JEE and NEET CRASH COURSE

PHYSICS

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Problem Solving Class

(Gravitation)

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P-Q942

If the distance between two masses is doubled, the gravitational attraction between them

(A) Is doubled

(B) Becomes four times

(C) Is reduced to half

(D) Is reduced to a quarter

$$F = \frac{G m_1 m_2}{r^2}$$

$$F \propto \frac{1}{r^2}$$

$$F' = \frac{1}{4} F$$

$$r' = 2r$$

$$F' \propto \frac{1}{r'^2} = \frac{1}{4r^2}$$

$$F' = \frac{1}{4} F$$

P-Q942-Solution

Ans [D]

$F \propto \frac{1}{r^2}$. If r becomes double then F reduces to $\frac{F}{4}$

Gravitational force

$$= \frac{GM_1M_2}{R^2}$$

P-Q943

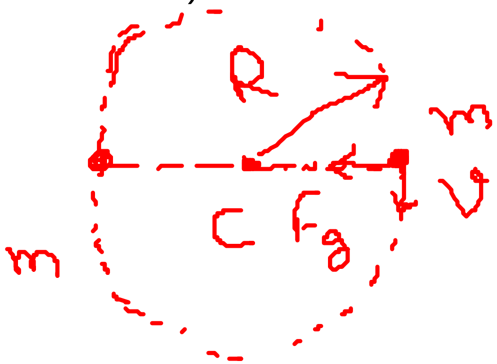
Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(A) $v = \frac{1}{2R}\sqrt{\frac{1}{Gm}}$

(B) $v = \sqrt{\frac{Gm}{2R}}$

(C) $v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$

(D) $v = \sqrt{\frac{4Gm}{R}}$



$$F_g = \frac{mv^2}{R}$$

$$\frac{Gm^{\cancel{2}}}{(2R)^2} = \frac{\cancel{m}v^2}{R}$$

$$\frac{Gm}{4R^{\cancel{2}}} = \frac{v^2}{\cancel{R}}$$

$$\boxed{v = \frac{1}{2}\sqrt{\frac{Gm}{R}}}$$

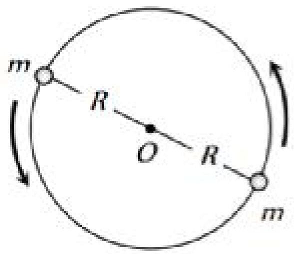
P-Q943-Solution

Ans [C]

Centripetal force is provided by the gravitational force of attraction between two particles

$$i.e.\ \frac{mv^2}{R} = \frac{Gm \times m}{(2R)^2}$$

$$\Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$



P-Q944

The earth (mass $= 6 \times 10^{24}\ kg$)) revolves round the sun with angular velocity $2 \times 10^{-7}\ rad/s$ in a circular orbit of radius $1.5 \times 10^{8}\ km$. The force exerted by the sun on the earth in *newtons*, is

(A) 18×10^{25}

(B) Zero

(C) 27×10^{39}

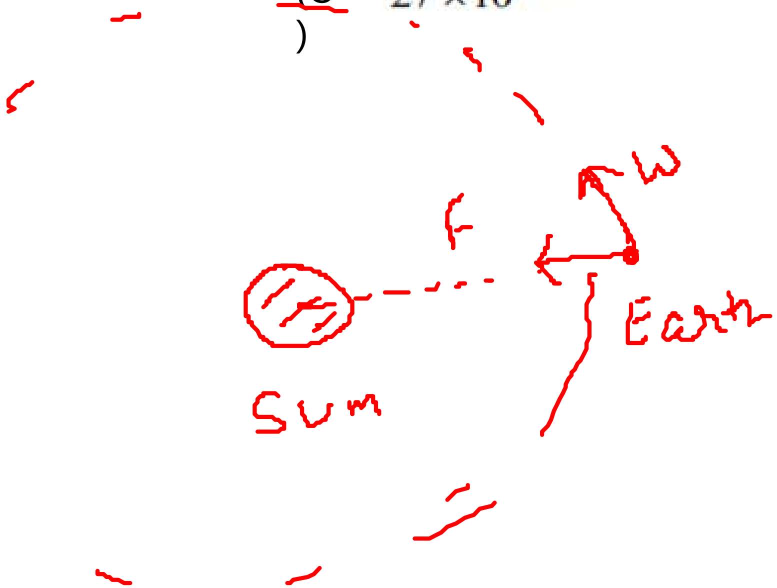
(D) 36×10^{21} ✓

$F = m\omega^2 r$

$= m_e \omega^2 r$

$= 6 \times 10^{24} \times 4 \times 10^{-14} \times \frac{3}{2} \times 10^{8} \times 10^{3}$

$= 36 \times 10^{21}\ N$



P-Q944-Solution

Ans [D]

$$m = 6\times 10^{24}\,kg,\quad \omega = 2\times 10^{-7}\,rad/s,\quad R = 1.5\times 10^{11}\,m$$

The force exerted by the sun on the earth $F = m\omega^2 R$

By substituting the value we can get, $F = 36\times 10^{21}\,N$

Earth revolving around sun so there is centripetal force

P-Q945

The distance of the centres of moon and earth is D. The mass of earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitational force will be zero

(A) $\frac{D}{2}$

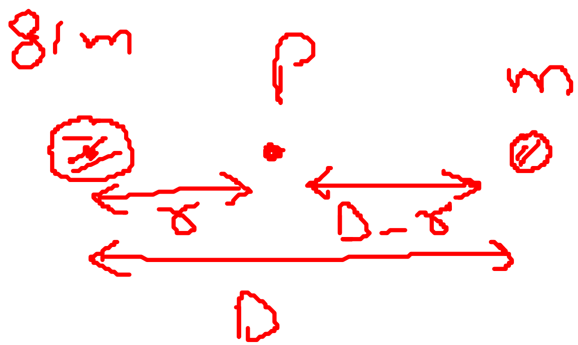
(B) $\frac{2D}{3}$

(C) $\frac{4D}{3}$

(D) $\frac{9D}{10}$

$E_P = 0$

$\frac{\cancel{G} \cdot 81 \cancel{m}}{r^2} = \frac{\cancel{G} \cancel{m}}{(D-r)^2}$



$\frac{9}{r} = \frac{1}{D-r}$

$9D - 9r = r$

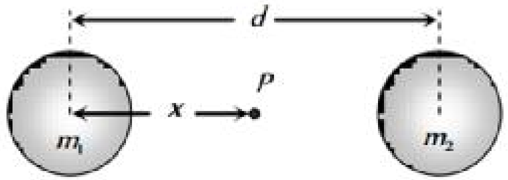
$9D = 10r$

$r = \frac{9D}{10}$

P-Q945-Solution

Ans [D]

Force will be zero at the point of zero intensity



$$x = \frac{\sqrt{m_1}}{\sqrt{m_1} + \sqrt{m_2}} d$$

$$= \frac{\sqrt{81M}}{\sqrt{81M} + \sqrt{M}} D = \frac{9}{10} D.$$

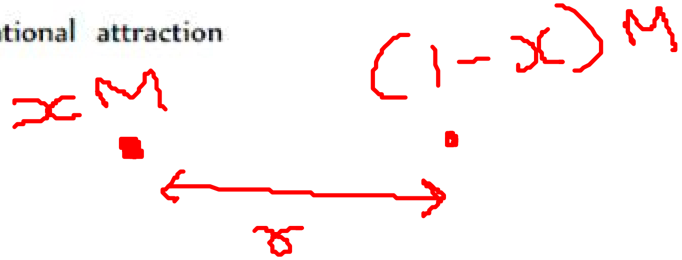
Mass M is divided into two parts xM and $(1-x)M$. For a given separation, the value of x for which the gravitational attraction between the two pieces becomes maximum is

(A) $\frac{1}{2}$

(B) $\frac{3}{5}$

(C) 1

(D) 2



$$F = \frac{G\,xM\,(1-x)M}{r^2}$$

$$= \frac{GM^2}{r^2}\left[x - x^2\right]$$

$$f(x) = x - x^2$$

$$f'(x) = 1 - 2x = 0$$

$$\boxed{x = \frac{1}{2}}$$

P-Q946-Solution

Ans [A]

$$F \propto xm \times (1-x)m = xm^2(1-x)$$

For maximum force $\frac{dF}{dx} = 0$

At this point the slope of the force will be zero

$$\Rightarrow \frac{dF}{dx} = m^2 - 2xm^2 = 0 \Rightarrow x = 1/2$$

P-Q947

If the change in the value of 'g' at a height h above the surface of the earth is the same as at a depth x below it, then (both x and h being much smaller than the radius of the earth)

(A) $x = h$

(B) $x = 2h$

(C) $x = \frac{h}{2}$

(D) $x = h^2$

$g_h = g\left(1 - \frac{2h}{R}\right)$; $h << R$

$g_d = g\left(1 - \frac{d}{R}\right)$

$g_h = g_d \Rightarrow \cancel{g}\left(1 - \frac{2h}{R}\right) = \cancel{g}\left(1 - \frac{x}{R}\right)$

$\boxed{2h = x}$

P-Q947-Solution

Ans [B]

The value of g at the height h from the surface of earth

$$g' = g\left(1 - \frac{2h}{R}\right)$$

The value of g at depth x below the surface of earth

$$g' = g\left(1 - \frac{x}{R}\right)$$

Variation of g with height and depth respectively

These two are given equal, hence $\left(1 - \frac{2h}{R}\right) = \left(1 - \frac{x}{R}\right)$

On solving, we get $x = 2h$

P-Q948

An iron ball and a wooden ball of the same radius are released from a height ‘h’ in vacuum. The time taken by both of them to reach the ground is

(A) Unequal

(B) Exactly equal

(C) Roughly equal

(D) Zero

$$t = \sqrt{\frac{2h}{g}}$$

$$a = \frac{GM}{r^2}$$

P-Q948-Solution

Ans [B]

Time of decent $t = \sqrt{\frac{2h}{g}}$

In vacuum no other force works except gravity so time period will be exactly equal.

P-Q949

A body weighs 700 *gm wt* on the surface of the earth. How much will it weigh on the surface of a planet whose mass is $\frac{1}{7}$ and radius is half that of the earth

(A)	200 *gm wt*	(B)	400 *gm wt*
(C)	50 *gm wt*	(D)	300 *gm wt*

$$W = mg \Rightarrow W \propto g \Rightarrow W \propto \frac{GM}{R^2}$$

$$W \propto \frac{M}{R^2} \qquad W' = \frac{1}{7} \times 4\,W = \frac{4}{7} W$$

$$= \frac{4}{7} \times 700 = 400$$

P-Q949-Solution

Ans [B]

We know that $g = \dfrac{GM}{R^2}$ ← Gravitation on Earth

On the planet $g_p = \dfrac{GM/7}{R^2/4} = \dfrac{4g}{7} = \dfrac{4}{7}g$ ← Gravitation on planet

Hence weight on the planet $= 700 \times \dfrac{4}{7} = 400 \; gm\,wt$

P-Q950

The mass and diameter of a planet have twice the value of the corresponding parameters of earth. Acceleration due to gravity on the surface of the planet is

(A) $9.8\ m/\sec^2$

(B) $4.9\ m/\sec^2$

(C) $980\ m/\sec^2$

(D) $19.6\ m/\sec^2$

$$g_e = \frac{GM}{R^2}$$

$$g_p = \frac{G\,2M}{4R^2} = \frac{1}{2} g_e = 4.9\ m/s^2$$

P-Q950-Solution

Ans [B]

$$\frac{g'}{g} = \frac{M'}{M}\left(\frac{R}{R'}\right)^2 = \left(\frac{2M}{M}\right)\left(\frac{R}{2R}\right)^2 = \frac{1}{2}$$

Gravity varies with mass and radius both

$$\Rightarrow g' = \frac{g}{2} = \frac{9.8}{2} = 4.9\ m/s^2$$

P-Q951

If the earth suddenly shrinks (without changing mass) to half of its present radius, the acceleration due to gravity will be

(A)	$g/2$	(B)	$4g$
(C)	$g/4$	(D)	$2g$

$$g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$$

$$R' = \frac{R}{2} \Rightarrow g' = 4g$$

P-Q951-Solution

Ans [B]

$$g = \frac{GM}{R^2}$$

If radius shrinks to half of its present value then g will becomes four times

$$R' = \frac{R}{2}$$

$$\mathbf{g' = 4g}$$

P-Q952

The depth d at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value at the surface, is [R = radius of the earth]

(A) $\frac{R}{n}$

(B) $R\left(\frac{n-1}{n}\right)$

(C) $\frac{R}{n^2}$

(D) $R\left(\frac{n}{n+1}\right)$

$$g_d = g\left(1-\frac{d}{R}\right) \Rightarrow \frac{\cancel{g}}{n} = \cancel{g}\left(1-\frac{d}{R}\right)$$

$$\frac{1}{n} = 1-\frac{d}{R} \Rightarrow \frac{d}{R} = 1-\frac{1}{n}$$

$$\Rightarrow d = R\left(1-\frac{1}{n}\right)$$

$$= R\left(\frac{n-1}{n}\right)$$

P-Q952-Solution

Ans [B]

$$g' = g\left(1 - \frac{d}{R}\right)$$

Gravitation varies with depth like this

$$\Rightarrow \frac{g}{n} = g\left(1 - \frac{d}{R}\right)$$

Gravitation decreases as we go depth

$$\Rightarrow d = \left(\frac{n-1}{n}\right)R$$

P-Q953

At what height over the earth's pole, the free fall acceleration decreases by one percent (assume the radius of earth to be 6400 *km*)

(A) 32 *km*

(B) 80 *km*

(C) 1.253 *km*

(D) 64 *km*

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

$$\%\ \text{change} = \frac{g_h - g}{g} \times 100 = \frac{\cancel{g}\left(\cancel{1} - \frac{2h}{R} - \cancel{1}\right)}{\cancel{g}} \times 100$$

$$= -\frac{2h}{R} \times 100$$

$$\cancel{-}1 = \cancel{-}\frac{2h}{R} \times 100$$

$$h = \frac{R}{200} = \frac{640\cancel{0}}{2\cancel{0}\cancel{0}}$$

$$= 32\ \text{Km}$$

P-Q953-Solution

Ans [A]

$$g \propto \frac{GM}{r^2} \therefore g \propto \frac{1}{r^2} \text{ or } r \propto \frac{1}{\sqrt{g}}$$

If g decrease by 1 % then R should be increase by ½ %

$$i.e.\ R = \frac{1}{2 \times 100} \times 6400 = 32\ km$$

P-Q954

height

At what altitude in metre will the acceleration due to gravity be 25% of that at the earth's surface (Radius of earth = R *metre*)

(A) $\frac{1}{4}R$

~~(B)~~ R

(C) $\frac{3}{8}R$

(D) $\frac{R}{2}$

$$g_h = \frac{25}{100} \times g$$

$$= g/4 = \frac{1}{4}\frac{GM}{R^2}$$

$$\boxed{g_h = \frac{GM}{(R+h)^2}}$$

$$\frac{1}{4}\frac{\cancel{GM}}{R^2} = \frac{\cancel{GM}}{(R+h)^2}$$

$$(R+h)^2 = 4R^2$$

$$R+h = 2R$$

$$\boxed{h = R}$$

P-Q954-Solution

Ans [B]

$$g' = g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \frac{g}{4} = g\left(\frac{R}{R+h}\right)^2$$

Given that gravity is only 25% of that on earth

$$\Rightarrow \frac{1}{2} = \frac{R}{R+h}$$

$$\Rightarrow R + h = 2R \therefore \ h = R$$

P-Q955

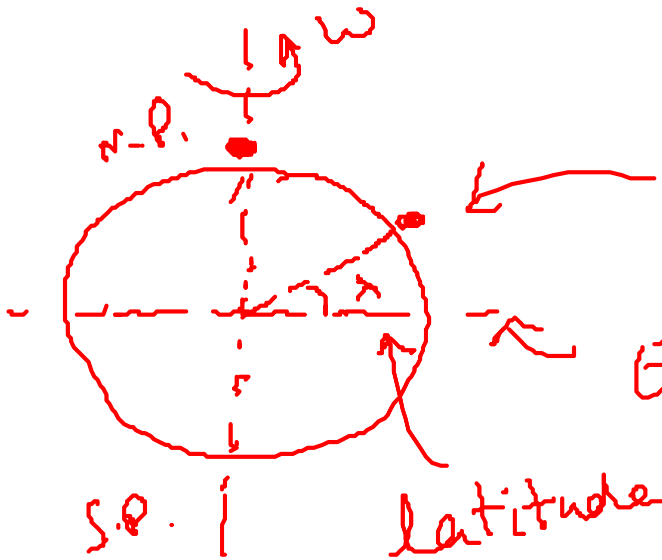
If the angular speed of the earth is doubled, the value of acceleration due to gravity (g) at the north pole

(A) Doubles

(B) (b) Becomes half

(C) Remains same ✓

(D) Becomes zero



Equator

$$\boxed{g_{eff} = g - \omega^2 R \cos^2\lambda}$$

$$g_{equator} = g - \omega^2 R$$

Minimum

$$\boxed{g_{pole} = g}$$

P-Q955-Solution

Ans [C]

Remains same

Acceleration due to gravity at poles is independent of the angular speed of earth

P-Q956

At the surface of a certain planet, acceleration due to gravity is one quarter of that on earth. If a brass ball is transported to this planet, then which one of the following statements is not correct

(A) The mass of the brass ball on this planet is a quarter of its mass as measured on earth

(B) The weight of the brass ball on this planet is a quarter of the weight as measured on earth

(C) The brass ball has the same mass on the other planet as on earth

(D) The brass ball has the same volume on the other planet as on earth

P-Q956-Solution

A

Ans [B]

~~The mass of the brass ball on this planet is a quarter of its mass as measured on earth~~

Mass of the ball always remain constant. It does not depend upon the acceleration due to gravity. Weight depends on acceleration due to gravity

P-Q957

Weight of 1 *kg* becomes 1/6 on moon. If radius of moon is $1.768 \times 10^{6}\ m$, then the mass of moon will be

(A) $1.99 \times 10^{30}\ kg$ (B) $7.56 \times 10^{22}\ kg$

(C) $5.98 \times 10^{24}\ kg$ (D) $7.65 \times 10^{22}\ kg$

$$g_{moon} = \frac{1}{6} g_{earth}$$

$$\frac{G M_{moon}}{R_m^2} = \frac{1}{6} \times \cancel{9.81}\ 1.63$$

$$M_{moon} = \frac{1.63 \times \cancel{3.24}\ 1.62 \times 3 \times 10^{12}}{\cancel{20}\ 10 \times 10^{-11}}$$

$$= 2.6 \times 3 \times 10^{22}$$

1.8

1.1164

3.24

P-Q957-Solution

Ans [D]

$$g_m = \frac{GM_m}{R_m^2} \text{ and } g_m = \frac{g_e}{6} = \frac{9.8}{6}\ m/s^2 = 1.63\ m/s^2$$

Substituting $R_m = 1.768 \times 10^6\ m,\ g_m = 1.63\ m/s^2$

and $G = 6.67 \times 10^{-11}\ N\text{-}m^2/kg^2$ We get

$$M_m = 7.65 \times 10^{22}\ kg$$

G is gravitational constant and will remain same at moon.

P-Q958

Radius of earth is around 6000 *km*. The weight of body at height of 6000 *km* from earth surface becomes

(A) Half

(B) One-fourth ✓

(C) One third

(D) No change

$$W = mg$$

$$g_h = \frac{GM}{(R+h)^2} = \frac{GM}{(R+R)^2} = \frac{GM}{4R^2} = \frac{g}{4}.$$

$$W' = \frac{W}{4}$$

P-Q958-Solution

Ans [B]

$$g' = g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \text{when } h = R \text{ then } g' = \frac{g}{4}$$

So the weight of the body at this height will become one fourth.

Question

JEE 10-April-16 Online

6. An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is :

(1) Zero since astronaut feels weightless

(2) $0 < F_G < \dfrac{GMm}{R^2}$

(3) $\dfrac{GMm}{(R+h)^2} < F_G < \dfrac{GMm}{R^2}$

(4) $F_G = \dfrac{GMm}{(R+h)^2}$

$$F_G = \frac{GMm}{(R+h)^2}$$

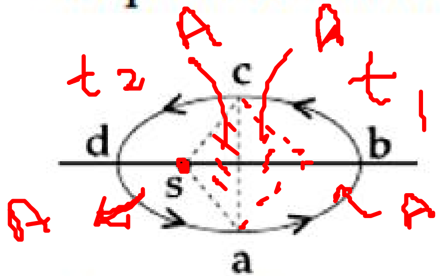
Solution

Ans [4]

Question

JEE 9-April-16 Online

6. Figure shows elliptical path abcd of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (See figure) With db as the semimajor axis, and ca as the semiminor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then :



(1) $t_1 = t_2$
(2) $t_1 = 2t_2$
(3) $t_1 = 3t_2$
(4) $t_1 = 4t_2$

$$\frac{A_1}{t_1} = \frac{A_2}{t_2}$$

$$\frac{dA}{dt} = \text{const}$$

$$\frac{\Delta A_1}{\Delta t_1} = \frac{\Delta A_2}{\Delta t_2}$$

$$\frac{3A}{t_1} = \frac{A}{t_2}$$

$$\boxed{t_1 = 3t_2}$$

Solution

Ans [3]

Question

JEE 15-April-18 Morning

2. A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$, and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is :

(1) $+\frac{GMm}{6R}$ (2) $\frac{GMm}{2R}$

(3) $-\frac{GMm}{2R}$ (4) $-\frac{GMm}{6R}$ ✓

$T.E = -\frac{GMm}{2R}$

$T_i = -\frac{GMm}{2R}$

$T_f = -\frac{GM\frac{m}{2}}{\cancel{2}\,R/\cancel{2}} + \left(-\frac{GMm/2}{\cancel{2}\cdot\frac{3R}{\cancel{2}}}\right)$

$T_f - T_i$

$= -\frac{GMm}{R}\left(\frac{2}{3} - \frac{1}{2}\right)$ $\frac{4-3}{6} = \frac{1}{6}$

$= -\frac{GMm}{2R}\left(1 + \frac{1}{3}\right)$

$= -\frac{2GMm}{3R}$

Solution

Ans [4]

Question

18. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is :

(1) $\frac{4L}{m}$ (2) $\frac{L}{m}$ ~~(3)~~ $\frac{L}{2m}$ (4) $\frac{2L}{m}$

Areal velocity $= \frac{dA}{dt} = \frac{L}{2m} =$ constant

Solution

Ans [3]

Question

14. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth $= 6.4 \times 10^3$ km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is:

(1) 1.28×10^4 km

(2) 6.4×10^3 km

(3) 3.2×10^3 km

(4) 1.6×10^3 km

$$E_1 = U_f - U_i$$

$$= -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$

$$E_2 = \frac{GMm}{2(R+h)}$$

$$\cancel{G}\cancel{M}\cancel{m}\left(\frac{1}{R} - \frac{1}{R+h}\right) = \frac{\cancel{GMm}}{2(R+h)}$$

$$\frac{1}{R} = \frac{1}{2(R+h)} + \frac{1}{R+h} = \frac{3}{2(R+h)}$$

$$2R + 2h = 3R$$

$$2h = R$$

$$h = R/2$$

Solution

Ans [3]