Problem Solving on AOD

By

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If the tangent to the curve, $y = x^3 + ax - b$ at the point $(1, -5)$ is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve ?

(2019 Main, 9 April I)

(a) $(-2, 2)$
(b) $(2, -2)$
(c) $(-2, 1)$
(d) $(2, -1)$

If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is **(2018 Main)**

(a) 6

(b) $\frac{7}{2}$

(c) 4

(d) $\frac{9}{2}$

The normal to the curve $y(x-2)(x-3) = x + 6$ at the point, where the curve intersects the Y-axis passes through the point **(2017 Main)**

(a) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

(b) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{2}, -\frac{1}{3}\right)$

(d) $\left(\frac{1}{2}, \frac{1}{3}\right)$

Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall\, x \in R$. Then, the set of all $x \in R$, where the function $h(x) = (fog)\,(x)$ is increasing, is **(2019 Main, 10 April II)**

(a) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$

(b) $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

(c) $[0, \infty)$

(d) $\left[\frac{-1}{2}, 0\right] \cup [1, \infty)$

A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is **(2019 Main, 10 April II)**

(a) $\frac{1}{9\pi}$ (b) $\frac{1}{18\pi}$ (c) $\frac{1}{36\pi}$ (d) $\frac{5}{6\pi}$

If the function f given by

$$f(x) = x^3 - 3(a-2)x^2 + 3ax + 7,$$

for some $a \in R$ is increasing in $(0, 1]$ and decreasing in $[1, 5)$, then a root of the equation, $\dfrac{f(x)-14}{(x-1)^2} = 0 \ (x \neq 1)$ is

(2019 Main, 12 Jan II)

(a) -7 (b) 6 (c) 7 (d) 5

The length of a longest interval in which the function $3 \sin x - 4 \sin^3 x$ is increasing, is **(2002, 2M)**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ (d) π

The function $f(x) = \sin^4 x + \cos^4 x$ increases, if

(a) $0 < x < \frac{\pi}{8}$

(b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$

(1999, 2M)

(c) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$

(d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

If m is the minimum value of k for which the function $f(x) = x\sqrt{kx - x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in the interval [0, 3] when $k = m$, then the ordered pair (m, M) is equal to

(2019 Main, 12 April I)

(a) $(4, 3\sqrt{2})$
(b) $(4, 3\sqrt{3})$
(c) $(3, 3\sqrt{3})$
(d) $(5, 3\sqrt{6})$

If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then

(a) $S_1 = \{-2\}; S_2 = \{0, 1\}$ **(2019 Main, 8 April I)**

(b) $S_1 = \{-2, 0\}; S_2 = \{1\}$

(c) $S_1 = \{-2, 1\}; S_2 = \{0\}$

(d) $S_1 = \{-1\}; S_2 = \{0, 2\}$

The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is

(2019 Main, 8 April II)

(a) $\sqrt{6}$

(b) $2\sqrt{3}$

(c) $\sqrt{3}$

(d) $\frac{2}{3}\sqrt{3}$

The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is **(2019 Main, 8 April I)**

(a) 2 (b) $\frac{7}{8}$ (c) $\frac{7}{4\sqrt{2}}$ (d) $\frac{11}{4\sqrt{2}}$

The maximum value of the function

$$f(x) = 3x^3 - 18x^2 + 27x - 40$$

on the set $S = \{x \in R : x^2 + 30 \le 11x\}$ is **(2019 Main, 11 Jan I)**

(a) 122 (b) -122 (c) -222 (d) 222

If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log |x| + \beta x^2 + x$, then **(2014 Main)**

(a) $\alpha = -6,\ \beta = \frac{1}{2}$

(b) $\alpha = -6,\ \beta = -\frac{1}{2}$

(c) $\alpha = 2,\ \beta = -\frac{1}{2}$

(d) $\alpha = 2,\ \beta = \frac{1}{2}$

Let $f(x) = 5 - |x - 2|$ and $g(x) = |x + 1|$, $x \in R$. If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value of β, then $\lim_{x \to -\alpha\beta} \frac{(x-1)(x^2 - 5x + 6)}{x^2 - 6x + 8}$ is equal to

(2019 Main, 12 April II)

(a) 1/2 (b) $-3/2$ (c) $-1/2$ (d) 3/2