Definite Integration

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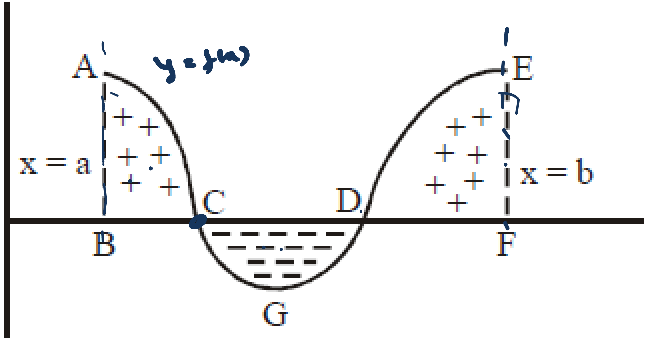
Definite Integral

A definite integral is denoted by $\int_a^b f(x)dx$ which represent the algebraic area bounded by the curve y = f(x), the ordinates x = a, x = b and the x axis.

$\int_a^b f(x)\,dx =$

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a) \text{ where } \int f(x)\,dx = F(x) + c}$$

$$\boxed{\int_a^b f(x)\,dx = \textit{Area}\ ABC - \textit{Area}\ CGD + \textit{Area}\ DEF}$$



VERY IMPORTANT NOTE : If $\int_a^b f(x)\,dx = 0 \Rightarrow$ then the equation $f(x) = 0$ has atleast one root lying in (a , b) provided f is a continuous function in (a , b) .

Evaluating definite integrals by finding antiderivatives :

Evaluate : $\int_0^2 \frac{dx}{4+x^2}$ $= \left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^2$

$$\int_0^2 \frac{dx}{4+x^2} = \frac{1}{2}\left[\tan^{-1}\frac{x}{2}\right]_0^2 = \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - 0\right)$$

$$= \frac{1}{2}\left[\tan^{-1}1 - 0\right] = \frac{\pi}{8}.\ \textbf{Ans.} = \left(\frac{\pi}{8}\right)$$

PROPERTIES OF DEFINITE INTEGRAL :

P–1 $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ provided f is same,

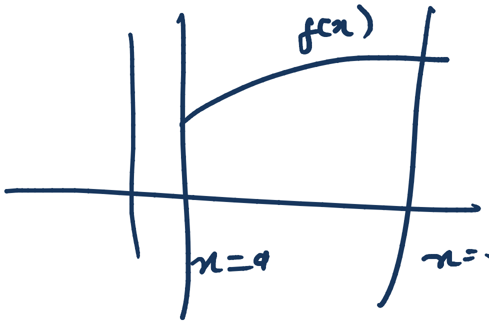
$$\int_1^2 x^2 dx = -\int_2^1 x^2 dx$$

P – 2 $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

P–3 $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where c may lie inside or outside the interval [a, b].

$$\int_1^3 f(x)dx = \int_1^4 f(x)dx + \int_4^3 f(x)dx$$

$$\int_1^4 f(x)dx - \int_3^4 f(x)dx$$



Problems

Evaluate : $\int_0^3 |5x-9|\,dx$.

$$\int_0^3 |5x-9|\,dx = \int_0^{9/5}(9-5x)\,dx + \int_{9/5}^3 (5x-9)\,dx$$

$$= \left[9x-\frac{5}{2}x^2\right]_0^{9/5} + \left[\frac{5x^2}{2}-9x\right]_{9/5}^3 =$$

$$= \frac{81}{5} - \frac{81}{10}$$

$$= \cancel{\frac{81}{10}} + \left(\frac{45}{2} - 27\right) - \left(\cancel{\frac{81}{10}} - \frac{81}{5}\right)$$

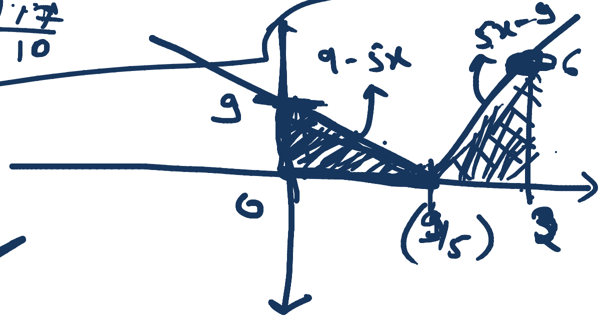
$$=$$

$$\frac{1}{2}\times\frac{9}{5}\times 9 + \frac{1}{2}\times\frac{6}{5}\times 6$$

$$\frac{81+36}{10}$$

$$\frac{117}{10}$$

$$\frac{117}{10}$$



$$\int_0^3 |5x-9|\,dx = \int_0^{9/5}(9-5x)\,dx + \int_{9/5}^3 (5x-9)\,dx$$

$$=$$

PROPERTIES OF DEFINITE INTEGRAL :

P – 4 : $\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} (f(x)+f(-x))\,dx = \begin{cases} 0 & \text{if } f(x) \text{ is odd} \\ 2\int_{0}^{a} f(x)\,dx & \text{if } f(x) \text{ is even} \end{cases}$

$f(-x) = -f(x)$

$f(-x) = f(x)$

Q) Evaluate : $\int_{-1}^{1} (x^3 + x)\,dx$ $= \int_0^1 \cancel{f(x)} + \cancel{f(-x)} = \underline{0}$

Q) The value of the integral $\int_{-4}^{4} (ax^3 + bx + c)\,dx$ depend on

$2\int_0^4 c\,dx = (8c)$

(A) b and c (B) a, b and c (C) only c (D) a and c

PROPERTIES OF DEFINITE INTEGRAL :

Kings Rule

P–5 $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$, In particular $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Evaluate : $\int_{50}^{100} \frac{\ln x}{\ln x + \ln(150-x)}\,dx = I$ — (1)

$\int_{50}^{100} \frac{\ln(150-x)}{\ln(150-x)+\ln x} = I$ — (2)

$\int_{50}^{100} \frac{\ln x + \ln(150-x)}{\ln(150-x)+\ln x} = 2I$

$50 = 2I$

$\boxed{I = 25}$

Evaluate : $\int_{-\pi/4}^{\pi/4} \frac{\tan^2 x}{1+e^x}\,dx = I$

$(a+b-x)$

$\int_{-\pi/4}^{\pi/4} \frac{\tan^2 x}{1+(e^{-x})} = I$

$\int_{-\pi/4}^{\pi/4} \frac{e^x \tan^2 x}{1+e^x} = I$

$\int_{-\pi/4}^{\pi/4} \tan^2 x = 2I$

PROPERTIES OF DEFINITE INTEGRAL :

P–6 $\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx = 2\int_0^{a} f(x)\,dx$ if $\boxed{f(2a-x) = f(x)}$

$= 0$ if $\boxed{f(2a-x) = -f(x)}$

$$\int_0^{2a} f(x)\,dx = \boxed{\int_0^{a}[f(x)+f(2a-x)]\,dx}$$

$I = \int_0^{2\pi} \sin^4 x\,dx = k\int_0^{\pi/2} \cos^4 x\,dx$ find value of $\boxed{k? = 4}$

$$\int_0^{2\pi}\sin^4 x\,dx = \int_0^{\pi}(\sin^4 x + \sin^4 x)\,dx$$

$$= 2\int_0^{\pi}\sin^4 x\,dx$$

$$= 2\int_0^{\pi/2}(\sin^4 x + \sin^4 x)\,dx$$

$$= \boxed{4\int_0^{\pi/2}\sin^4 x\,dx}$$

$$4\int_0^{\pi/2}\cos^4 x\,dx$$

PROPERTIES OF DEFINITE INTEGRAL :

Periodic functions

* **P–7** $\int_0^{na} f(x)\,dx = n \times \int_0^{a} f(x)\,dx$; where 'a' is the period of the function i.e. $\boxed{f(a+x) = f(x)}$ a = period

P–8 $\int_{a+nT}^{b+nT} f(x)\,dx = \int_a^b f(x)\,dx$ where f(x) is periodic with period T & $n \in I$.

P–9 $\int_{ma}^{na} f(x)\,dx = (n-m)\int_0^a f(x)\,dx$ if f(x) is periodic with period 'a'.

$\int_{1\cdot(2\pi)}^{3\cdot(2\pi)} \; = (3-1)\int_0^{2\pi} \sin x\,dx$

$\int_0^{6\pi} \sin x\,dx = 3\boxed{\int_0^{2\pi} \sin x\,dx}$

$= 2\int_0^{2\pi} \sin$

$\int_{a+4\pi}^{b+4\pi} f(x)\,dx = \int_a^b f(x)\,dx$

Show that $\int_0^{1000} e^{x-[x]}\,dx = 1000(e-1)$

$\{x\} \to 1$

$\int_0^{1000\times 1} e^{\{x\}}\,dx = 1000\int_0^1 e^x\,dx$

$1000[e-1]$

Properties Of Definite Integral :

P–7 $\int_0^{na} f(x)\,dx = n\int_0^{a} f(x)\,dx$; where 'a' is the period of the function i.e. $f(a+x) = f(x)$

P–8 $\int_{a+nT}^{b+nT} f(x)\,dx = \int_a^b f(x)\,dx$ where $f(x)$ is periodic with period T & $n \in I$.

P–9 $\int_{ma}^{na} f(x)\,dx = (n-m)\int_0^{a} f(x)\,dx$ if $f(x)$ is periodic with period 'a'.

If $g(x) = \int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ is equal to -

(A) $g(x) + g(\pi)$ (B) $g(x) - g(\pi)$ (C) $g(x)\,g(\pi)$ (D) $g(x)/g(\pi)$