Permutation & Combination

Problem Solving (JEE Mains)

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JEE Mains Problems

The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is **(2019 Main, 8 April II)**

(a) 306 ✓(b) 310
(c) 360 (d) 288

0, 1, 2, 3, 4, 5

$\underline{5}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad} = 6\times6\times6 = 216$
$1\quad 6\quad 6\quad 6$

$\underline{4}\ \underline{(4,5)}\ \underline{\quad}\ \underline{\quad} = 72$
$1\quad 2\quad 6\quad 6$

$\underline{4}\ \underline{3}\ \underline{(3,4,5)}\ \underline{\quad} = 18$
$1\quad 1\quad 3\quad 6$

$\underline{4}\ \underline{3}\ \underline{2}\ \underline{(2,3,4,5)} = 4$
$1\quad 1\quad 1\quad 4$

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JEE Mains Problems

The number of integers greater than 6000 that can be formed using the digits 3, 5, 6, 7 and 8 without repetition is (2015 Main)

(a) 216
(b) 192
(c) 120
(d) 72

(6,7,8)
$\underset{3}{\_}\ \underset{4}{\_}\ \underset{3}{\_}\ \underset{2}{\_} = 72 \quad + \quad \underset{5}{\_}\ \underset{4}{\_}\ \underset{3}{\_}\ \underset{2}{\_}\ \underset{1}{\_} = 120$

$72 + 120 = 192.$

JEE Mains Problems

The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is (2019 Main, 12 April I)

(a) $2^{20} - 1$ (b) 2^{21} (c) 2^{20} (d) $2^{20} + 1$

$\underbrace{{}^{21}C_0 + {}^{21}C_1 \cdots {}^{21}C_{10}}_{S} + \underbrace{{}^{21}C_{11} + {}^{21}C_{12} \cdots {}^{21}C_{21}}_{S} = 2^{21}$

$2S = 2^{21}$

$S = 2^{20}$

31 → 10 alike.
31 → 21 different

Alike (10)	Diff. (21)		
0	10	=	$1 \times {}^{21}C_{10}$
1	9	=	$1 \times {}^{21}C_9$
2	8	=	$1 \times {}^{21}C_8$
⋮	⋮		⋮
10	0		$1 \times {}^{21}C_0$

Ans.: ${}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 \cdots {}^{21}C_{10} = S = 2^{20}$.

JEE Mains Problems

$x^2+4-4x = \frac{x^2+x+198}{2}$ $\quad 19\times 10$

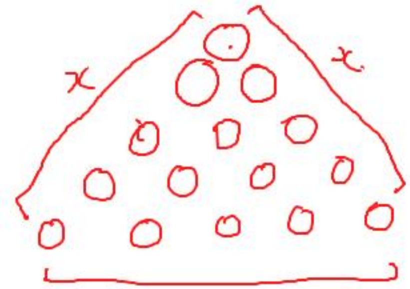
$x^2-9x-190=0$

$x^2-19x+10x-190=0 \quad x=\cancel{-10}, 19$

Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then, the number of balls used to form the equilateral triangle is 'n'

(2019 Main, 9 April II)

(a) 262 (b) 190 (c) 225 (d) 157



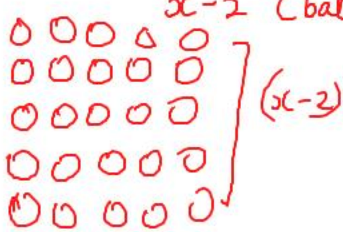
x balls.

$1+2+3+4 - - x = \boxed{\frac{x(x+1)}{2} = n}$

$n = \frac{19\times 20}{2}$
$= 190$

New Situation = $n+99$. (total balls)

$x-2$ (balls on each side)



Total balls = $(x-2)(x-2) = n+99$.

$(x-2)^2 = n+99$

$(x-2)^2 = \frac{x(x+1)}{2} + 99 \Rightarrow x = 19$

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JEE Mains Problems

There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is **(2019 Main, 12 Jan II)**

(a) 12 (b) 11 (c) 9 (d) 7

Men − Men $= 2 \times {}^{m}C_2$

Men − Women $= 2 \times {}^{m}C_1 \times {}^{2}C_1$

$2 \times {}^{m}C_2 - 2 \times {}^{m}C_1 \times {}^{2}C_1 = 84$

$\cancel{2} \times \frac{m(m-1)}{\cancel{2}} - 2m \times 2 = 84$

$m^2 - m - 4m = 84$

$m^2 - 5m - 84 = 0$

$m^2 - 12m + 7m - 84 = 0$

$(m-12)(m+7) = 0$

$m = 12, \cancel{-7}$

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JEE Mains Problems

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Men – Men $= 2 \times {}^mC_2$

Men – Women $= 2 \times {}^mC_1 \times {}^2C_1$

$2 \times {}^mC_2 - 2 \times {}^mC_1 \times {}^2C_1 = 84$

$\cancel{2} \times \frac{m(m-1)}{\cancel{2}} - 2m \times 2 = 84$

$m^2 - m - 4m = 84$

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JEE Mains Problems

A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then, the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is **(2017 Main)**

(a) 485 (b) 468 (c) 469 (d) 484

X: 3M, 4F

Y: 4M, 3F

X Y

X Y

$(3M)(3F) + (2M\,1F)(1M\,2F) + (1M\,2F)(2M\,1F) + (3F)(3M)$

$^3C_3 \times {}^3C_3 + {}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2 + {}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 + {}^4C_3 \times {}^4C_3$

$1 + 3\times4\times4\times3 + 3\times6\times6\times3 + 4\times4$

$1 + 144 + 324 + 16 = 485$

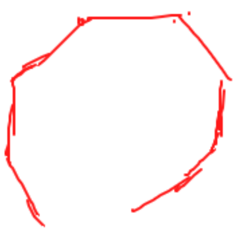
JEE Mains Problems

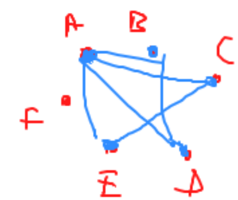
$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\boxed{{}^{n+1}C_r - {}^nC_r = {}^nC_{r-1}}$

Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is **(2013 Main)**

(a) 7 (b) 5 (c) 10 (d) 8





$^6C_3$

n vertices

Total triangles that can be formed = $^nC_3$

$T_n = {}^nC_3$, $T_{n+1} = {}^{n+1}C_3$

$T_{n+1} - T_n = 10$

$^{n+1}C_3 - {}^nC_3 = 10$

$^nC_2 = 10 \Rightarrow \frac{n(n-1)}{2} = 10 \Rightarrow n^2 - n - 20 = 0$

$(n-5)(n+4) = 0$

$n = 5, -4$

JEE Mains Problems

The number of 6 digits numbers that can be formed using the digits 0, 1, 2,5, 7 and 9 which are divisible by 11 and no digit is repeated, is **(2019 Main, 10 April I)**

(a) 60 (b) 72

(c) 48 (d) 36

6, 15

7, 17

$0+1+2+5+7+9 = 24$

$a+c+e = 12$

$b+d+f = 12$

abcdef

div of 11 : $(a+c+e) - (b+d+f)$ should be div by 11

Case-1 (0,5,7), (1,2,9)

odd even

2 3 2 2 1 1 = 24.

Case-2 (0,5,7) (1,2,9)

even odd

3 3 2 2 1 1 = 36

24+36 = 60

JEE Mains Problems

The number of 6 digits numbers that can be formed using the digits 0, 1, 2,5, 7 and 9 which are divisible by 11 and no digit is repeated, is **(2019 Main, 10 April I)**

(a) 60 (b) 72

(c) 48 (d) 36

abcdef

div of 11 : $(a+c+e) - (b+d+f)$ should be div by 11

$0+1+2+5+7+9 = 24$

6, 18

7, 17

$a+c+e = 12$

$b+d+f = 12$

Case-1 (0,5,7) odd , (1,2,9) even.

2 3 2 2 1 1 = 24.

Case-2 (0,5,7) even (1,2,9) odd.

3 3 2 2 1 1 = 36

$24 + 36 = 60$

JEE Mains Problems

8 M   5F

A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with atleast 3 females, then (2019 Main, 9 April I)

(a) $m = n = 68$
(b) $m + n = 68$
(c) $m = n = 78$
(d) $n = m - 8$

At least 6 males : 6M 5F + 7M 4F + 8M 3F

$m = {}^8C_6 \times {}^5C_5 + {}^8C_7 \times {}^5C_4 + {}^8C_8 \times {}^5C_3$

$= 28 + 40 + 10 = 78$

At least 3 females : 8M 3F + 7M 4F + 6M 5F

$n = 78$

JEE Mains Problems

Consider three boxes, each containing 10 balls labelled 1, 2, ..., 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the ith box, $(i = 1, 2, 3)$. Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is **(2019 Main, 12 Jan I)**

(a) 82 (b) 120 (c) 240 (d) 164

$${}^{10}C_3 = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

$n_1 < n_2 < n_3$

→ $\boxed{1, 5, 7}$ ✓

→ 4, 8, 9 ✓

→ 6, 7, 8 ✓

JEE Mains Problems

The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repitition of digits allowed) is equal to **(2019 Main, 9 Jan II)**

(a) 374 (b) 375 (c) 372 (d) 250

$\underset{3}{\underline{0,1,3}}\ \underset{5}{\underline{\ \ }}\ \underset{5}{\underline{\ \ }}\ \underset{5}{\underline{\ \ }} = 3\times5\times5\times5 = 375$

$375-1 = 374$

M-2

$\underset{2}{\underline{1,3}}\ \underset{5}{\underline{\ \ }}\ \underset{5}{\underline{\ \ }}\ \underset{5}{\underline{\ \ }} = 250$

$\underset{4}{\underline{1,3,7,9}}\ \underset{5}{\underline{\ \ }}\ \underset{5}{\underline{\ \ }} = 100$

$\underset{4}{\underline{1,3,7,9}}\ \underset{5}{\underline{\ \ }} = 20$

$\underline{1,3,7,9} = 4$

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JEE Mains Problems

Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is **(2019 Main, 9 Jan I)**

(a) 350 (b) 500 (c) 200 (d) 300

7 Boys - 5 Girls

5 x y

None of x & y. + x Present y Absent + x Absent y Present

$${}^5C_3 \times {}^5C_2 + {}^1C_1 \times {}^5C_2 \times {}^5C_2 + {}^1C_1 \times {}^5C_2 \times {}^5C_2$$

$100 + 100 + 100 = 300$

JEE Mains Problems

If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary, then the position of the word SMALL is **(2016 Main)**

(a) 46th (b) 59th
(c) 52nd (d) 58th

A L L M S

Total words in Dictionary = $\frac{5!}{2!} = \frac{120}{2} = 60$

A _ _ _ _ $\frac{4!}{2!} = 12$

L $\underbrace{\_\ \_\ \_\ \_}_{A, L, M, S}$ $4! = 24$

M $\underbrace{\_\ \_\ \_\ \_}_{A, L, L, S}$ $\frac{4!}{2!} = 12$

S A $\underbrace{\_\ \_\ \_}_{L, L, M}$ $= \frac{3!}{2!} = 3$

S L $\underbrace{\_\ \_\ \_}_{A, L, M}$ $= 3! = 6$

S M A L L = 1

58th word

JEE Mains Problems

A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to **(2019 Main, 12 April II)**

(a) 28 (b) 27

(c) 25 (d) 24

2B 1G + 1B 2G

$${}^5C_2 \times {}^nC_1 + {}^5C_1 \times {}^nC_2 = 1750$$

$$\cancel{10}^{\,2}\, n + \cancel{5}\,\frac{n(n-1)}{2} = \cancel{1750}_{\,350}$$

$$4n + n^2 - n = 700$$

$$n^2 + 3n - 700 = 0$$

$$n^2 + 28n - 25n - 700 = 0$$

$$(n-25)(n+28) = 0$$

$$n = 25,\ \cancel{-28}$$

JEE Mains Problems

Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is

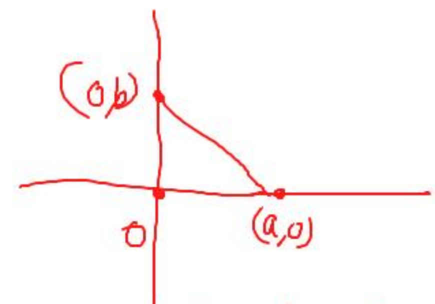
(2019 Main, 9 Jan II)

(a) 36 ✓ (b) 32

(c) 18 (d) 9

9 triangles in 1st quad.

Ans: $9 \times 4 = 36$



$\frac{1}{2} \times a \times b = 50$

$a \times b = 100$

1×100

2×50

4×25

5×20

10×10 100×1

20×5

25×4

50×2

JEE Mains Problems

From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf, so that the dictionary is always in the middle. The number of such arrangements is

(2018 Main)

(a) atleast 1000
(b) less than 500
(c) atleast 500 but less than 750
(d) atleast 750 but less than 1000

6 Novels 3 Dict.

$^6C_4 \times {}^3C_1 \times 4\times3\times2\times1$

$\frac{6\times5}{\cancel{2\times1}} \times 3\times4\times3\times2$

4 3 1 2 1

$30\times36 = 1080$

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JEE Mains Problems

The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball, is **(2012)**

(a) 75 (b) 150 (c) 210 (d) 243

$\boxed{B_1 \ B_2 \ B_3 \ B_4 \ B_5}$

X Y Z

$3,1,1 \quad + \quad 2,2,1$

$$\left(\frac{5!}{3!\,1!\,1!\,2!} + \frac{5!}{2!\,2!\,1!\,2!}\right) \times 3!$$

$$\left(\frac{120}{6\times 2} + \frac{120}{2\times2\times2}\right) \times 6$$

$$(10 + 15) \times 6$$

$$25 \times 6 = 150$$

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JEE Mains Problems

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(a) 75 (b) 150 (c) 210 (d) 243

$\boxed{B_1\ B_2\ B_3\ B_4\ B_5}$

X Y Z

$(3,1,1) + (2,2,1)$

$\left(\frac{5!}{3!1!1!2!} + \frac{5!}{2!2!1!2!}\right) \times 3!$

$\left(\frac{120}{6\times2} + \frac{120}{2\times2\times2}\right) \times 6$

$(10+15) \times 6$

$25 \times 6 = 150$

$3!$

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JEE Mains Problems

The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently, is (2002, 1M)

(a) 40 (b) 60 (c) 80 (d) 100

B NN AAA

M-1 Complimentary Approach

Total ways − (those ways which are not required)
(words in which N & N are together)

B $\boxed{NN}$ A A A

$\frac{6!}{2!3!} - \frac{5!}{3!}$

$\frac{\cancel{720}^{120}}{2\times 6} - \frac{120}{6}$

$60 - 20 = 40$

M-2 Fill N, N in the Gaps of B, A, A, A

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$\frac{4!}{3!} \times {}^5C_2 \times \frac{2!}{2!} = 4 \times 10 \times 1 = 40$

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