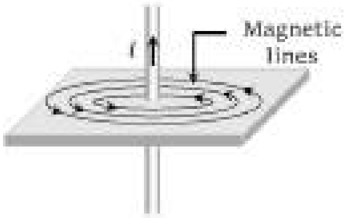
q at rest
E ✓, B = 0

q → v
E ✓, B ✓

Oersted found that a magnetic field is established around a current carrying conductor.

Magnetic field exists as long as there is current in the wire.

The direction of magnetic field was found to be changed when direction of current was reversed.



Note : ❒ A moving charge produces magnetic as well as electric field, unlike a stationary charge which only produces electric field.

Biot Savart's Law

Biot-Savart's law is used to determine the magnetic field at any point due to a current carrying conductors.

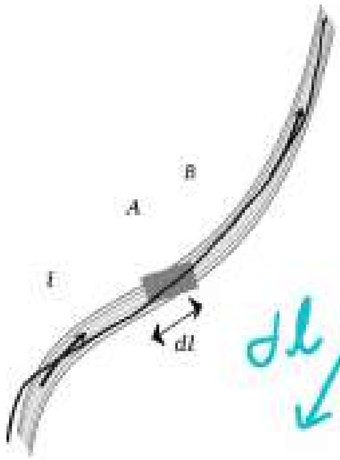
This law is although for infinitesimally small conductors yet it can be used for long conductors. In order to understand the Biot-Savart's law, we need to understand the term current-element.

Current element

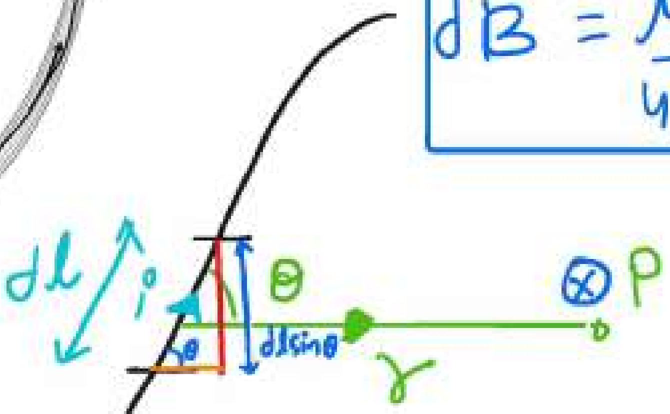
It is the product of current and length of infinitesimal segment of current carrying wire.

The current element is taken as a vector quantity. Its direction is same as the direction of current.

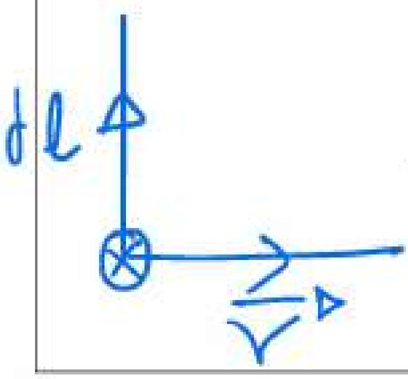
Current element $AB = i\vec{dl}$



$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\vec{dl}\times\hat{r}}{r^2}}$$



$$\boxed{dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}}$$



$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\vec{dl}\times\vec{r}}{r^3}}$$

$\frac{\mu_0}{4\pi} = 10^{-7}$

In C.G.S. : $k = 1 \Rightarrow dB = \frac{idl\sin\theta}{r^2}$ Gauss

In S.I. : $k = \frac{\mu_0}{4\pi} \Rightarrow dB = \frac{\mu_0}{4\pi} \cdot \frac{idl\sin\theta}{r^2}$ Tesla

where μ_0 = Absolute permeability of air or vacuum $= 4\pi \times 10^{-7} \frac{Wb}{Amp - metre}$. It's other units are $\frac{Henry}{metre}$

or $\frac{N}{Amp^2}$ or $\frac{Tesla - metre}{Ampere}$

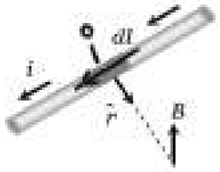
(2) **Similarities and differences between Biot-Savart law and Coulomb's Law**

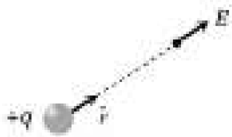
(i) The current element produces a magnetic field, whereas a point charge produces an electric field.

(ii) The magnitude of magnetic field varies as the inverse square of the distance from the current element, as does the electric field due to a point charge.

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{l} \times \hat{r}}{r^2}$ Biot-Savart Law $\quad \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$ Coulomb's Law

(iii) The electric field created by a point charge is radial, but the magnetic field created by a current element is perpendicular to both the length element $d\vec{l}$ and the unit vector $\hat{r}$.





$dB = \frac{\mu_0}{4\pi} \frac{idl\sin\theta}{r^2}$ $\qquad E = \frac{kq}{r^2}$

$dB \propto \frac{1}{r^2}$ $\qquad \vec{E} = \frac{kq\vec{r}}{r^3}$

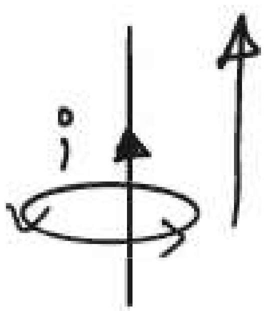
$dB = \frac{\mu_0}{4\pi} \frac{i\vec{dl} \times \vec{r}}{r^3}$

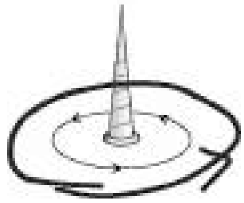
Direction of Magnetic Field

The direction of magnetic field is determined with the help of the following simple laws :

(1) Maxwell's cork screw rule

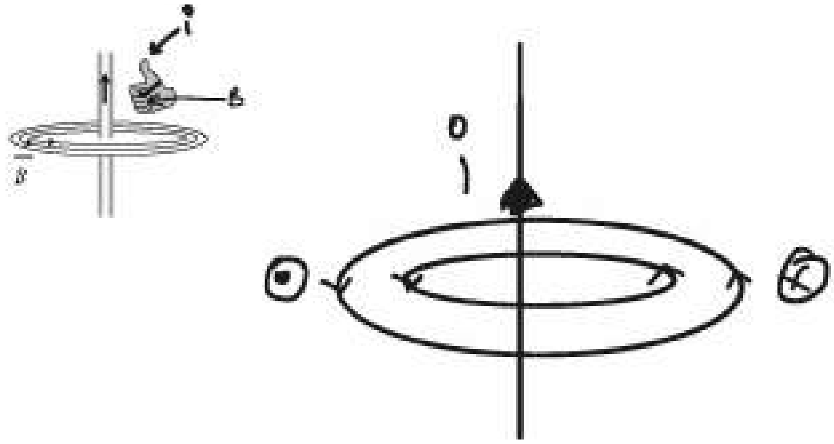
According to this rule, if we imagine a right handed screw placed along the current carrying linear conductor, be rotated such that the screw moves in the direction of flow of current, then the direction of rotation of the thumb gives the direction of magnetic lines of force.





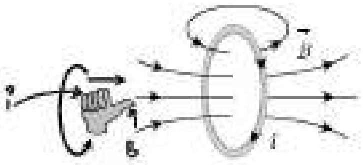
(2) Right hand thumb rule

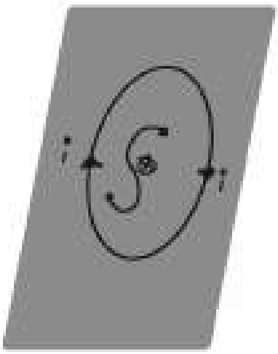
According to this rule if a current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow, then the direction of folding fingers will represent the direction of magnetic lines of force.



(3) Right hand thumb rule of circular currents

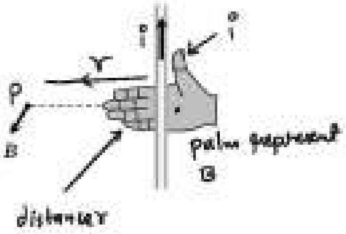
According to this rule if the direction of current in circular conducting coil is in the direction of folding fingers of right hand, then the direction of magnetic field will be in the direction of stretched thumb.

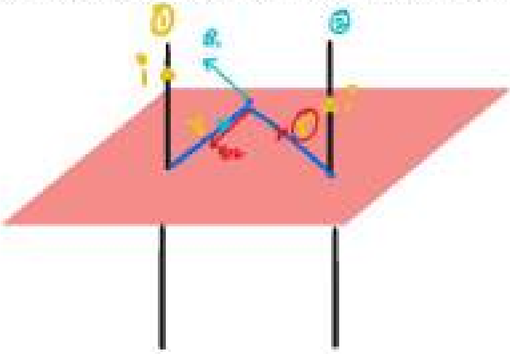


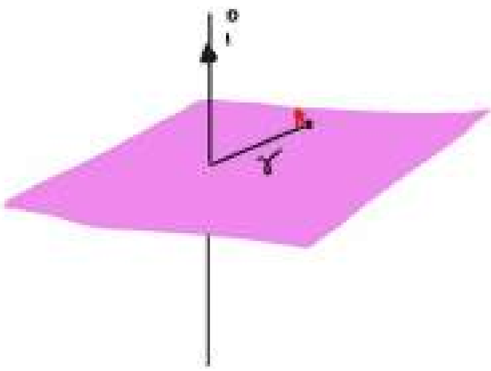


(4) **Right hand palm rule** Best rule.

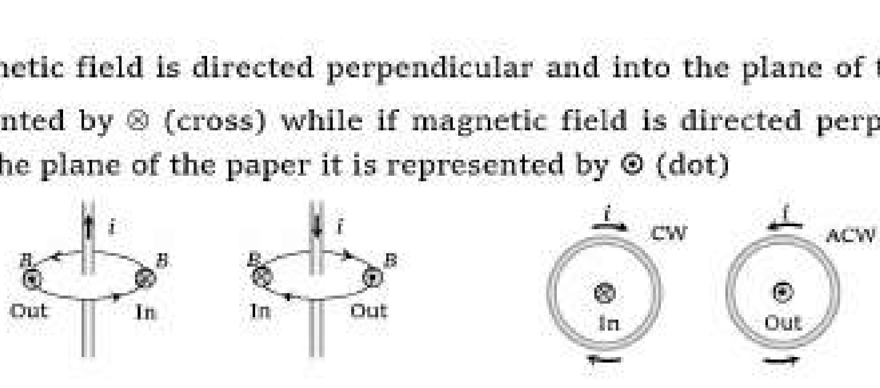
If we stretch our right hand such that fingers point towards the point. At which magnetic field is required while thumb is in the direction of current then normal to the palm will show the direction of magnetic field.





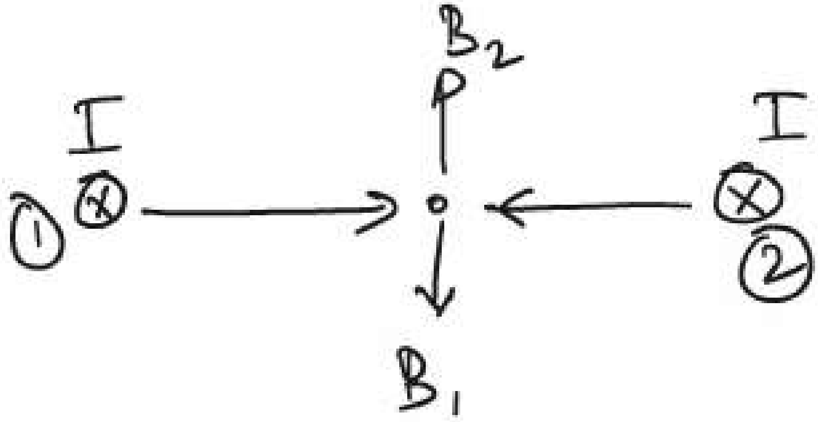


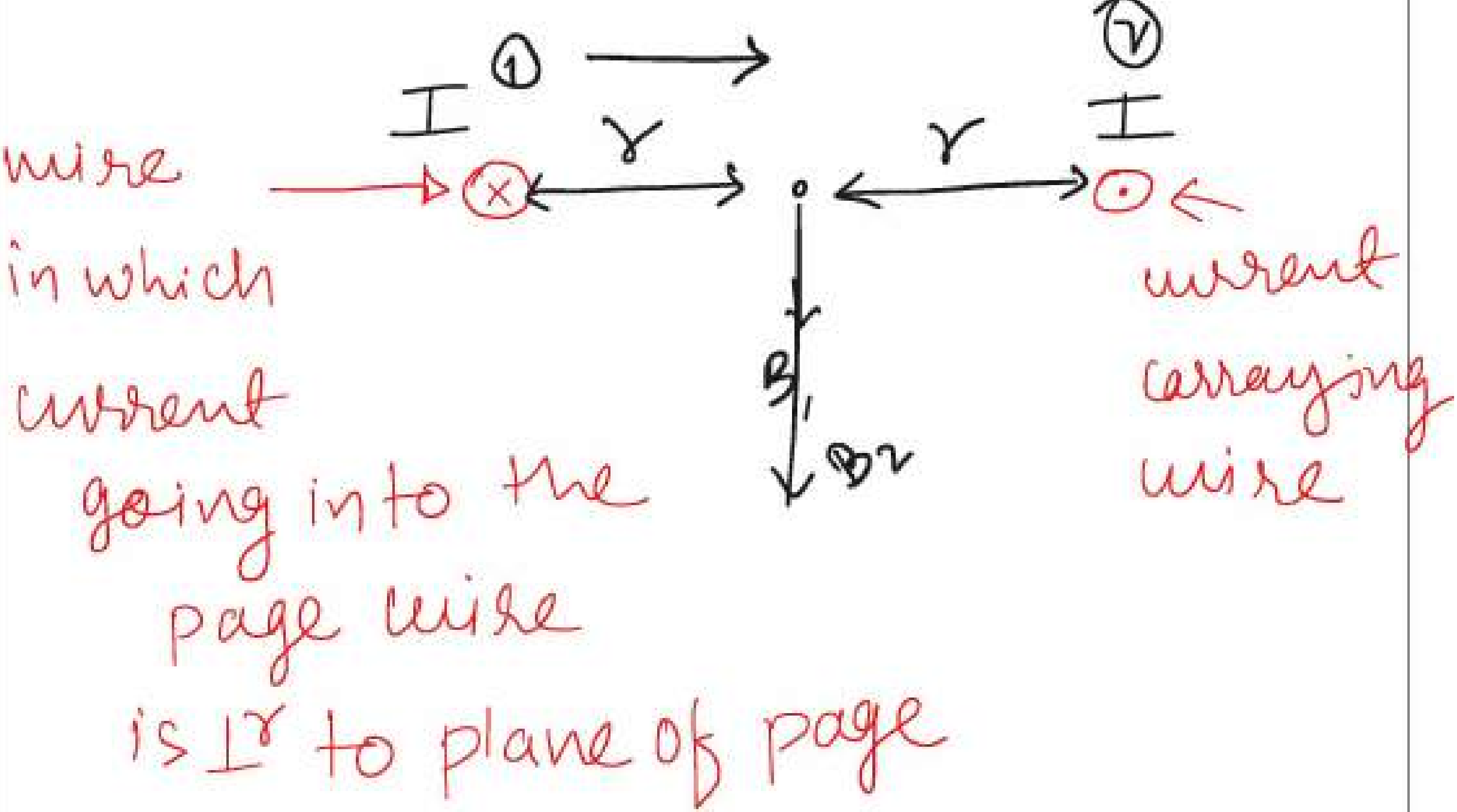
Note : ❒ If magnetic field is directed perpendicular and into the plane of the paper it is represented by ⊗ (cross) while if magnetic field is directed perpendicular and out of the plane of the paper it is represented by ⊙ (dot)



In : Magnetic field is away from the observer or perpendicular inwards.

Out : Magnetic field is towards the observer or perpendicular outwards.





Application of Biot-Savarts Law

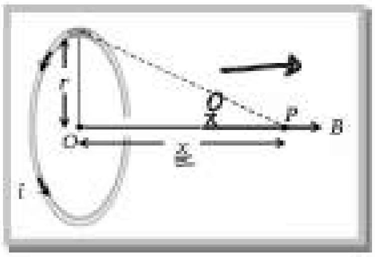
(1) Magnetic field due to a circular current

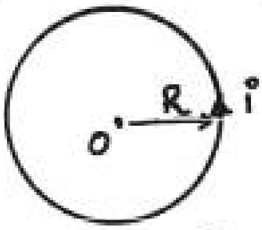
If a coil of radius r, carrying current i then magnetic field on it's axis at a distance x from its centre given by

$x = 0$

$$B_{axis} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N i r^2}{(x^2 + r^2)^{3/2}};$$ where N = number of turns in coil.

$$B_{axis} = N \frac{\mu_0}{4\pi} \frac{2\pi r^2 i}{(x^2+r^2)^{3/2}}, \quad B_{centre} = \frac{\mu_0 N 2\pi r^2 i}{4\pi\, 2r^3}$$





$$\frac{N\mu_0 i}{2r}$$

$$B = \frac{\mu_0 i}{2r}, \quad B \propto \frac{1}{r}$$

maximum at centre.

Different cases

Case 1 : Magnetic field at the centre of the coil

(i) At centre $x = 0 \Rightarrow B_{centre} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N i}{r} = \frac{\mu_0 N i}{2r} = B_{max}$

(ii) For single turn coil $N = 1 \Rightarrow B_{centre} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r} = \frac{\mu_0 i}{2r}$

(iii) In C.G.S. $\frac{\mu_0}{4\pi} = 1 \Rightarrow B_{centre} = \frac{2\pi i}{r}$

Note : ❐ $B_{centre} \propto N$ (i, r constant), $B_{centre} \propto i$ (N, r constant), $B_{centre} \propto \frac{1}{r}$ (N, i constant)

Case 2 : Ratio of B_{centre} and B_{axis}

~~The ratio~~ of magnetic field at the centre of circular coil and on it's axis is given by

$$\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$$

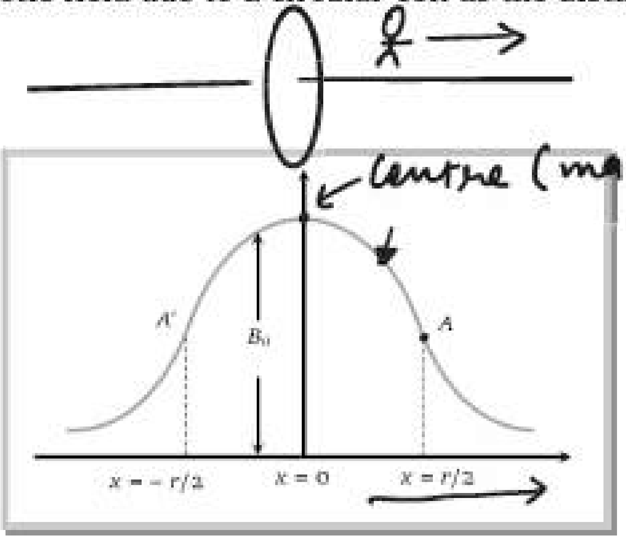
$$\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}.$$

where x is distance from centre of

ring.

Case 3 : *B-x* curve

The variation of magnetic field due to a circular coil as the distance x varies as shown in the figure.



centre (maximum $\frac{\mu_0 i}{2r}$)

(2) **Helmholtz coils** (IIT main 2020)

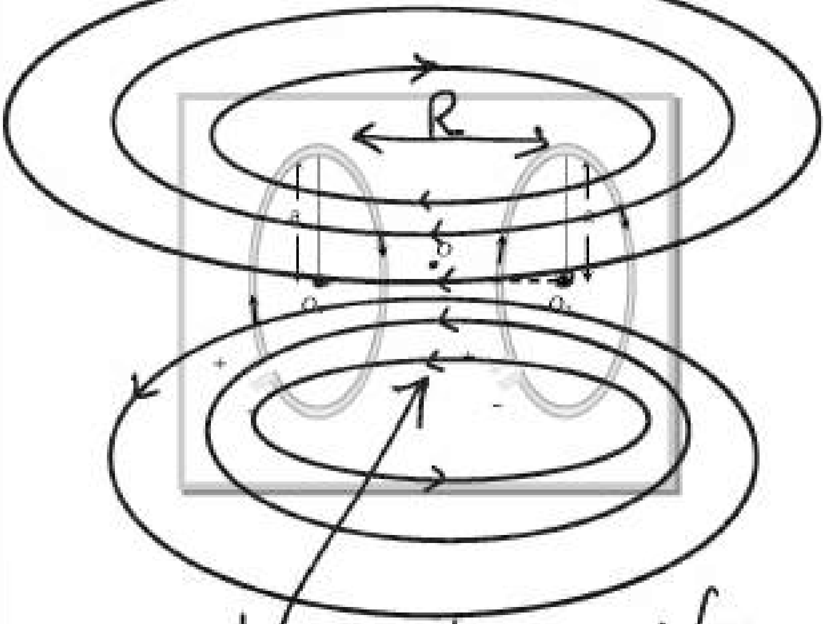
(i) This is the set-up of two coaxial coils of same radius such that distance between their centres is equal to their radius.

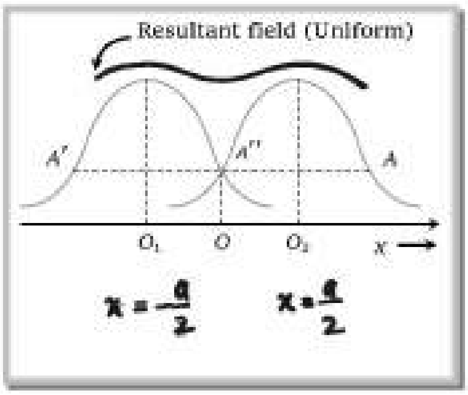
(ii) These coils are used to obtain uniform magnetic field of short range which is obtained between the coils.

(iii) At axial mid point O, magnetic field is given by $B = \frac{8\mu_0 Ni}{5\sqrt{5}R} = 0.716\frac{\mu_0 Ni}{R} = 1.432\,B$, where $B = \frac{\mu_0 Ni}{2R}$

(iv) Current direction is same in both coils otherwise this arrangement is not called Helmholtz's coil arrangement.

(v) Number of points of inflextion ⇒ Three (A, A', A'')



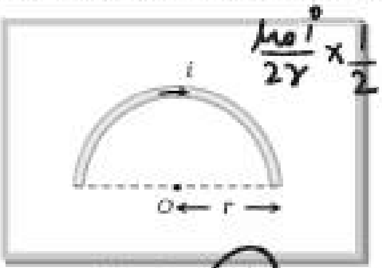


almost uniform magnetic field

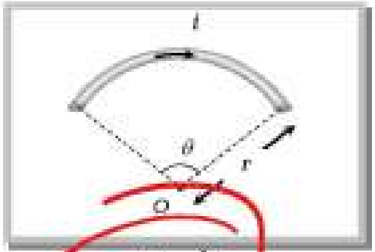
$\frac{\mu_0 i}{2r}$

(3) **Magnetic field due to current carrying circular arc :** Magnetic field at centre O

$\frac{\mu_0 i}{2r} \times \frac{1}{2}$

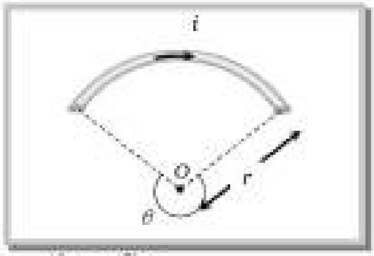


$B = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{r} = \frac{\mu_0 i}{4r}$



$B = \frac{\mu_0}{4\pi} \cdot \frac{\theta i}{r}$

$\frac{\mu_0 i}{2r} \times \frac{1}{2\pi} \times \theta$



$B = \frac{\mu_0}{4\pi} \cdot \frac{(2\pi - \theta) i}{r}$

Special results

If magnetic field at the centre of circular coil is denoted by B_0 $\left(= \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r}\right)$

Magnetic field at the centre of arc which is making an angle θ at the centre is $B_{arc} = \left(\frac{B_0}{2\pi}\right)\theta$

Angle at centre	Magnetic field at centre in term of B_0
360° (2π)	B_0
180° (π)	$B_0 / 2$
120° $(2\pi/3)$	$B_0 / 3$
90° $(\pi/2)$	$B_0 / 4$
60° $(\pi/3)$	$B_0 / 6$
30° $(\pi/6)$	$B_0 / 12$

(4) **Concentric circular loops** $(N = 1)$

(i) Coplanar and concentric : It means both coils are in same plane with common centre

(a) Current in same direction

$B_1 = \frac{\mu_0}{4\pi} 2\pi i \left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

Note : ❐ $\frac{B_1}{B_2} = \left(\frac{r_2 + r_1}{r_2 - r_1}\right)$

$B_1 = \frac{\mu_0 i}{2r_1} + \frac{\mu_0 i}{2r_2}$

$B_1 = \frac{\mu_0 i}{2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

(b) Current in opposite direction

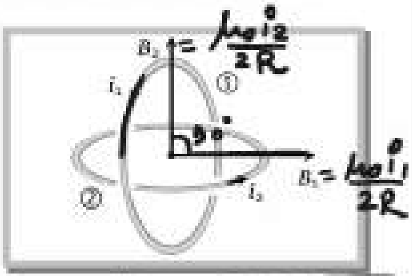
$B_2 = \frac{\mu_0}{4\pi} 2\pi i \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$B_{Net} = \frac{\mu_0 i}{2r_1} - \frac{\mu_0 i}{2r_2}$

(ii) Non-coplanar and concentric : Plane of both coils are perpendicular to each other

Magnetic field at common centre

$$\boxed{B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2}}$$



$\sqrt{a^2 + b^2 + 2ab\cos 90^\circ}$

$\sqrt{a^2 + b^2}$

$\sqrt{B_1^2 + B_2^2}$

(5) **Magnetic field due to a straight current carrying wire**

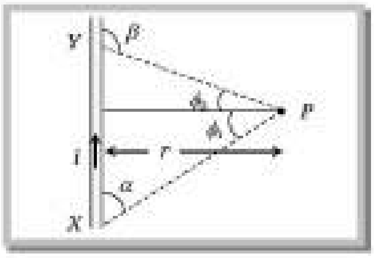
Magnetic field due to a current carrying wire at a point P which lies at a perpendicular distance r from the wire as shown is given as

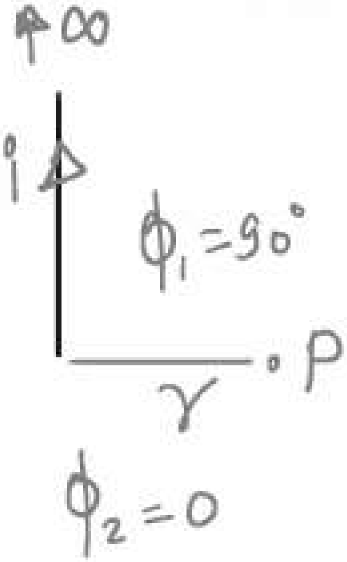
$B = \frac{\mu_0}{4\pi} \cdot \frac{i}{r}(\sin\phi_1 + \sin\phi_2)$

leave.

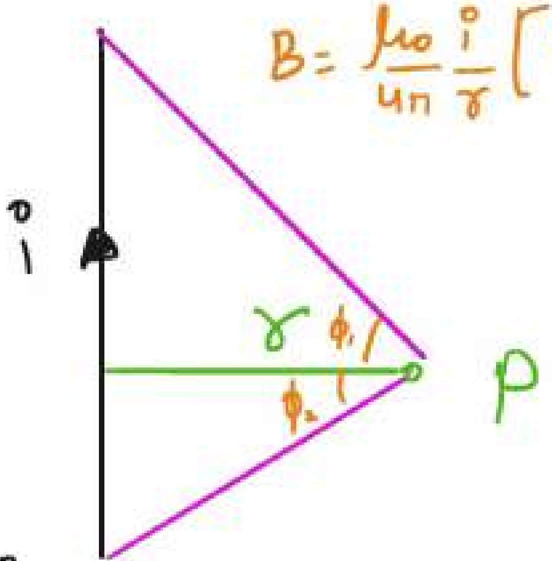
From figure $\alpha = (90^\circ - \phi_1)$ and $\beta = (90^\circ + \phi_2)$

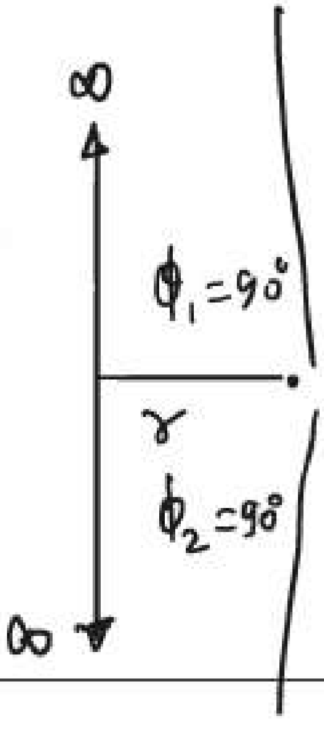
Hence $B = \frac{\mu_0}{4\pi} \cdot \frac{i}{r}(\cos\alpha - \cos\beta)$





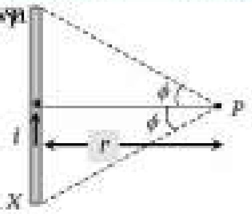
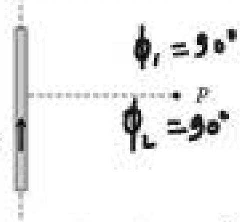
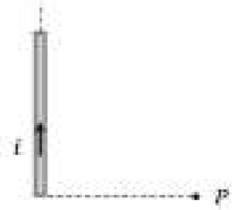
$B = \frac{\mu_0}{4\pi} \frac{i}{r}[\sin\phi_1 + \sin\phi_2]$





$B = \frac{\mu_0 i}{4\pi r}[\sin 90^\circ + \sin 90^\circ]$

$B = \frac{\mu_0 i}{4\pi r} \times 2 = \frac{\mu_0 i}{2\pi r}$

Case 1 : When the linear conductor XY is of finite length and the point P lies on it's perpendicular bisector as shown	**Case 2 :** When the linear conductor XY is of infinite length and the point P lies near the centre of the conductor	**Case 3 :** When the linear conductor is of infinite length and the point P lies near the end Y or X
	 $\phi_1 = 90°$ $\phi_2 = 90°$	
$\phi_1 = \phi_2 = \phi$	$\phi_1 = \phi_2 = 90°$.	$\phi_1 = 90°$ and $\phi_2 = 0°$.
So $B = \frac{\mu_0}{4\pi}\cdot\frac{i}{r}(2\sin\phi)$	So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90° + \sin 90°] = \frac{\mu_0}{4\pi}\frac{2i}{r}$	So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90° + \sin 0°] = \frac{\mu_0}{4\pi}\frac{i}{r}$

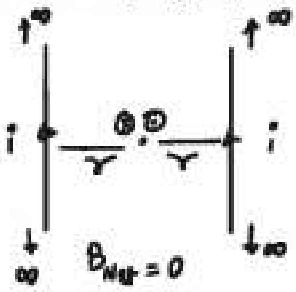
$B = \frac{\mu_0 i}{2\pi r}$ $B = \frac{\mu_0 i}{4\pi r}$

Different cases

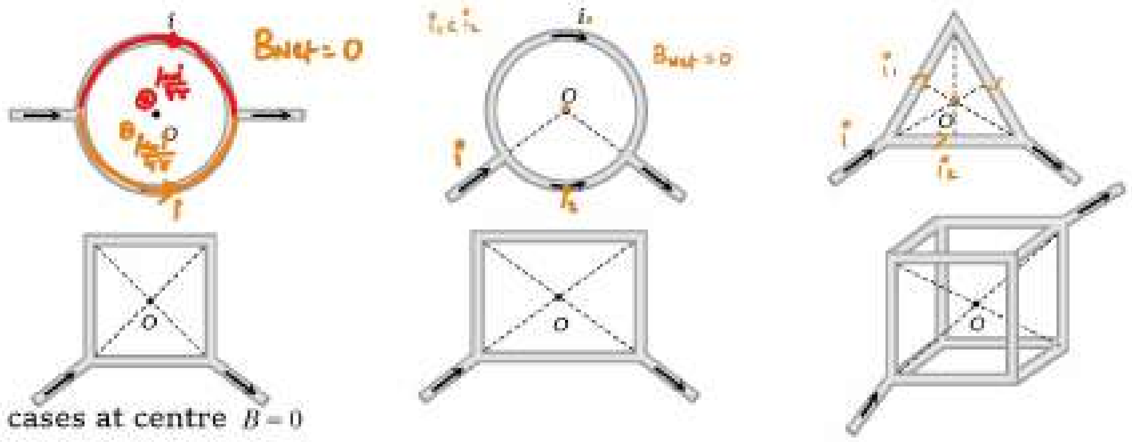
Note : ❐ When point P lies on axial position of current carrying conductor then magnetic field at P

$B = 0$

❐ The value of magnetic field induction at a point, on the centre of separation of two linear parallel conductors carrying equal currents in the same direction is zero.



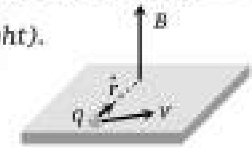
(6) **Zero magnetic field :** If in a symmetrical geometry, current enters from one end and exists from the other, then magnetic field at the centre is zero.



In all cases at centre $B = 0$

Concepts

- *If a current carrying circular loop (n = 1) is turned into a coil having n identical turns then magnetic field at the centre of the coil becomes n^2 times the previous field i.e.* $B_{(n\ turn)} = n^2\ B_{(single\ turn)}$
- *When a current carrying coil is suspended freely in earth's magnetic field, it's plane stays in* ***East-West*** *direction.*
- *Magnetic field ($\vec{B}$) produced by a moving charge q is given by* $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\hat{r})}{r^2}$ *; where v = velocity of charge and $v << c$ (speed of light).*



- *If an electron is revolving in a circular path of radius r with speed v then magnetic field produced at the centre of circular path* $B = \frac{\mu_0}{4\pi}.\frac{ev}{r^2}$.

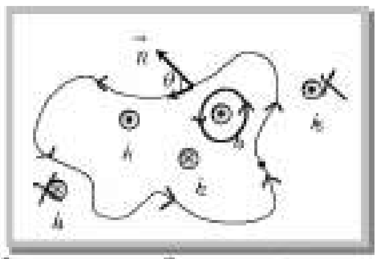
Amperes Law

Amperes law gives another method to calculate the magnetic field due to a given current distribution.

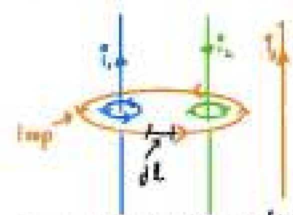
Line integral of the magnetic field $\vec{B}$ around any closed curve is equal to μ_0 times the net current I threading through the area enclosed by the curve

$\oint B d\ell = \mu_0 (+i_1 - i_2)$

i.e. $\oint \vec{B} d\vec{l} = \mu_0 \sum i = \mu_0 (i_1 + i_3 - i_2)$



$\oint B d\ell = \mu_0 (i_1 + i_3 - i_2)$



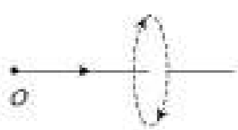
How to find magnetic field at any point of loop?

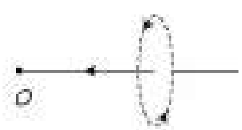
$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i_{enclosed}$

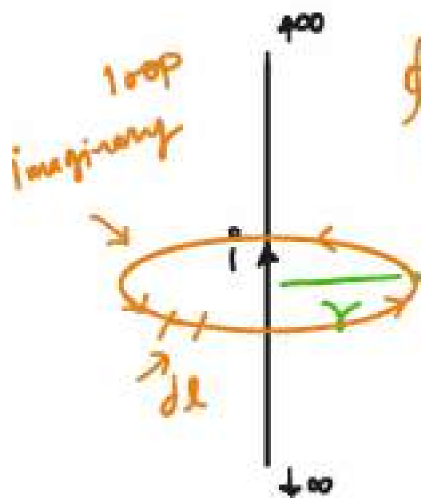
Close loop

Note: ❐ Total current crossing the above area is $(i_1 + i_3 - i_2)$. Any current outside the area is not included in net current. (Outward ⊙ → +*ve*, Inward ⊗ → − *ve*)

❐ When the direction of current is away from the observer then the direction of closed path is clockwise and when the direction of current is towards the observer then the direction of closed path is anticlockwise.







$\oint B d\ell = \mu_0 i$

$B \oint d\ell = \mu_0 i$

$B 2\pi r = \mu_0 i$

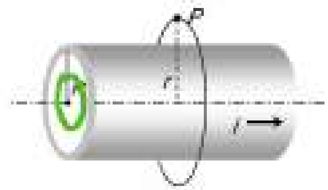
$B = \dfrac{\mu_0 i}{2\pi r}$

Application of Amperes law

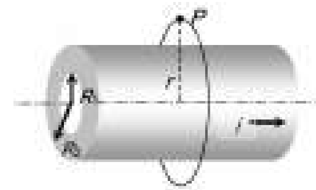
(1) Magnetic field due to a cylindrical wire

(i) Outside the cylinder IIT mains

Solid cylinder



Thin hollow cylinder



Thick hollow cylinder

In all above cases magnetic field outside the wire at P $\oint \vec{B}.\vec{dl} = \mu_0 i \Rightarrow B\int dl = \mu_0 i \Rightarrow B \times 2\pi r = \mu_0 i \Rightarrow$

$B_{out} = \frac{\mu_0 i}{2\pi r}$

In all the above cases $B_{surface} = \frac{\mu_0 i}{2\pi R}$

(ii) **Inside the cylinder :** Magnetic field inside the hollow cylinder is zero.

Cross sectional view Solid cylinder

Thin hollow cylinder

Thick hollow cylinder

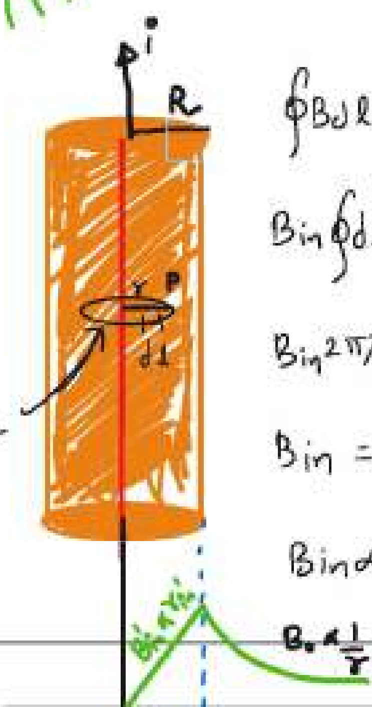
IIT

Current through πR^2 Area $= i$

current through unit area $= \frac{i}{\pi R^2}$

$i_{enclosed} = \frac{i}{\cancel{\pi} R^2} \times \cancel{\pi} r^2$

$i_{enclosed} = \frac{i r^2}{R^2}$



$\oint B dl = \mu_0 i_{enclosed}$

$B_{in} \oint dl = \mu_0 \frac{i r^2}{R^2}$

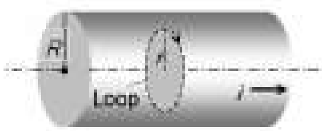
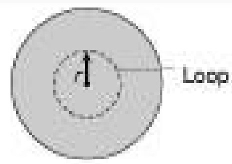
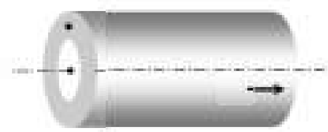
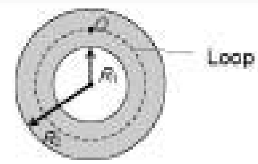
$B_{in} 2\pi \cancel{r} = \frac{\mu_0 i r^{\cancel{2}}}{R^2}$

$B_{in} = \frac{\mu_0 i}{2\pi R^2} r$

$B_{in} \propto r$

$B_{out} = \frac{\mu_0 i}{2\pi r}$

$r > R$

Solid cylinder	Inside the thick portion of hollow cylinder
	
Current enclosed by loop (i') is lesser then the total current (i) Current density is uniform *i.e.* $J = J' \Rightarrow \frac{i}{A} = \frac{i'}{A'}$ $\Rightarrow i' = i \times \frac{A'}{A} = i\left(\frac{r^2}{R^2}\right)$ Hence at point Q $\oint \vec{B} \cdot \vec{dl} = \mu_0 i' \Rightarrow B \times 2\pi r = \frac{\mu_0 i r^2}{R^2}$ $\Rightarrow B = \frac{\mu_0}{2\pi} \cdot \frac{ir}{R^2}$	Current enclosed by loop (i') is lesser then the total current (i) Also $i' = i \times \frac{A'}{A} = i \times \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$ Hence at point Q $\oint \vec{B} \cdot \vec{dl} = \mu_0 i' \Rightarrow B \times 2\pi r = \mu_0 i \times \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$ $\Rightarrow B = \frac{\mu_0 i}{2\pi r} \cdot \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$. If $r = R_1$ (inner surface) $B = 0$ If $r = R_2$ (outer surface) $B = \frac{\mu_0 i}{2\pi R_2}$ (max.)

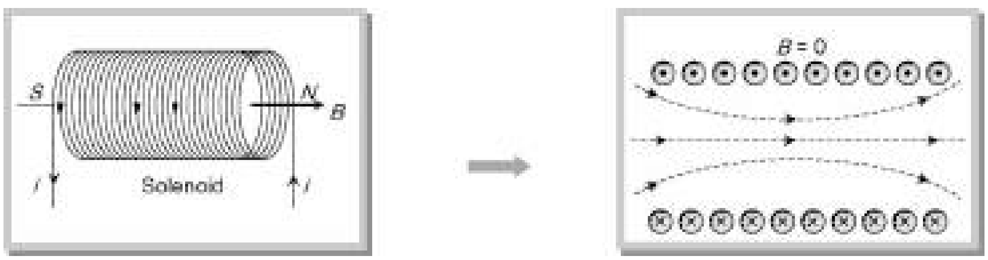
❒ For all cylindrical current distributions

$B_{axis} = 0$ (min.), $B_{surface}$ = max (distance r always from axis of cylinder), $B_{out} \propto 1/r$.

(2) Solenoid

A cylinderical coil of many tightly wound turns of insulated wire with generally diameter of the coil smaller than its length is called a solenoid.

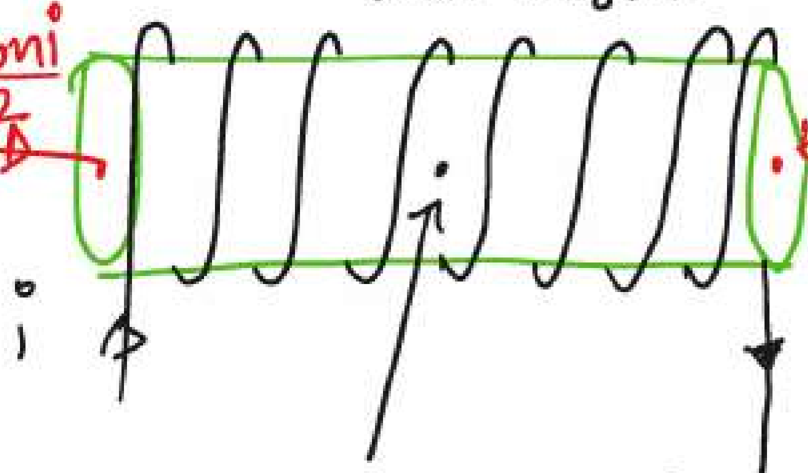
One end of the solenoid behaves like the north pole and opposite end behaves like the south pole. As the length of the solenoid increases, the interior field becomes more uniform and the external field becomes weaker.



A magnetic field is produced around and within the solenoid. The magnetic field within the solenoid is uniform and parallel to the axis of solenoid.

no. of turns per unit length $\rightarrow n = \frac{N}{l}$

$B_{end} = \frac{\mu_0 n i}{2}$



$B_{end} = \frac{\mu_0 n i}{2}$

$B_{centre} = \mu_0 n i$

Infinite length solenoid : If the solenoid is of infinite length and the point is well inside the solenoid *i.e.* $\alpha = \beta = (\pi/2)$.

So $\quad B_{in} = \mu_0 n i$

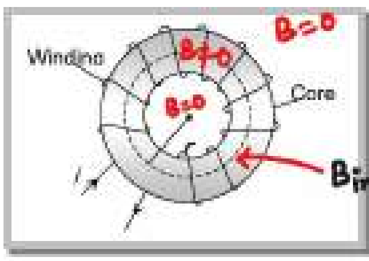
(ii) If the solenoid is of infinite length and the point is near one end *i.e.* $\alpha = 0$ and $\beta = (\pi/2)$

So $$B_{end} = \frac{1}{2}(\mu_0 ni)$$

Note : ❒ Magnetic field outside the solenoid is zero.

❒ $B_{end} = \frac{1}{2} B_{in}$

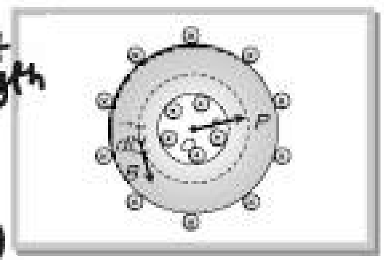
(4) **Toroid :** A toroid can be considered as a ring shaped closed solenoid. Hence it is like an endless cylindrical solenoid.



no. of turns per unit length

$B_{in} = \mu_0 n i$

$\frac{N}{2\pi r} = n$



Consider a toroid having *n* turns per unit length

Let *i* be the current flowing through the toroid (figure). The magnetic lines of force mainly remain in the core of toroid and are in the form of concentric circles. Consider such a circle of mean radius *r*. The circular closed path surrounds *N* loops of wire, each of which carries a current *i* therefore from $\oint \vec{B}.d\vec{l} = \mu_0 i_{net}$

$\Rightarrow B \times (2\pi r) = \mu_0 Ni \quad \Rightarrow B = \frac{\mu_0 Ni}{2\pi r} = \mu_0 ni$ where $n = \frac{N}{2\pi r}$

For any point inside the empty space surrounded by toroid and outside the toroid, magnetic field *B* is zero because the net current enclosed in these spaces is zero.

Concepts

☞ *The line integral of magnetising field $(\vec{H})$ for any closed path called magnetomotive force (MMF). It's S.I. unit is amp.*

☞ *Ratio of dimension of e.m.f. to MMF is equal to the dimension of resistance.*

- *Biot-Savart law is valid for asymmetrical current distributions while Ampere's law is valid for symmetrical current distributions.*
- *Biot-Savart law is based only on the principle of magnetism while Ampere's laws is based on the principle of electromagnetism.*

Motion of Charged Particle in a Magnetic Field

If a particle carrying a positive charge q and moving with velocity v enters a magnetic field B then it experiences a force F which is given by the expression

$$F = q(\vec{v} \times \vec{B}) \Rightarrow \boxed{F = qvB\sin\theta}$$ Vel. ⊥ B

Here $\vec{v} =$ velocity of the particle, $\vec{B} =$ magnetic field

(1) Zero force

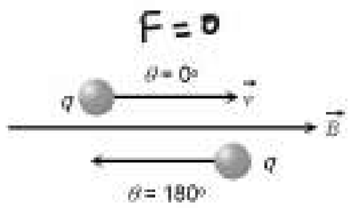
Force on charged particle will be zero (*i.e.* $F = 0$) if

(i) No field *i.e.* $B = 0 \Rightarrow F = 0$

(ii) Neutral particle *i.e.* $q = 0 \Rightarrow F = 0$

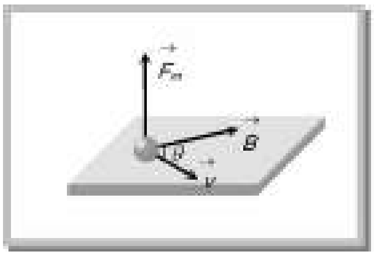
(iii) Rest charge *i.e.* $v = 0 \Rightarrow F = 0$

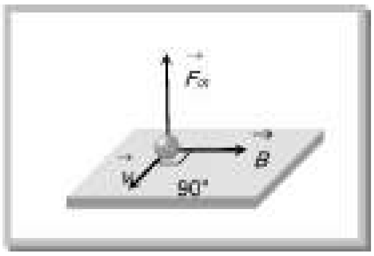
(iv) Moving charge *i.e.* $\theta = 0°$ or $\theta = 180° \Rightarrow F = 0$



(2) **Direction of force**

The force $\vec{F}$ is always perpendicular to both the velocity $\vec{v}$ and the field $\vec{B}$ in accordance with Right Hand Screw Rule, through $\vec{v}$ and $\vec{B}$ themselves may or may not be perpendicular to each other.



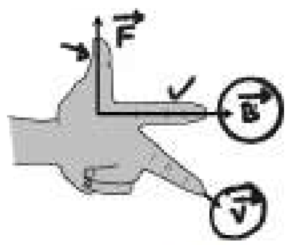


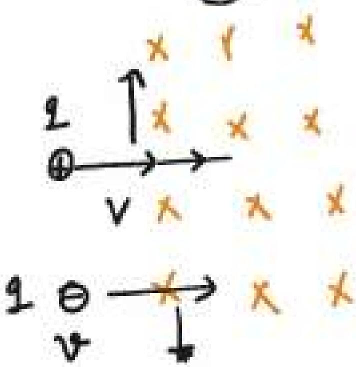
Direction of force on charged particle in magnetic field can also be find by Flemings Left Hand Rule (FLHR).

Here, *First finger* (indicates) $\rightarrow$ Direction of magnetic field

Middle finger $\rightarrow$ Direction of motion of positive charge or direction, opposite to the motion of negative charge.

Thumb $\rightarrow$ Direction of force



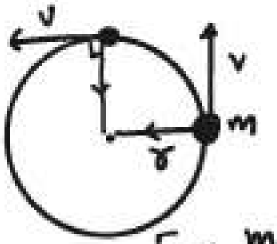


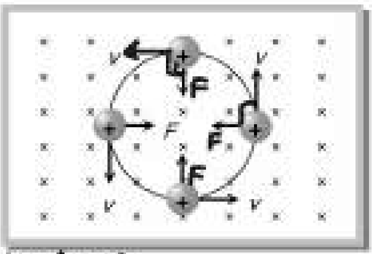
(3) **Circular motion of charge in magnetic field**

Consider a charged particle of charge q and mass m enters in a uniform magnetic field B with an initial velocity v perpendicular to the field.

$\vec{F} = q\vec{v} \times \vec{B}$

$\vec{F} \perp \vec{v}$





$F_c = \frac{mv^2}{r} = qvB\sin 90^\circ$

$\theta = 90^\circ$, hence from $F = qvB\sin\theta$ particle will experience a maximum magnetic force $F_{max} = qvB$ which act's in a direction perpendicular to the motion of charged particle. (By Flemings left hand rule).

(i) Radius of the path

$\frac{mv^2}{r} = qvB$

$$\boxed{r = \frac{mv}{qB}}$$

$r = \frac{p}{qB}$

$p = mv$

$KE = \frac{mv^2}{2} \times \frac{m}{m}$

$KE = \frac{p^2}{2m}$

$p = \sqrt{2mKE}$

$r = \frac{\sqrt{2mKE}}{qB}$

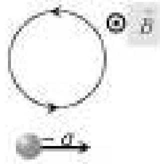
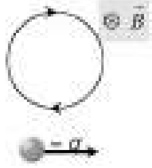
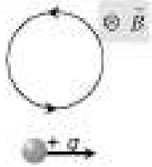
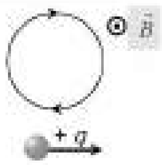
Potential diff.

$\Delta KE = q\Delta V$

$r = \frac{\sqrt{2mq\Delta V}}{qB}$

(ii) **Direction of path :** If a charge particle enters perpendicularly in a magnetic field, then direction of path described by it will be

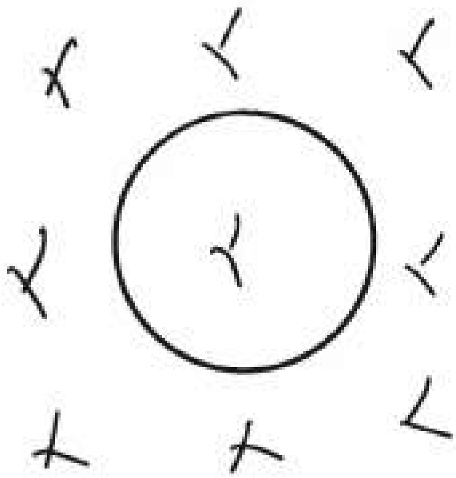
Type of charge	Direction of magnetic field	Direction of it's circular motion

Negative	Outwards ⊙	 Anticlockwise
Negative	Inward ⊗	 Clockwise
Positive	Inward ⊗	 Anticlockwise
Positive	Outward ⊙	 Clockwise

(iii) **Time period** : As in uniform circular motion $v = r\omega$, so the angular frequency of circular motion, called cyclotron or gyro-frequency, will be given by $\omega = \frac{v}{r} = \frac{qB}{m}$ and hence the time period, $T = \frac{2\pi}{\omega} = 2\pi\frac{m}{qB}$

i.e., time period (or frequency) is independent of speed of particle and radius of the orbit and depends only on the field B and the nature, *i.e.*, specific charge $\left(\frac{q}{m}\right)$, of the particle.

$$T = \frac{2\pi m}{qB}$$



(4) Motion of charge on helical path

T is Free From radius & vel. both.

Note : ❒ 1 rotation = 2π = T and 1 pitch = 1 T

❒ Number of pitches = Number of rotations = Number of repetition = Number of helical turns

❒ If pitch value is p, then number of pitches obtained in length l given as

Number of pitches = $\frac{l}{p}$ and time reqd. $t = \frac{l}{v\cos\theta}$

Some standard results read imp.

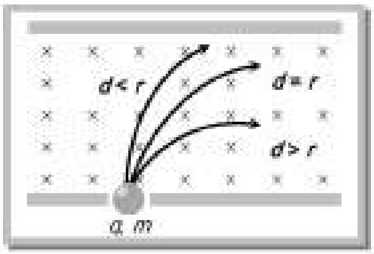
Ratio of radii of path described by proton and α-particle in a magnetic field (particle enters perpendicular to the field)

Constant quantity	Formula	Ratio of radii	Ratio of curvature (c)
v - same	$r = \frac{mv}{qB} \Rightarrow r \propto \frac{m}{q}$	$r_p : r_\alpha = 1:2$	$c_p : c_R = 2:1$
p - same	$r = \frac{p}{qB} \Rightarrow r \propto \frac{1}{q}$	$r_p : r_\alpha = 2:1$	$c_p : c_R = 1:2$

k - same	$r=\frac{\sqrt{2mk}}{qB}\Rightarrow r\propto\frac{\sqrt{m}}{q}$	$r_p:r_\alpha=1:1$	$c_p:c_B=1:1$
V - same	$r\propto\sqrt{\frac{m}{q}}$	$r_p:r_\alpha=1:\sqrt{2}$	$c_p:c_B=\sqrt{2}:1$

imp.

& Particle motion between two parallel plates $(\vec{v}\perp\vec{B})$



(i) To strike the opposite plate it is essential that $d<r$.

(ii) Does not strike the opposite plate $d>r$.

(iii) To touch the opposite plate $d=r$.

(iv) To just not strike the opposite plate $d\geq r$.

(v) To just strike the opposite plate $d\leq r$.

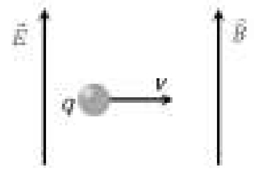
(5) Lorentz force

$$F = qE + qVB\sin\theta$$

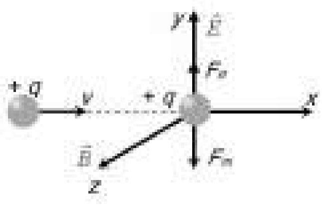
$$\vec{F} = \boxed{q\vec{E} + q\vec{V}\times\vec{B}}$$

(i) When $\vec{v},\vec{E}$ and $\vec{B}$ all the three are collinear :

(ii) When $\vec{E}$ is parallel to $\vec{B}$ and both these fields are perpendicular to $\vec{v}$ then :



(iii) $\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular :



Note : ❒ From the above discussion, conclusion is as follows

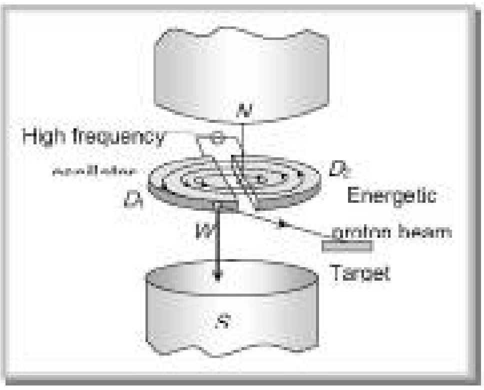
❒ If $E = 0$, $B = 0$, so $F = 0$.

❒ If $E = 0$, $B \neq 0$, so F may be zero (if $\theta = 0^\circ$ or 180°).

❒ If $E \neq 0$, $B \neq 0$, so $F = 0$ (if $|\vec{F}_e| = |\vec{F}_m|$ and their directions are opposite)

❒ If $E \neq 0$, $B = 0$, so $F \neq 0$ (because $\vec{v} \neq \text{constant}$).

Cyclotron



Note : ❐ The positive ions are produced in the gap between the two dees by the ionisation of the gas. To produce proton, hydrogen gas is used; while for producing alpha-particles, helium gas is used.

(1) **Cyclotron frequency :** Time taken by ion to describe *q* semicircular path is given by $t = \frac{\pi r}{v} = \frac{\pi m}{qB}$

If *T* = time period of oscillating electric field then $T = 2t = \frac{2\pi m}{qB}$ the cyclotron frequency $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

(2) **Maximum energy of position :** Maximum energy gained by the charged particle $E_{max} = \left(\frac{q^2B^2}{2m}\right)r^2$

where r_0 = maximum radius of the circular path followed by the positive ion.

Note : ❐ Cyclotron frequency is also known as magnetic resonance frequency.

❐ Cyclotron can not accelerate electrons because they have very small mass.

Motion of Charged Particle in a Magnetic Field

If a particle carrying a positive charge q and moving with velocity v enters a magnetic field B then it experiences a force F which is given by the expression

$$F = q(\vec{v} \times \vec{B}) \Rightarrow F = qvB\sin\theta$$

Here $\vec{v} =$ velocity of the particle, $\vec{B} =$ magnetic field

(1) **Zero force**

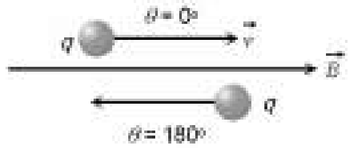
Force on charged particle will be zero (*i.e.* $F = 0$) if

(i) No field *i.e.* $B = 0 \Rightarrow F = 0$

(ii) Neutral particle *i.e.* $q = 0 \Rightarrow F = 0$

(iii) Rest charge *i.e.* $v = 0 \Rightarrow F = 0$

(iv) Moving charge *i.e.* $\theta = 0^\circ$ or $\theta = 180^\circ \Rightarrow F = 0$



(2) **Direction of force**

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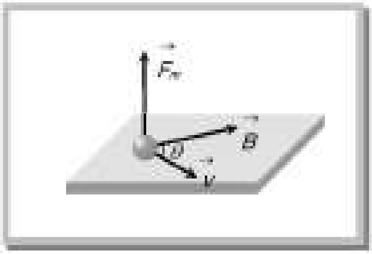

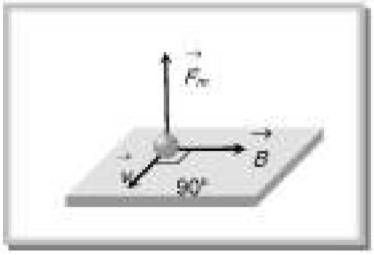

Direction of force on charged particle in magnetic field can also be find by Flemings Left Hand Rule (FLHR).

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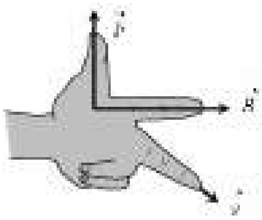
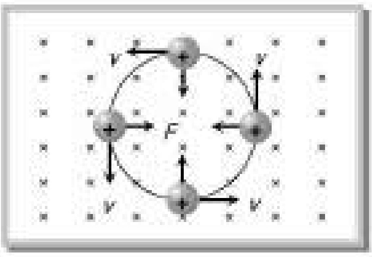
Middle finger $\rightarrow$ Direction of motion of positive charge or direction, opposite to the motion of negative charge.

Thumb $\rightarrow$ Direction of force



(3) **Circular motion of charge in magnetic field**

Consider a charged particle of charge q and mass m enters in a uniform magnetic field B with an initial velocity v perpendicular to the field.

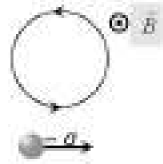
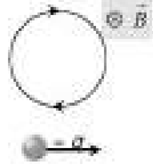
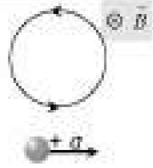
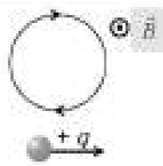


$\theta = 90^{\circ}$, hence from $F = qvB\sin\theta$ particle will experience a maximum magnetic force $F_{max} = qvB$ which act's in a direction perpendicular to the motion of charged particle. (By Flemings left hand rule).

(i) Radius of the path

(ii) **Direction of path :** If a charge particle enters perpendicularly in a magnetic field, then direction of path described by it will be

Type of charge	Direction of magnetic field	Direction of it's circular motion

Negative	Outwards ⊙	 Anticlockwise
Negative	Inward ⊗	 Clockwise
Positive	Inward ⊗	 Anticlockwise
Positive	Outward ⊙	 Clockwise

(iii) **Time period** : As in uniform circular motion $v = r\omega$, so the angular frequency of circular motion, called cyclotron or gyro-frequency, will be given by $\omega = \frac{v}{r} = \frac{qB}{m}$ and hence the time period, $T = \frac{2\pi}{\omega} = 2\pi \frac{m}{qB}$

i.e., time period (or frequency) is independent of speed of particle and radius of the orbit and depends only on the field B and the nature, *i.e.*, specific charge $\left(\frac{q}{m}\right)$, of the particle.

(4) **Motion of charge on helical path**

Note : ❒ 1 rotation = 2π = T and 1 pitch = 1 T

❒ Number of pitches = Number of rotations = Number of repetition = Number of helical turns

❒ If pitch value is p, then number of pitches obtained in length l given as

Number of pitches = $\frac{l}{p}$ and time reqd. $t = \frac{l}{v\cos\theta}$

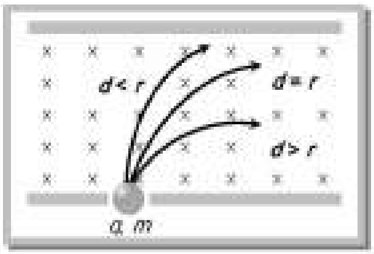
Some standard results

Ratio of radii of path described by proton and α-particle in a magnetic field (particle enters perpendicular to the field)

Constant quantity	Formula	Ratio of radii	Ratio of curvature (c)
v - same	$r = \frac{mv}{qB} \Rightarrow r \propto \frac{m}{q}$	$r_p : r_\alpha = 1 : 2$	$c_p : c_R = 2 : 1$
p - same	$r = \frac{p}{qB} \Rightarrow r \propto \frac{1}{q}$	$r_p : r_\alpha = 2 : 1$	$c_p : c_R = 1 : 2$

k - same	$r = \frac{\sqrt{2mk}}{qB} \Rightarrow r \propto \frac{\sqrt{m}}{q}$	$r_p : r_\alpha = 1:1$	$c_p : c_B = 1:1$
V - same	$r \propto \sqrt{\frac{m}{q}}$	$r_p : r_\alpha = 1:\sqrt{2}$	$c_p : c_B = \sqrt{2}:1$

& Particle motion between two parallel plates $(\vec{v} \perp \vec{B})$



(i) To strike the opposite plate it is essential that $d < r$.

(ii) Does not strike the opposite plate $d > r$.

(iii) To touch the opposite plate $d = r$.

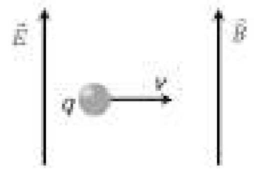
(iv) To just not strike the opposite plate $d \geq r$.

(v) To just strike the opposite plate $d \leq r$.

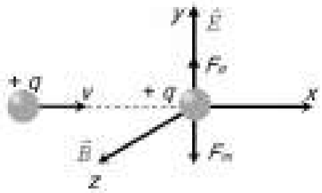
(5) Lorentz force

(i) When $\vec{v}, \vec{E}$ and $\vec{B}$ all the three are collinear :

(ii) When $\vec{E}$ is parallel to $\vec{B}$ and both these fields are perpendicular to $\vec{v}$ then :



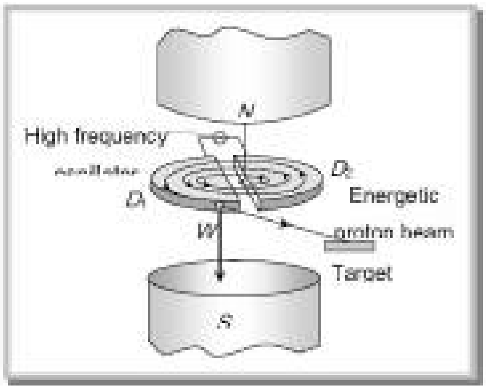
(iii) $\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular :



Note : ❒ From the above discussion, conclusion is as follows

- ❒ If $E = 0$, $B = 0$, so $F = 0$.
- ❒ If $E = 0$, $B \neq 0$, so F may be zero (if $\theta = 0^\circ$ or 180°).
- ❒ If $E \neq 0$, $B \neq 0$, so $F = 0$ (if $|\vec{F}_e| = |\vec{F}_m|$ and their directions are opposite)
- ❒ If $E \neq 0$, $B = 0$, so $F \neq 0$ (because $\vec{v} \neq \text{constant}$).

Cyclotron



Note : ❐ The positive ions are produced in the gap between the two dees by the ionisation of the gas. To produce proton, hydrogen gas is used; while for producing alpha-particles, helium gas is used.

(1) **Cyclotron frequency :** Time taken by ion to describe q semicircular path is given by $t = \frac{\pi r}{v} = \frac{\pi m}{qB}$

If T = time period of oscillating electric field then $T = 2t = \frac{2\pi m}{qB}$ the cyclotron frequency $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

(2) **Maximum energy of position :** Maximum energy gained by the charged particle $E_{max} = \left(\frac{q^2B^2}{2m}\right)r^2$

where r_0 = maximum radius of the circular path followed by the positive ion.

Note : ❐ Cyclotron frequency is also known as magnetic resonance frequency.

❐ Cyclotron can not accelerate electrons because they have very small mass.