Problem Solving on Indefinite Integration

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Let $\alpha \in (0, \pi / 2)$ be fixed. If the integral

$$\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx = A(x) \cos 2\alpha + B(x)$$

$\sin 2\alpha + C$, where C is a constant of integration, then the functions $A(x)$ and $B(x)$ are respectively

(2019 Main, 12 April II)

(a) $x + \alpha$ and $\log_e |\sin(x + \alpha)|$
(b) $x - \alpha$ and $\log_e |\sin(x - \alpha)|$
(c) $x - \alpha$ and $\log_e |\cos(x - \alpha)|$
(d) $x + \alpha$ and $\log_e |\sin(x - \alpha)|$

The integral $\int \frac{2x^3 - 1}{x^4 + x}\, dx$ is equal to

(here C is a constant of integration) **(2019 Main, 12 April I)**

(a) $\frac{1}{2}\log_e \frac{|x^3 + 1|}{x^2} + C$ (b) $\frac{1}{2}\log_e \frac{(x^3 + 1)^2}{|x^3|} + C$

(c) $\log_e \left|\frac{x^3 + 1}{x}\right| + C$ (d) $\log_e \frac{|x^3 + 1|}{x^2} + C$

If $\int \frac{dx}{(x^2 - 2x + 10)^2} = A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2 - 2x + 10}\right) + C$,

where, C is a constant of integration, then

(2019 Main, 10 April I)

(a) $A = \frac{1}{27}$ and $f(x) = 9(x-1)$

(b) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$

(c) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

(d) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$

If $\int \frac{dx}{x^3 (1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$

where, C is a constant of integration, then the function $f(x)$ is equal to **(2019 Main, 8 April II)**

(a) $-\frac{1}{6x^3}$

(b) $-\frac{1}{2x^3}$

(c) $-\frac{1}{2x^2}$

(d) $\frac{3}{x^2}$

The integral $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$ is equal to (where C is a constant of integration) **(2019 Main, 12 Jan II)**

(a) $\frac{x^4}{6(2x^4 + 3x^2 + 1)^3} + C$

(b) $\frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$

(c) $\frac{x^4}{(2x^4 + 3x^2 + 1)^3} + C$

(d) $\frac{x^{12}}{(2x^4 + 3x^2 + 1)^3} + C$

$$\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx \text{ is equal to}$$

(where, C is a constant of integration)

(2019 Main, 8 April I)

(a) $2x + \sin x + 2 \sin 2x + C$

(b) $x + 2 \sin x + 2 \sin 2x + C$

(c) $x + 2 \sin x + \sin 2x + C$

(d) $2x + \sin x + \sin 2x + C$

If $\int \frac{x+1}{\sqrt{2x-1}}\,dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then $f(x)$ is equal to

(2019 Main, 11 Jan II)

(a) $\frac{2}{3}(x+2)$

(b) $\frac{1}{3}(x+4)$

(c) $\frac{2}{3}(x-4)$

(d) $\frac{1}{3}(x+1)$

If $\int \frac{\sqrt{1-x^2}}{x^4}\, dx = A(x)(\sqrt{1-x^2})^m + C$,

for a suitable chosen integer m and a function $A(x)$, where C is a constant of integration, then $(A(x))^m$ equals **(2019 Main, 11 Jan I)**

(a) $\frac{1}{9x^4}$ (b) $\frac{-1}{3x^3}$

(c) $\frac{-1}{27x^9}$ (d) $\frac{1}{27x^6}$

If $f(x)=\int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}\,dx, (x \geq 0)$, and $f(0)=0$, then the value of $f(1)$ is **(2019 Main, 9 Jan I)**

(a) $-\frac{1}{2}$

(b) $-\frac{1}{4}$

(c) $\frac{1}{4}$

(d) $\frac{1}{2}$

The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

is equal to **(2018 Main)**

(a) $\frac{1}{3(1+\tan^3 x)} + C$

(b) $\frac{-1}{3(1+\tan^3 x)} + C$

(c) $\frac{1}{1+\cot^3 x} + C$

(d) $\frac{-1}{1+\cot^3 x} + C$

. The value of $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ is

(2015 Main)

(a) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

(b) $(x^4+1)^{\frac{1}{4}} + c$

(c) $-(x^4+1)^{\frac{1}{4}} + c$

(d) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

If $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + C$, where C is a constant of integration, then $g(-1)$ is equal to **(2019 Main, 10 April II)**

(a) -1 (b) 1

(c) $-\frac{1}{2}$ (d) $-\frac{5}{2}$

If $\int e^{\sec x}$

$(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x))$

$dx = e^{\sec x} f(x) + C$, then a possible choice of $f(x)$ is

(2019 Main, 9 April II)

(a) $x \sec x + \tan x + \frac{1}{2}$

(b) $\sec x + \tan x + \frac{1}{2}$

(c) $\sec x + x \tan x - \frac{1}{2}$

(d) $\sec x - \tan x - \frac{1}{2}$

If $\int x^5 e^{-4x^3}\, dx = \frac{1}{48} e^{-4x^3} f(x) + C$,

where C is a constant of integration, then $f(x)$ is equal to

(2019 Main, 10 Jan II)

(a) $-4x^3 - 1$

(b) $4x^3 + 1$

(c) $-2x^3 - 1$

(d) $-2x^3 + 1$

The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$ is equal to (here C is a constant of integration) **(2019 Main, 9 April I)**

(a) $3\tan^{-1/3} x + C$

(b) $-3\tan^{-1/3} x + C$

(c) $-3\cot^{-1/3} x + C$

(d) $-\frac{3}{4}\tan^{-4/3} x + C$