(a) ROLLE'S THEOREM :

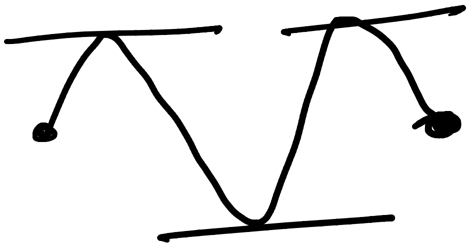
Let f(x) be a function of x subject to the following conditions : $y = f(x)$

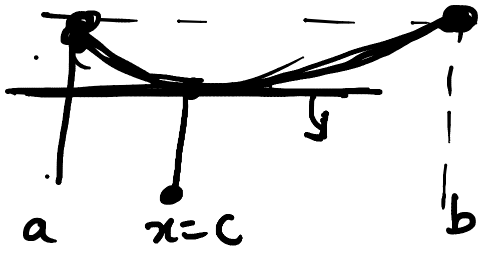
(i) f(x) is a continuous function of x in the closed interval of $a \le x \le b$.

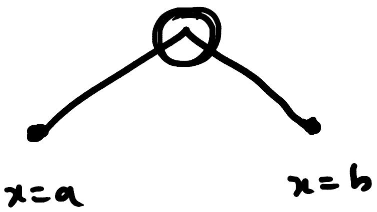
(ii) $f'(x)$ exists for every point in the open interval $a < x < b$.

(iii) $f(a) = f(b)$.

Then there exists at least one point $x = c$ such that $a < c < b$ where $\boxed{f'(c) = 0}$.







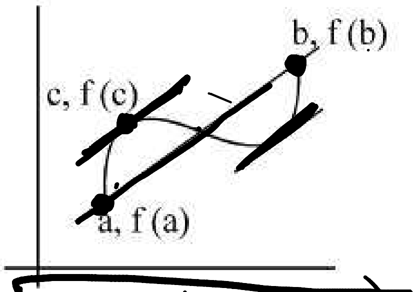
(b) LMVT THEOREM : (LAGRANGE'S MEAN VALUE THEOREM)

Let f(x) be a function of x subject to the following conditions :

(i) f(x) is a continuous function of x in the closed interval of $a \le x \le b$.

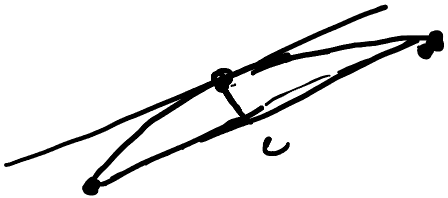
(ii) $f'(x)$ exists for every point in the open interval $a < x < b$.

(iii) $f(a) \neq f(b)$.



Then there exists at least one point $x = c$ such that $a < c < b$ where $\boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$

Slope line joining a & b



Maxima & Minima

(a) Maxima (Local/Relative maxima) :

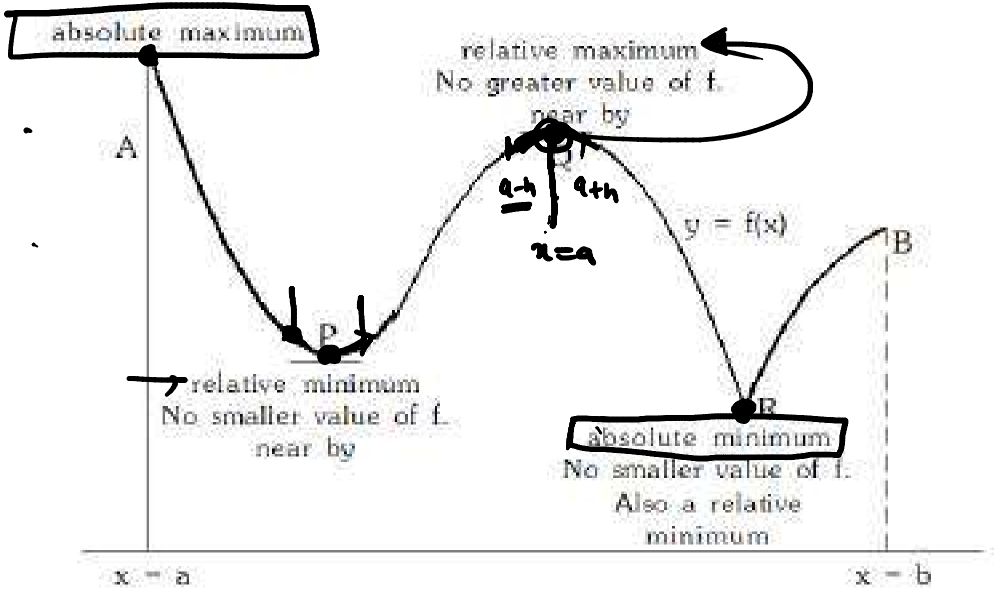
A function f(x) is said to have a maximum at $x = a$ if there exist a neighbourhood $(a - h, a + h) - \{a\}$ such that

$\boxed{f(a) > f(x)}\ \forall\ x \in (a - h, a + h) - \{a\}$

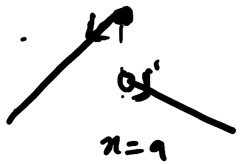
(b) Minima (Local/Relative minima):

A function f(x) is said to have a minimum at $x = a$ if there exist a neighbourhood $(a - h, a + h) - \{a\}$ such that

$f(a) < f(x)\ \forall\ x \in (a - h, a + h) - \{a\}$



yes



($x = a$ is a point of Maxima)

DERIVATIVE TEST FOR ASCERTAINING MAXIMA/MINIMA :

(a) **First derivative test** :

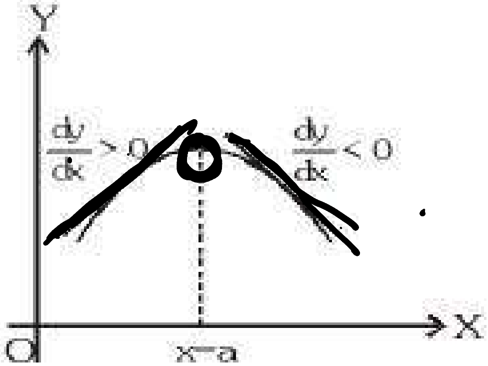
If $f'(x) = 0$ at a point (say $x = a$) and

$f(x)$ → Point of Extremum

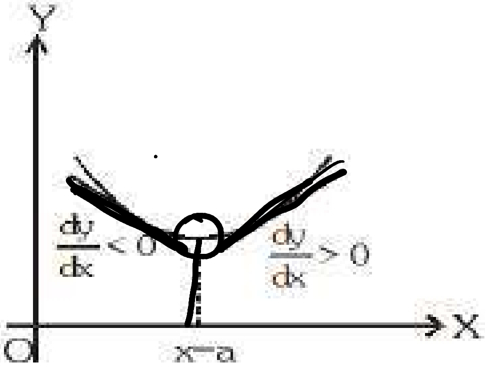
(i) If $f'(x)$ changes sign from positive to negative while graph of the function passes through $x = a$ then $x = a$ is said to be a point **local maxima.**

(ii) If $f'(x)$ changes sign from negative to positive while graph of the function passes through $x = a$ then $x = a$ is said to be a point **local minima.**

$f'(x)$



Local Maxima



Local Minima

(b) Second derivative test : (Imp)

If f(x) is continuous and differentiable at x = a where $\boxed{f'(a) = 0}$ and f''(a) also exists then for ascertaining maxima/minima at x = a, 2nd derivative test can be used -

$f'(x)=0$ $x = a_1, a_2, a_3$

(i) If $f''(a) > 0$ $\Rightarrow$ x = a is a point of local minima

(ii) If $f''(a) < 0$ $\Rightarrow$ x = a is a point of local maxima

(iii) If $f''(a) = 0$ $\Rightarrow$ second derivative test fails. To identify maxima/minima at this point either first derivative test or higher derivative test can be used.

$f(x) = x^2 - x$

$f'(x) = 2x - 1$

$\boxed{x = 1/2}$ $f'(x)=0$

$f''(x) = 2$

$f''(1/2) = 2$

Point of Minima

n^{th} derivative test : (Higher Test)

Let f(x) function such that $f'(a) = f''(a) = f'''(a) = \ldots\ldots = f^{n-1}(a) = 0$ & $\boxed{f^n(a) \neq 0}$, then

(i) n = even

(1) $f^n(a) > 0 \Rightarrow$ Minima

(2) $f^n(a) < 0 \Rightarrow$ Maxima

(ii) n = odd

Neither maxima nor minima at $x = a$

$\hookrightarrow$ Point of Inflection

$f(x) = x^3$

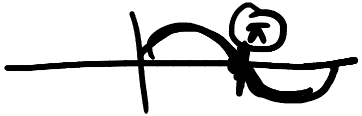
Increasing $\boxed{f'(x) = 3x^2}$ $\boxed{x = 0}$

$\boxed{f''(x) = 6x}$, $\boxed{f''(0) = 0}$

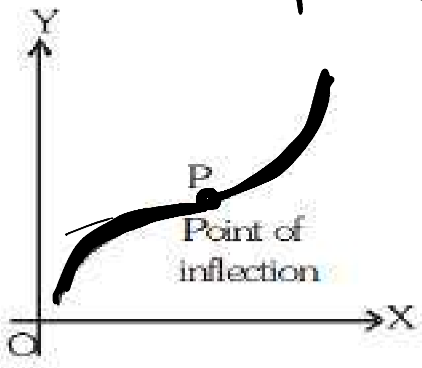
$\boxed{f'''(x) = 6}$

$n = 3$ (odd)

POINT OF INFLECTION :



A point where the graph of a function has a tangent line and where the concavity changes is called a point of inflection.



$y = \sin x$

$\frac{dy}{dx} = \cos x$

$= -\sin x$

If function $y = f(x)$ is double differentiable then the point at which $\left(\frac{d^2y}{dx^2} = 0\right)$ & changes its sign is the point of inflection.

$f''(x) = 0$

$f''(x)$

Note : If at any point $\frac{d^2y}{dx^2}$ does not exist but sign of $\frac{d^2y}{dx^2}$ changes about this point then it is also called point of inflection. e.g. for $y = x^{1/3}$, $x = 0$ is point of inflection.

SHORTEST DISTANCE BETWEEN TWO CURVES :

Shortest distance between two non-intersecting curves always lies along the common normal. (Wherever defined)