Problem Solving on Definite Integration



JEE - 2020

Ankush Garg (B.Tech, IIT Jodhpur)

The value of
$$\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx \text{ is } = \bigcirc$$
(2019 Main, 9 April I)

(a)
$$\frac{\pi - 1}{2}$$

(b)
$$\frac{\pi - 3}{8}$$

(e)
$$\frac{\pi - 4}{4}$$

(a)
$$\frac{\pi - 1}{2}$$
 (b) $\frac{\pi - 2}{8}$ (c) $\frac{\pi - 1}{4}$ (d) $\frac{\pi - 2}{4}$

$$\frac{7}{2} \int \frac{\cos^{3}x}{\cos^{2}x + \sin^{3}x} = \frac{1}{2} \qquad \frac{\pi}{2} + \frac{\cos^{2}x}{4} \Big|_{0}^{\pi/2}$$

$$\frac{\sin^{3}x + \cos^{3}x}{\sin^{3}x + \cos^{3}x} = 2T \qquad \frac{\pi}{2} + \left(-\frac{1}{4} - \frac{1}{4}\right) = 2T$$

$$\frac{\sin^{3}x + \cos^{3}x}{\sin^{3}x + \cos^{3}x} = 2T \qquad \frac{\pi}{2} - \frac{1}{4} = 2T$$

$$\frac{\sin^{3}x + \cos^{3}x}{\sin^{3}x + \cos^{3}x} = 2T$$

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$$\frac{\pi}{2} - \frac{1}{4} = 2T$$

$$\frac{7}{2} + \left(-\frac{1}{4} - \frac{1}{4}\right) = 27$$

$$\Rightarrow \boxed{1 = \frac{\pi - 1}{4}}$$

If
$$\int_0^{\pi/2} \frac{\cot x}{\cot x + \csc x} dx = \underline{m(\pi + n)}$$
, then $m \cdot n$ is equal to (2019 Main, 12 April I)

- (a) $-\frac{1}{2}$ (b) 1 (c) $\frac{1}{2}$

$$\frac{\pi_{1}}{\log x + 1} = m\pi + m\pi$$

$$\frac{\pi_{2}}{\log x + 1} = m\pi + m\pi$$

$$\frac{\pi_{2}}{2} = m\pi$$

If
$$f(x) = \frac{2 - x \cos x}{2 + x \cos x}$$
 and $g(x) = \log_e x$, $(x > 0)$ then the

value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x))dx$ is

(2019 Main, 8 April I)

(a)
$$\log_e 3$$

(b)
$$\log_e e$$

(c)
$$\log_e 2$$

(d)
$$\log_e 1$$

$$\frac{\pi}{2} = \frac{\pi}{2} \left[\frac{g(f(\pi)) + g(f(-\pi))}{2 + \kappa \cos \kappa} \right]$$

$$\frac{g(f(\pi)) = \frac{\log \left(\frac{2 + \kappa \cos \kappa}{2 + \kappa \cos \kappa} \right)}{2 + \kappa \cos \kappa}$$

$$= -\frac{\log \left(\frac{2 - \kappa \cos \kappa}{2 + \kappa \cos \kappa} \right)}{2 + \kappa \cos \kappa}$$

The value of the integral
$$\int_{-2}^{2} \frac{\sin^{2} x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$$

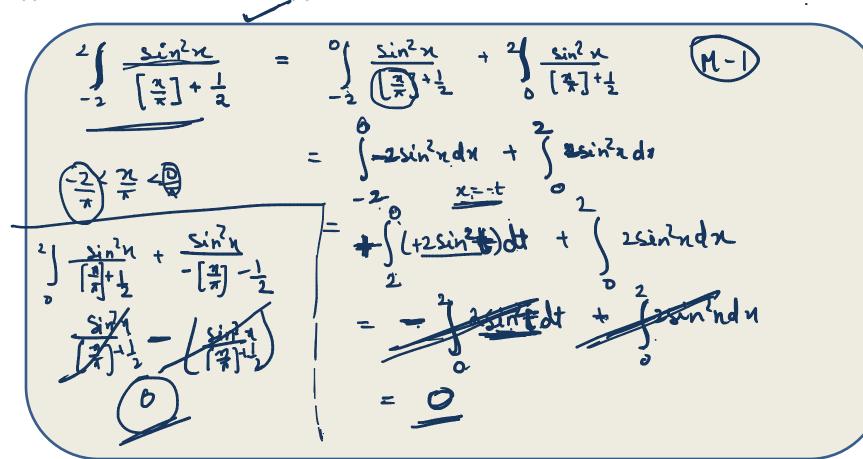
 $= \sqrt{\frac{\sin^2 x}{\left[\frac{\pi}{2}\right]^{\frac{1}{2}}} + \frac{\sin^2 x}{\left[\frac{\pi}{2}\right]^{\frac{1}{2}}} + \frac{\sin^2 x}{\left[\frac{\pi}{2}\right]^{\frac{1}{2}}}$

(where, [x] denotes the greatest integer less than or equal to x) is (2019 Main, 11 Jan I

(a) $4 - \sin 4$

(b) 4

(c) sin 4

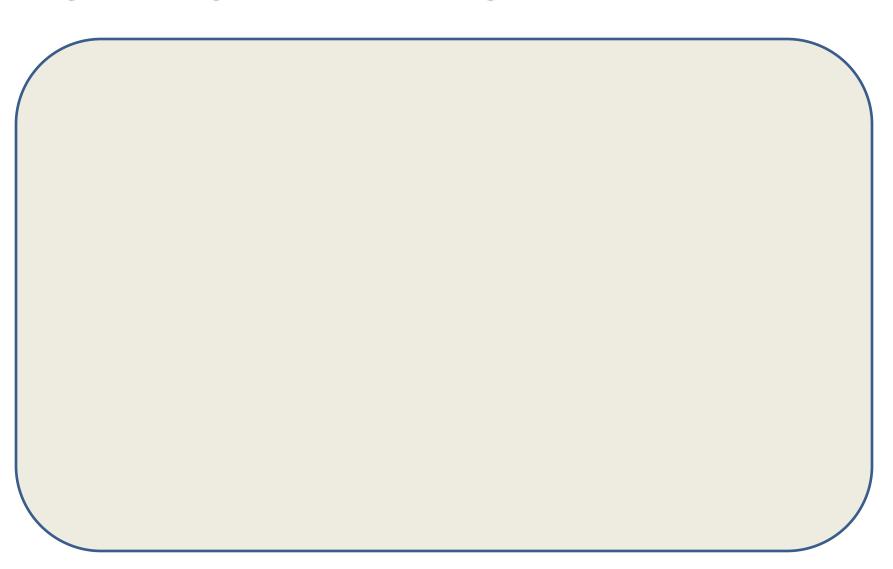


The value of $\int_0^{\pi} |\cos x|^3 dx$ is

(a) $\frac{2}{3}$ (b) $-\frac{4}{3}$ (c) 0

(2019 Main, 9 Jan I)

(d) $\frac{4}{3}$



The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} \, dx$ is

(2018 Main)

(a)
$$\frac{\pi}{8}$$
 (b) $\frac{\pi}{2}$ (c) 4π

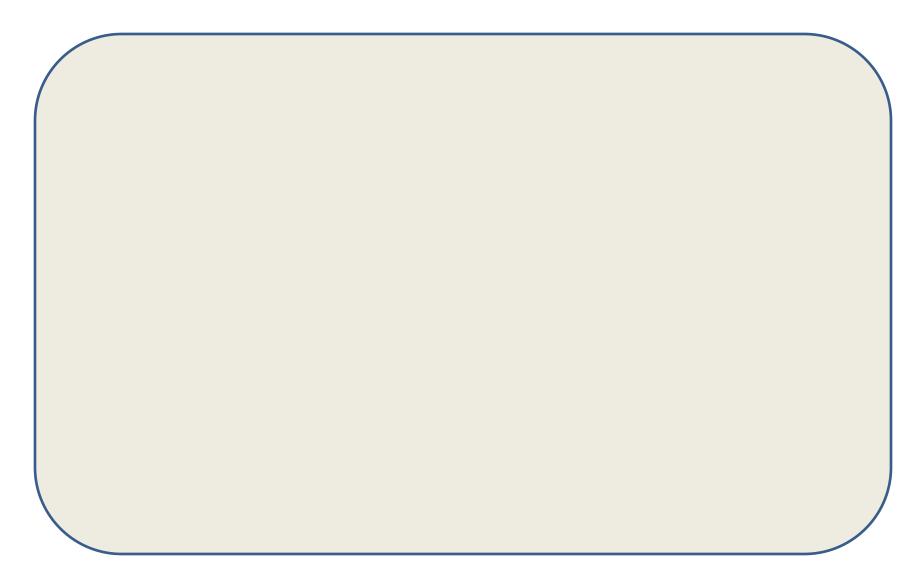
(b)
$$\frac{\pi}{2}$$

(c)
$$4\pi$$

(d)
$$\frac{\pi}{4}$$

The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} \, dx$ is equal to (2015, Main) (a) 2 (b) 4 (c) 1 (d) 6





If for a real number y, [y] is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2} [2\sin x] \, dx$ is (1999, 2M)

 $(a) - \pi$

(b) 0

(c) $-\frac{\pi}{2}$

(d) $\frac{\pi}{2}$

The value of
$$\int_{-\pi/2}^{\pi/2} \frac{dx}{[x] + [\sin x] + 4}$$
, where [t] denotes

the greatest integer less than or equal to t, is

(2019 Main, 10 Jan II)

(a)
$$\frac{1}{12} (7\pi - 5)$$

(c) $\frac{3}{10} (4\pi - 3)$

(b)
$$\frac{1}{12} (7\pi + 5)$$

(c)
$$\frac{3}{10}$$
 (4 π – 3)

(b)
$$\frac{1}{12} (7\pi + 5)$$

(d) $\frac{3}{20} (4\pi - 3)$

If
$$\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt$$
, then $f'\left(\frac{1}{2}\right)$ is

(a) $\frac{24}{25}$ (b) $\frac{18}{25}$ (c) $\frac{6}{25}$

(2019 Main, 10 Jan II)

$$\lim_{n \to \infty} \left(\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \frac{n}{n^2 + 3^2} + \dots + \frac{1}{5n} \right)$$
is equal to

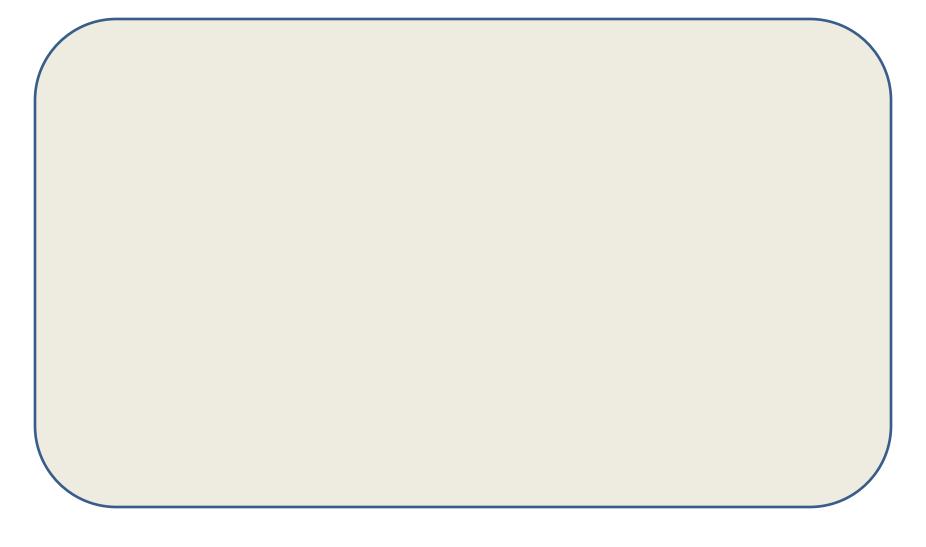
(2019 Main, 12 Jan II)

(a)
$$tan^{-1}(3)$$

(b)
$$tan^{-1}(2)$$

(c)
$$\pi/4$$

(d)
$$\pi/2$$



$$\lim_{n\to\infty} \left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + \dots + \frac{(2n)^{1/3}}{n^{4/3}} \right) \text{is equal to}$$

(2019 Main, 10 April I)

(a)
$$\frac{4}{3}(2)^{4/3}$$

(b)
$$\frac{3}{4}(2)^{4/3} - \frac{4}{3}$$

(a)
$$\frac{4}{3}(2)^{4/3}$$

(c) $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$

(d)
$$\frac{4}{3}(2)^{3/4}$$

The integral
$$\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x + \cot^5 x)}$$
 equals

(2019 Main, 11 Jan II)

(a)
$$\frac{1}{5} \left(\frac{\pi}{4} - \tan^{-1} \left(\frac{1}{3\sqrt{3}} \right) \right)$$
 (b) $\frac{1}{20} \tan^{-1} \left(\frac{1}{9\sqrt{3}} \right)$

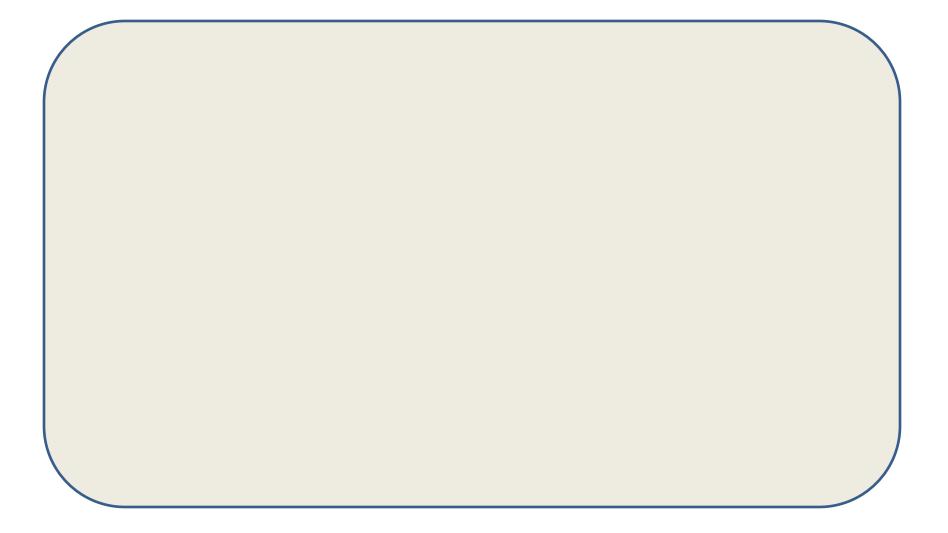
(c)
$$\frac{1}{10} \left(\frac{\pi}{4} - \tan^{-1} \left(\frac{1}{9\sqrt{3}} \right) \right)$$
 (d) $\frac{\pi}{40}$

Let $I = \int_{a}^{b} (x^4 - 2x^2) dx$. If *I* is minimum, then the ordered

pair (a, b) is

(2019 Main, 10 Jan I)

- (a) $(-\sqrt{2}, 0)$ (c) $(\sqrt{2}, -\sqrt{2})$
- (b) $(0, \sqrt{2})$ (d) $(-\sqrt{2}, \sqrt{2})$



Let
$$I = \int_{0}^{1} \frac{\sin x}{\sqrt{x}} dx$$
 and $J = \int_{0}^{1} \frac{\cos x}{\sqrt{x}} dx$. Then which one of

the following is true?

[AIEEE 2008]

(A)
$$I < \frac{2}{3}$$
 and $J < 2$ (B) $I < \frac{2}{3}$ and $J > 2$

(B) I <
$$\frac{2}{3}$$
 and J > 2

(C)
$$I > 2/3$$
 and $J < 2$ (D) $I > 2/3$ and $J > 2$