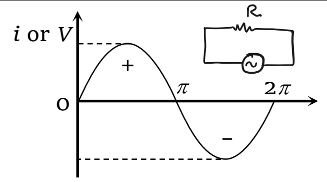
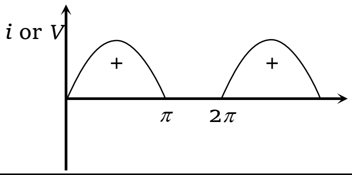
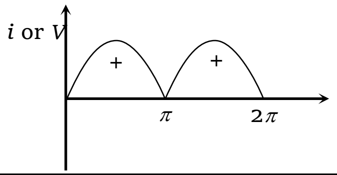
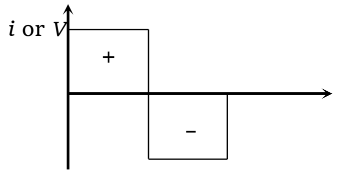
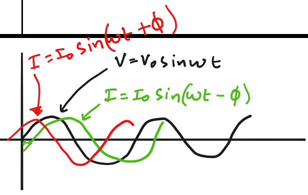
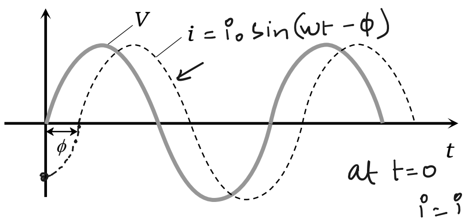
Nature of wave form	Wave form	r.m.s. value	average value	Form factor $R_f = \frac{\text{r.m.s. value}}{\text{Average value}}$	Peak factor $R_p = \frac{\text{Peak value}}{\text{r.m.s. value}}$
Sinusoidal ✓ imp.		$\frac{i_0}{\sqrt{2}}$	$\frac{2}{\pi} i_0$	$\frac{\pi}{2\sqrt{2}} = 1.11$	$\sqrt{2} = 1.41$
Half wave rectified		$\frac{i_0}{2}$	$\frac{i_0}{\pi}$	$\frac{\pi}{2} = 1.57$	2
Full wave rectified ✓		$\frac{i_0}{\sqrt{2}}$	$\frac{2i_0}{\pi}$	$\frac{\pi}{2\sqrt{2}}$	$\sqrt{2}$
Square or Rectangular		i_0	i_0	1	1

Phase difference



current lagging voltage by φ

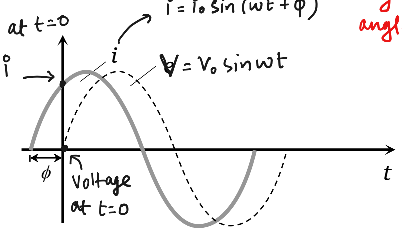
current leading voltage by φ angle



at t=0

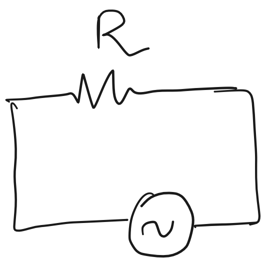
$i = i_0 \sin(-\phi)$

$i = -i_0 \sin\phi$

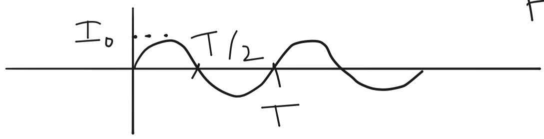


Voltage (V) = $V_0 \sin \omega t$
Current (i) = $i_0 \sin(\omega t - \phi)$
Phase difference = 0 – (– ϕ) = + ϕ
i.e. voltage is leading by an angle (+ ϕ) *w.r.t.* current

Voltage (V) = $V_0 \sin \omega t$
Current (i) = $i_0 \sin(\omega t + \phi)$
Phase difference = 0 – (+ ϕ) = – ϕ
i.e. voltage is leading by an angle (– ϕ) *w.r.t.* current



$I = I_0 \sin\omega t$



Heat = Power × time

Heat = $\int \text{Power}\, dt$

Heat = $\int_0^T i^2 R\, dt$

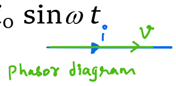
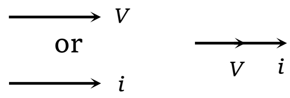
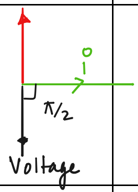
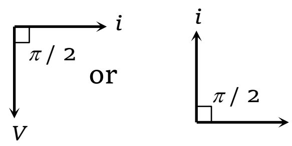
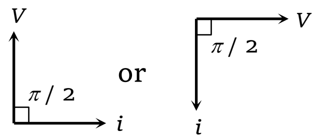
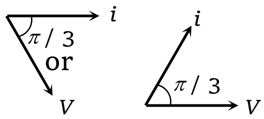
Heat = $\int_0^T R I_0^2 \sin^2\omega t\, dt$

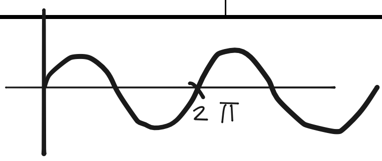
$J_0 = \frac{I_0^2 RT}{2}$

$I_{rm}^2 \cancel{RT} = \frac{I_0^2 \cancel{RT}}{2}$

$I_{rm} = \frac{I_0}{\sqrt{2}}$

Phasor and phasor diagram

Equation of *V* and *i*	Phase difference ϕ	Time difference T.D.	Phasor diagram
$V = V_0 \sin\omega t$ $i = i_0 \sin\omega t$  	0	0	
$V = V_0 \sin\omega t$ $i = i_0 \sin(\omega t + \frac{\pi}{2})$  	$-\frac{\pi}{2}$	$\frac{T}{4}$	
$V = V_0 \sin\omega t$ $i = i_0 \sin(\omega t - \frac{\pi}{2})$	$+\frac{\pi}{2}$	$\frac{T}{4}$	
$V = V_0 \sin\omega t$ $i = i_0 \sin(\omega t + \frac{\pi}{3})$	$-\frac{\pi}{3}$	$\frac{T}{6}$	



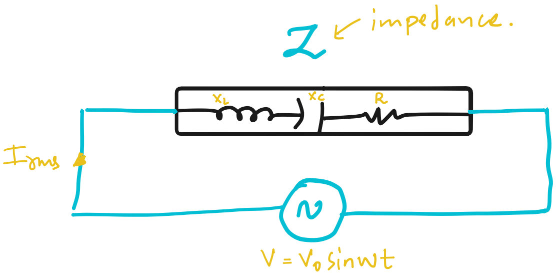
$2\pi \longrightarrow T$

$\pi/2 \longrightarrow T/4$

(1) **Resistance (*R*) :** The opposition offered by a conductor to the flow of current through it is defined as the resistance of that conductor. Reciprocal of resistance is known as conductance (*G*) *i.e.* $G = \frac{1}{R}$

(2) **Impedance (*Z*) :** The opposition offered by the capacitor, inductor and conductor to the flow of ac through it is defined as impedance. It's unit is *ohm*(Ω). $Z = \frac{V_0}{i_0} = \frac{V_{rms}}{i_{rms}}$

impedance represent equivalant resistance

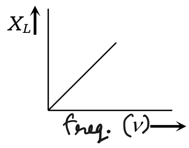
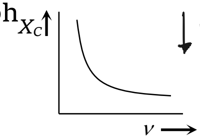


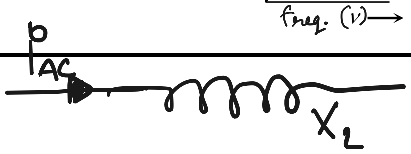
unit of impedance also ohm.

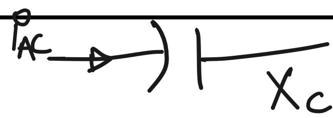
$V = IZ$

$Z = \frac{V_{rms}}{I_{rms}}$

(3) **Reactance (X) :** The opposition offered by inductor or capacitor or both to the flow of ac through it is defined as reactance. It is of following two type –

Inductive reactance (X_L)	**Capacitive reactance (X_C)**
(i) Offered by inductive circuit	(i) Offered by capacitive circuit
(ii) $X_L = \omega L = 2\pi \nu L$ $X_L = wL = 2\pi f L$ ← self inductance. For DC $f = 0$ $X_L = 0$ inductor is just a wire for DC.	(ii) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi \nu C}$
(iii) $\nu_{dc} = 0$ so for dc, $X_L = 0$	(iii) For dc $X_C = \infty$ $X_c = \frac{1}{2\pi f c} = \frac{1}{0} = \infty$ it means cap. blocks DC
(iv) X_L-ν Graph  $x_L \propto$ freq.	(iv) X_C - ν Graph  $\downarrow X_c \propto \frac{1}{\text{Freq}\uparrow}$





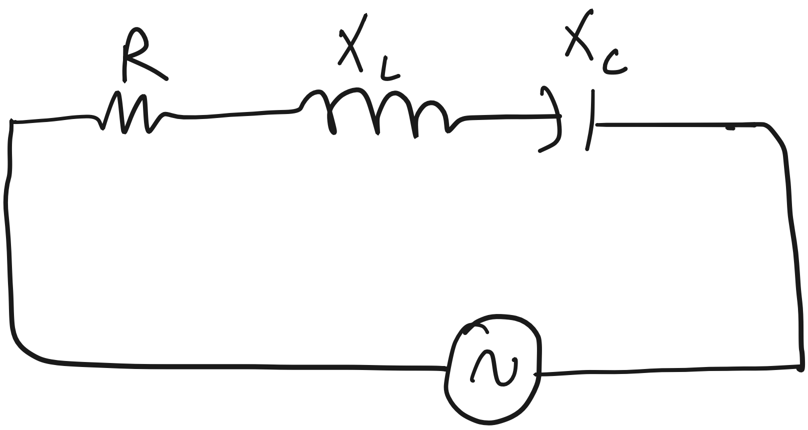
$X_C = \frac{1}{wC} = \frac{1}{2\pi f c}$

Note : ❑ Resultant reactance of LC circuit is defined as $X = X_L \sim X_C$.

(4) **Admittance (Y)** : Reciprocal of impedance is known as admittance $\left(Y = \dfrac{1}{Z}\right)$. It's unit is *mho*.

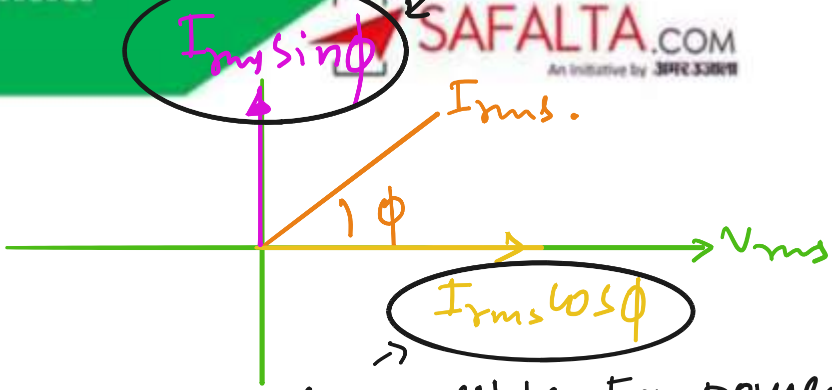
(5) **Susceptance (S)** : the reciprocal of reactance is defined as susceptance $\left(S = \dfrac{1}{X}\right)$. It is of two type

(i) inductive susceptance $S_L = \dfrac{1}{X_L} = \dfrac{1}{2\pi \nu L}$ and (ii) Capacitive susceptance, $S_C = \dfrac{1}{X_C} = \omega C = 2\pi \nu C$.



watt less current

Power = $I_{rms} V_{rms} \cos\phi$



responsible For power.

Types of power

(i) **Instantaneous power :** Suppose in a circuit $V = V_0 \sin\omega t$ and $i = i_0 \sin(\omega t + \phi)$ then $P_{\text{instantaneous}} = Vi = V_0 i_0 \sin\omega t \sin(\omega t + \phi)$

(ii) **Average power (True power) :** The average of instantaneous power in an ac circuit over a full cycle is called average power. It's unit is *watt i.e.*

$$P_{av} = \overline{P}_{inst} \Rightarrow P_{av} = V_{rms} i_{rms} \cos\phi = \frac{V_0}{\sqrt{2}} \cdot \frac{i_0}{\sqrt{2}} \cos\phi = \frac{1}{2} V_0 i_0 \cos\phi = i_{rms}^2 R = \frac{V_{rms}^2 R}{Z^2}$$

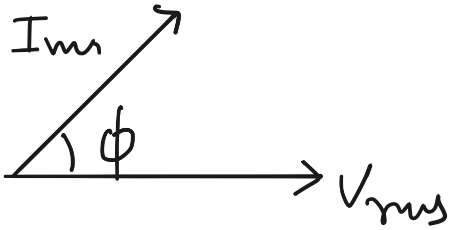
$P_{av} = i_{rms} V_{rms} \cos\phi$

where ϕ is angle between V_{rms} & I_{rms}.

Power factor : It may be defined as

(i) Cosine of the angle of lag or lead

$\cos\phi$ is known as power factor.



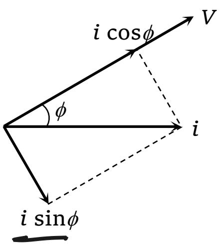
(ii) The ratio $\frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}} = \cos\phi$

(iii) The ratio $\frac{\text{True power}}{\text{Apparent power}} = \frac{W}{VA} = \frac{kW}{kVA} = \cos\phi = \frac{I_{rms} V_{rms} \cos\phi}{I_{rms} V_{rms}} = \cos\phi$

Note : ❑ Power factor is a dimensionless quantity and it's value lies between 0 and 1.

❑ For a pure resistive circuit $R = Z \Rightarrow$ p.f. $= \cos\phi = 1$

The component of current which does not contribute to the average power dissipation is called wattless current



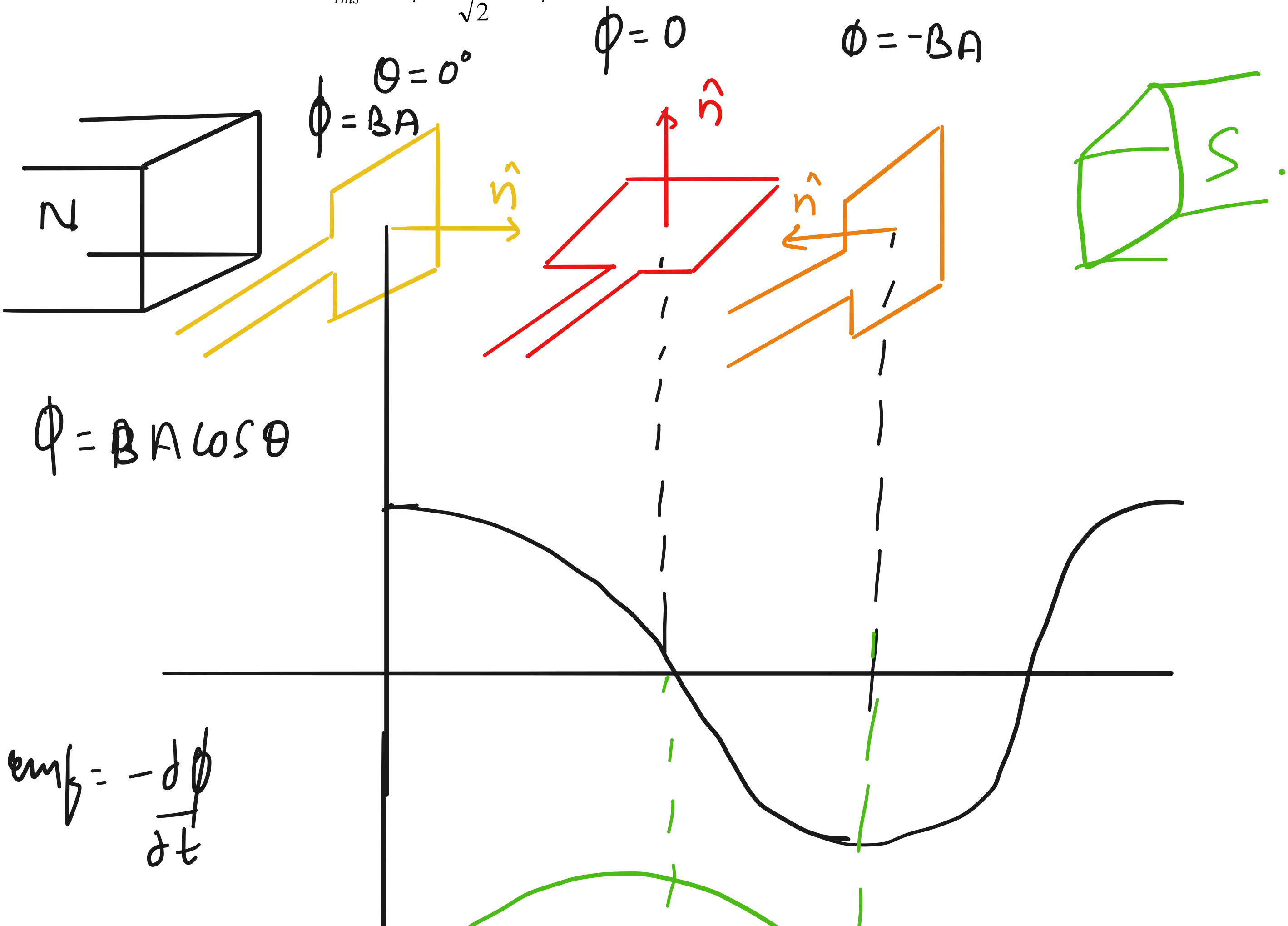
(i) The average of wattless component over one cycle is zero

(ii) Amplitude of wattless current = $i_0 \sin\phi$

and r.m.s. value of wattless current = $i_{rms} \sin\phi = \frac{i_0}{\sqrt{2}} \sin\phi$.

It is quadrature (90°) with voltage

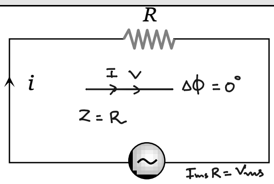
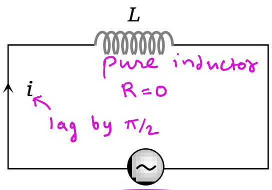
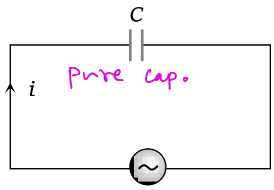
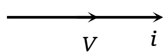
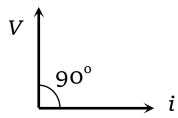
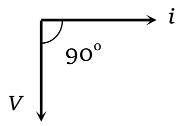
Note : ❑ The component of ac which remains in phase with the alternating voltage is defined as the effective current. The peak value of effective current is $i_0 \cos\phi$ and it's r.m.s. value is $i_{rms} \cos\phi = \frac{i_0}{\sqrt{2}} \cos\phi$.



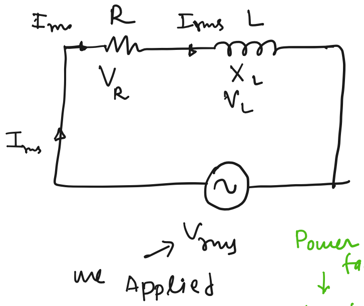
emf = $BA\omega \sin \omega t$

Different ac Circuit

(1) ***R*, *L* and *C* circuits**

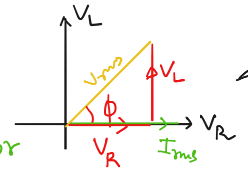
Circuit characteristics	Purely resistive circuit (*R*-circuit)	Purely inductive circuit (*L*-circuit)	Purely capacitive circuit (*C*-circuit)
(i) Circuit	 $V = V_0 \sin \omega t$	 $V = V_0 \sin \omega t$	 $V = V_0 \sin \omega t$
(ii) Current	$i = i_0 \sin \omega t$	$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$	$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$
(iii) Peak current	$i_0 = \frac{V_0}{R}$	$i_0 = \frac{V_0}{X_L} = \frac{V_0}{\omega_L} = \frac{V_0}{2\pi\nu L}$	$i_0 = \frac{V_0}{X_C} = V_0 \omega C = V_0 (2\pi\nu C)$
(iv) Phase difference	$\phi = 0°$	$\phi = 90°$ (or $+\frac{\pi}{2}$)	$\phi = 90°$ (or $-\frac{\pi}{2}$)
(v) Power factor	$\cos \phi = 1$	$\cos \phi = 0$	$\cos \phi = 0$
(vi) Power	$P = V_{rms} i_{rms} = \frac{V_0 i_0}{2}$	$P = I_{rms} V_{rms} \cos 90° = 0$ $P = 0$	$P = 0$
(vii) Time difference	TD = 0	TD = $\frac{T}{4}$	TD = $\frac{T}{4}$
(viii) Leading quantity	Both are in same phase	Voltage	Current
(ix) Phasor diagram			

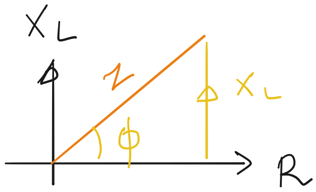
RL ckt



$I_{rms} Z = \sqrt{(I_{rms}R)^2 + (I_{rms}X_L)^2}$, $\quad Z = \sqrt{R^2 + X_L^2}$

$V_{rms} = \sqrt{V_R^2 + V_L^2}$





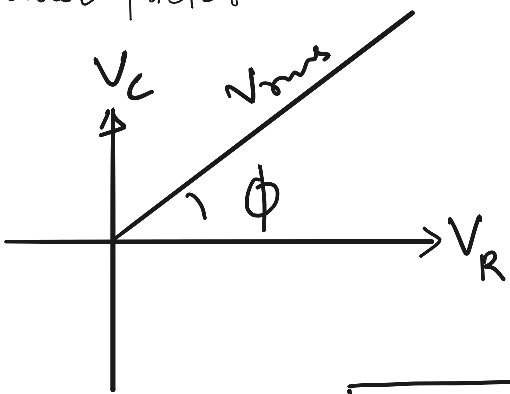
Power factor

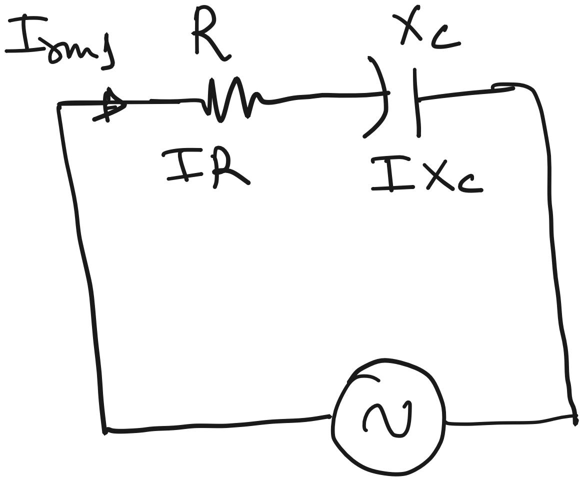
$\cos\phi = \frac{V_R}{V_{rms}} = \frac{\cancel{I_{rms}}R}{\cancel{I_{rms}}Z}$

$\cos\phi = \frac{R}{Z}$

power factor.

$$\tan\phi = \frac{V_L}{V_R} = \frac{X_L}{R}$$





$$V_{rms} = \sqrt{V_C^2 + V_R^2}$$

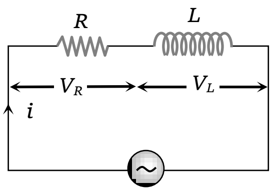
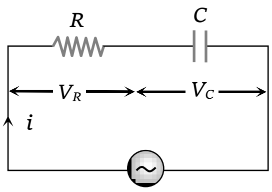
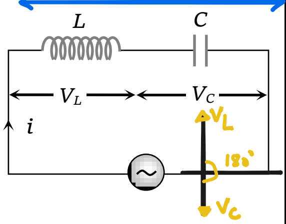
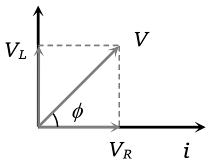
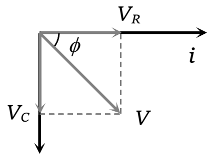
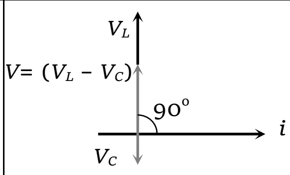
$$I_m Z = \sqrt{(I_m X_C)^2 + (I_m R)^2}$$

$$Z = \sqrt{X_C^2 + R^2}$$

$$\cos\phi = \frac{R}{Z}$$

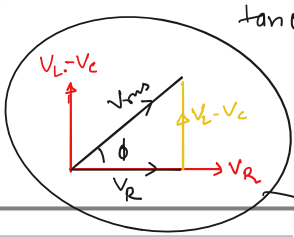
(2) ***RL*, *RC* and *LC* circuits**

$Z = X_L - X_C$ if $X_L = X_C$ $Z = 0$

Circuit characterstics	*RL*-circuit	*RC*-circuit	*LC*-circuit
(i) Circuit	 $V_R = iR$, $V_L = iX_L$ $V = V_0 \sin\omega t$	 $V_R = iR$, $V_C = iX_C$ $V = V_0 \sin\omega t$	$V_{rms} = V_L - V_C$  $V_L = iX_L$, $V_C = iX_C$ $V = V_0 \sin\omega t$
(ii) Current	$i = i_0 \sin(\omega t - \phi)$	$i = i_0 \sin(\omega t + \phi)$	$i = i_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$
(iii) Peak current	$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + X_L^2}}$ $= \frac{V_0}{\sqrt{R^2 + 4\pi^2\nu^2L^2}}$	$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + X_C^2}}$ $= \frac{V_0}{\sqrt{R^2 + \frac{1}{4\pi^2\nu^2C^2}}}$	$i_0 = \frac{V_0}{Z} = \frac{V_0}{X_L - X_C}$ $= \frac{V_0}{\omega L - \frac{1}{\omega C}}$
(iv) Phasor diagram			
(v) Applied voltage	$V = \sqrt{V_R^2 + V_L^2}$	$V = \sqrt{V_R^2 + V_C^2}$	$V = V_L - V_C$
(vi) Impedance	$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2L^2}$ $= \sqrt{R^2 + 4\pi^2\nu^2L^2}$	$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$	$Z = X_L - X_C = X$
(vii) Phase difference	$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{\omega L}{R}$	$\phi = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\frac{1}{\omega CR}$	$\phi = 90^\circ$
(viii) Power factor	$\cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}}$	$\cos\phi = \frac{R}{\sqrt{R^2 + X_C^2}}$	$\cos\phi = 0$

(ix) Leading quantity	Voltage	Current	Either voltage or current

Note : ❑ In LC circuit if $X_L = X_C \Rightarrow V_L = V_C$ then resonance occurs and resonant frequency (natural frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec or $\nu_0 = \frac{1}{2\pi\sqrt{LC}}$ Hz.

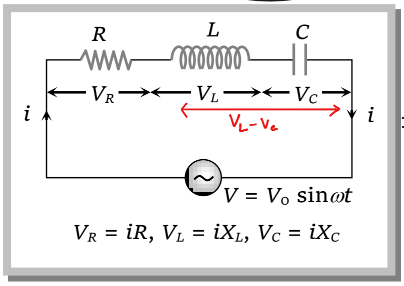


$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I_{rm} X_L - I_{rm} X_C}{I_{rm} R}$

$\tan\phi = \frac{X_L - X_C}{R}$

$\cos\phi = \frac{V_R}{\sqrt{V_R^2 + (V_L - V_C)^2}} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

Series *RLC* Circuit



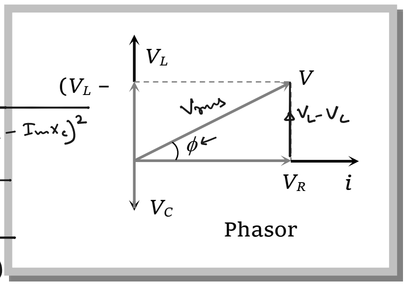
$V_{rms} = \sqrt{V_R^2 + (V_L - V_C)^2}$

$I_{rms} Z = \sqrt{(I_{rms} R)^2 + (X_L I_{rm} - I_{rm} X_C)^2}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})}$

$\omega = 2\pi f.$



(1) **Equation of current :** $i = i_0 \sin(\omega t \pm \phi)$; where $i_0 = \frac{V_0}{Z}$

(2) **Equation of voltage :** From phasor diagram $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

(3) **Impedance of the circuit :** $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

(4) **Phase difference :** From phasor diagram

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{2\pi\nu L - \frac{1}{2\pi\nu C}}{R}$$

(5) **If net reactance is inductive :** Circuit behaves as *LR* circuit

(6) **If net reactance is capacitive :** Circuit behave as *CR* circuit

(7) **If net reactance is zero :** Means $X = X_L - X_C = 0 \Rightarrow X_L = X_C$. This is the condition of resonance

$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

$X_L = X_C$ $\quad Z_{min} = R$ ← I_{rms} (maximum)

(8) **At resonance** (series resonant circuit)

(i) $X_L = X_C \Rightarrow Z_{min} = R$ *i.e.* circuit behaves as resistive circuit

(ii) $V_L = V_C \Rightarrow V = V_R$ *i.e.* whole applied voltage appeared across the resistance

(iii) Phase difference : $\phi = 0^\circ \Rightarrow$ p.f. $= \cos\phi = 1$

$\omega L = \frac{1}{\omega C}$ resonance condition

$2\pi f L = \frac{1}{2\pi f C}$

$4\pi^2 f^2 = \frac{1}{LC}$ $\quad f = \frac{1}{2\pi\sqrt{LC}}$

$\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$

at this Freq. Irms is maxi.
Z is min.

$\cos\phi$

at resonance Power $= I_m V_m \cos\phi$

(iv) Power consumption $P = V_{rms}\, i_{rms} = \frac{1}{2} V_0 i_0$

(v) Current in the circuit is maximum and it is $i_0 = \frac{V_0}{R}$

(vi) These circuit are used for voltage amplification and as selector circuits in wireless telegraphy.

(9) **Resonant frequency** (Natural frequency)

At resonance $X_L = X_C$

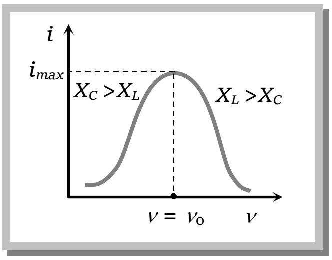
$$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}} \frac{rad}{sec} \quad \Rightarrow \nu_0 = \frac{1}{2\pi\sqrt{LC}} Hz \,(\text{or } cps)$$

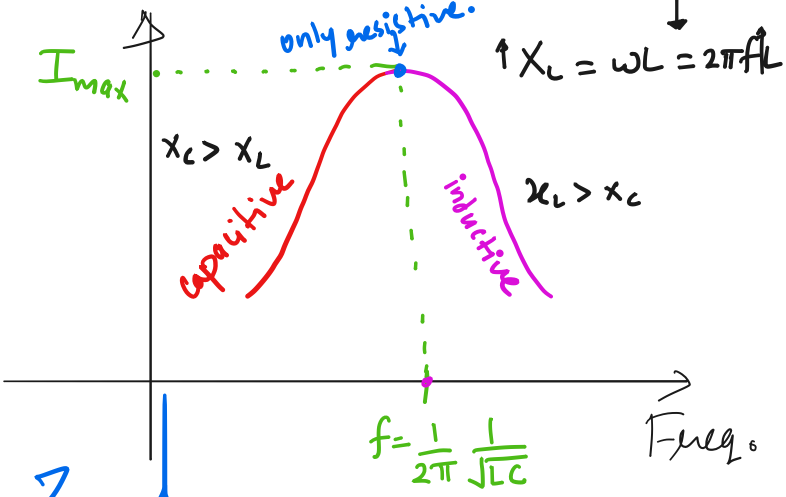
(Resonant frequency doesn't depend upon the resistance of the circuit)

$\uparrow X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

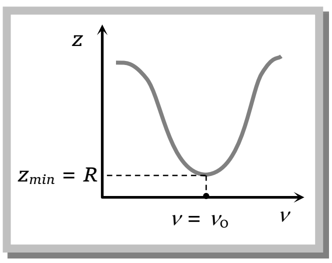
Different graphs

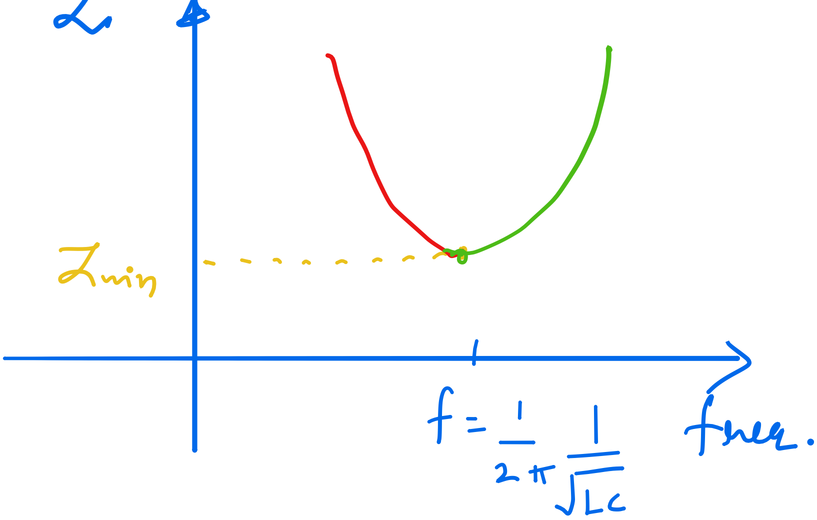
(i) i - ν graph





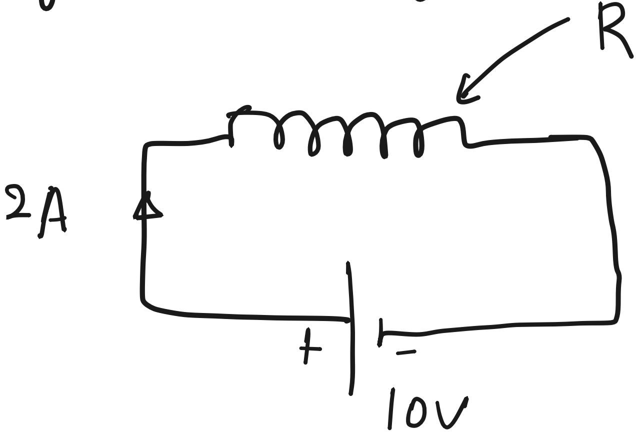
(ii) z - ν graph

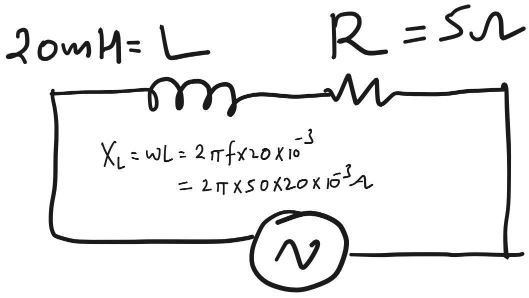




Note → Important facts

1. Inductor behaves like wire when we Apply DC supply.





$V = V_0 \sin \omega t$

$f = 50Hz.$

$$R = \frac{V}{I} = \frac{10}{2} = 5\Omega$$

$$Z = \sqrt{X_L^2 + R^2}$$

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{X_L^2 + R^2}} = \frac{5}{\sqrt{(2\pi)^2 + (5)^2}}$$

It shows sharpness of I vs Freq graph.

it is a defined term.

Quality factor (Q - factor) of series resonant circuit

The characteristic of a series resonant circuit is determined by the quality factor (Q - factor) of the circuit.

It defines sharpness of i - ν curve at resonance when Q - factor is large, the sharpness of resonance curve is more and vice-versa.

Q - factor also defined as follows

$$Q\text{ - factor} = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipation}} = \frac{2\pi}{T} \times \frac{\text{Maximum energy stored}}{\text{Mean power dissipated}}$$

$$= \frac{\text{Resonant frequency}}{\text{Band width}} = \frac{\omega_0}{\Delta\omega}$$

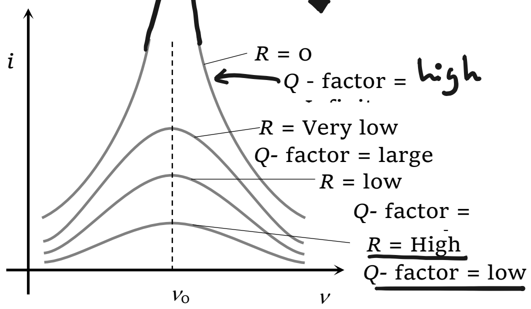
$$Q\text{ - factor} = \frac{V_L}{V_R} \text{ or } \frac{V_C}{V_R} = \frac{\omega_0 L}{R} \text{ or } \frac{1}{\omega_0 CR} \Rightarrow Q\text{ - factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

at resonance.

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega = \frac{1}{\sqrt{LC}}$$



Resonance

$$Q\text{ factor} = \frac{V_L}{V_R} = \frac{V_C}{V_R}$$

$$\frac{\cancel{I_m} X_C}{\cancel{I_m} R} = \frac{1}{\omega C R} = \frac{1}{\frac{1}{\sqrt{LC}} \times C R}$$

$$Q = \sqrt{\frac{L}{C}} \times \frac{1}{R}$$

$$= \frac{I_m X_L}{R}$$

$$= \frac{\cancel{I_m} \omega L}{\cancel{I_m} R} = \frac{L}{R} \times \frac{1}{\sqrt{LC}}$$

$$Q\text{ factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$