Integration

* DERIVATIVE OF ANTIDERIVATIVE FUNCTION : (Leibnitz Rule)

If h(x) & g(x) are differentiable functions of x then ,

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f[h(x)]\,.\,h'(x) - f[g(x)]\,.\,g'(x)$$

$$\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(t)\,dt\right) = f(h(x))h'(x) - f(g(x))g'(x)$$

Q) Let $G(x) = \int_{2}^{x^2} \frac{dt}{1+\sqrt{t}}$ $(x > 0)$. Find $G'(9)$.

$G(x) = \int_{2}^{x^2} \frac{dt}{1+\sqrt{t}}$ $G'(9)$

$G'(x) = \left(\frac{1}{1+x}\right)\cdot 2x - 0$

$G'(x) = \left(\frac{2x}{1+x}\right)$

$G'(9) = \frac{18}{10} = \frac{9}{5}$

Evaluate : $\lim_{x\to 0} \frac{\int_0^x (1-\cos 2x)\,dx}{x\int_0^x \tan x\,dx}$.

$\left(\frac{0}{0}\right)$

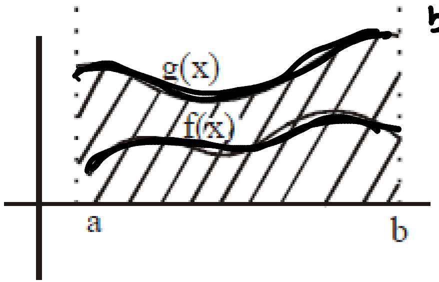
$\int_a^a$

PROPERTIES OF DEFINITE INTEGRAL :

P–10 If $f(x) \le g(x)$ for $a \le x \le b$ then $\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$

P–11 $\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$.

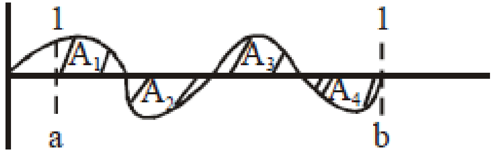
Geometrical Proof:



$g(x) > f(x)$

$\int_a^b g(x)dx > \int_a^b f(x)dx$

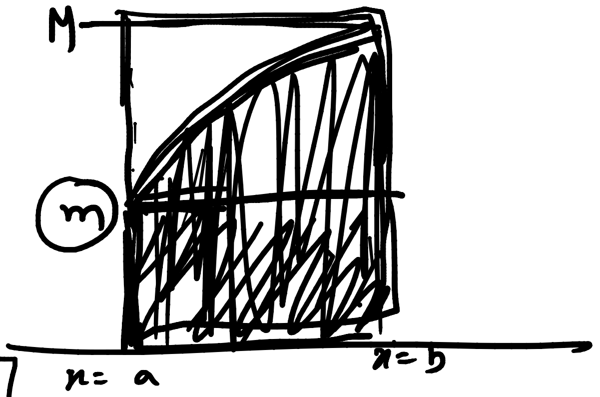
Let the graph of $y = (x)$ is as shown



then LHS = $|A_1 - A_2 + A_3 - A_4|$ & RHS = $A_1 + A_2 + A_3 + A_4$.

Property (Estimation of an Integral) If m is the least value and M is the greatest value of function f(x) in the interval [a, b] then,

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$



Prove that $\int_0^{2\pi} \frac{dx}{10+3\cos x}$ lies between $\frac{2\pi}{13}$ and $\frac{2\pi}{7}$.

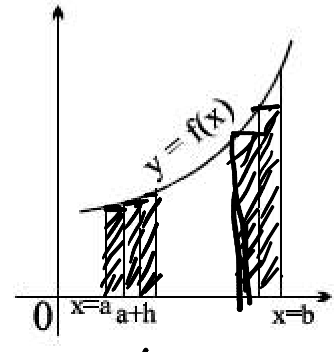
$$\boxed{2\pi\left(\frac{1}{13}\right) < \int_0^{2\pi} \frac{dx}{10+3\cos x} < 2\pi\left(\frac{1}{7}\right)}$$

DEFINITE INTEGRAL AS A LIMIT OF SUM

Fundamental theorem of integral calculus :

$$\int_a^b f(x)\,dx = \lim_{\substack{h \to 0 \\ n \to \infty}} h\left[f(a) + f(a+h) + f(a+2h) + \ldots\ldots + f(a + \overline{n-1}\,h) \right]$$

or $$\boxed{\int_a^b f(x)\,dx = \lim_{\substack{h \to 0 \\ n \to \infty}} h \times \sum_{r=0}^{n-1} f(a+rh)}$$ where $b - a = nh$



$h \to 0$

Definite Integral as the Limit of a Sum

(i) Express the given series in the form of $\boxed{\sum \frac{1}{n} f\left(\frac{r}{n}\right)}$

(ii) The limit when $n \to \infty$ is its sum

Replace r/n by x, 1/n by dx and $\boxed{\lim_{n\to\infty} \sum}$ by the sign of integration $\int$

(iii) The lower and upper limits of integration will be the value of r/n for the first and last term (or the limits of these values respectively).

$$\boxed{\sum_{r=k}^{m} \frac{1}{n} f\left(\frac{r}{n}\right)} \to \int_{k/n}^{m/n} f(x)\,dx$$

$\boxed{n \to \infty}$

Problems

Find the value of $\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\ldots\ldots+\frac{1}{4n}\right) = \ln 4$

$\frac{1}{n} f\left(\frac{r}{n}\right)$

$n+3n$

$$\lim_{n\to\infty}\sum_{r=0}^{3n}\frac{1}{n+r} = \int_0^3 \frac{1}{1+x}\,dx$$

$$\sum_{r=0}^{3n}\frac{1}{n\left(1+\frac{r}{n}\right)} = \ln(1+x)\Big|_0^3$$

$$\lim_{n\to\infty}\sum_{r=0}^{3n}\frac{1}{n}\times\left(\frac{1}{1+\frac{r}{n}}\right) = \ln 4$$

$\frac{3n}{n}$

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WALLI'S THEORM :

$$\int_0^{\pi/2} \sin^n x \cos^m x \, dx = \frac{[(n-1)(n-3)\ldots 1 \text{ or } 2]\,[(m-1)(m-3)\ldots 1 \text{ or } 2]}{(m+n)(m+n-2)\ldots 1 \text{ or } 2} K$$

(m, n are non-negative integer)

$$\text{where } K = \begin{cases} \left(\dfrac{\pi}{2}\right) & \text{if } m, n \text{ both are even} \\ (1) & \text{otherwise} \end{cases}$$

$$\int_0^{\pi/2} \sin^4 x \cos^6 x = \frac{K[(4-1)(4-3)]\,[(6-1)(6-3)(6-5)] \times 1}{(10)(8)(6)(4)(2)}$$

$$= \frac{(3)(\cancel{5})(\cancel{3})}{\cancel{10}_{2} \times 8 \times \cancel{6}_{2} \times 4 \times 2} \times \frac{\pi}{2}$$

$$= \left(\frac{3\pi}{512}\right)$$

Problems

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Evaluate : $\int_{0}^{\pi/2} \cos^7 x \, dx$

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Problems

Evaluate : $\int_0^{2\pi} x \sin^6 x \cos^4 x dx$