JEE and NEET CRASH COURSE

PHYSICS

SAFALTA.COM
An Initiative by अमर उजाला

Problem Solving Class

(Rotational Motion)

By, Ritesh Agarwal, B. Tech. IIT Bombay

PQ8Q20

Moment of inertia of a thin rod of mass m and length l about an axis passing through a point $l/4$ from one end and perpendicular to the rod is

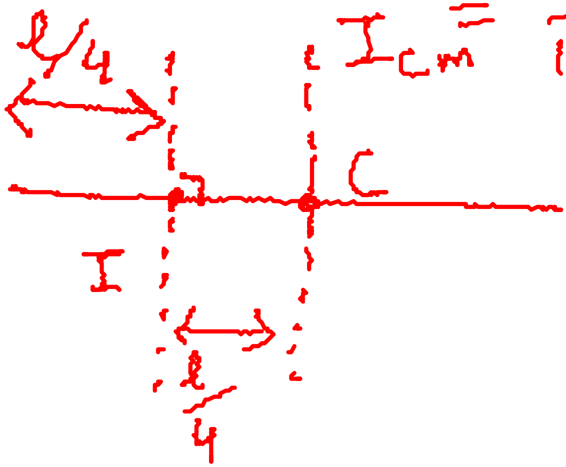
(A) $\frac{ml^2}{12}$

(B) $\frac{ml^2}{3}$

(C) $\frac{7ml^2}{48}$ ✓

(D) $\frac{ml^2}{9}$

$I_{cm} = \frac{ml^2}{12}$



$$I = I_{cm} + m\left(\frac{l}{4}\right)^2$$

$$= \frac{ml^2}{12} + \frac{ml^2}{16} = \frac{4+3}{48} ml^2$$

$$= \frac{7}{48} ml^2$$

PQ8S20

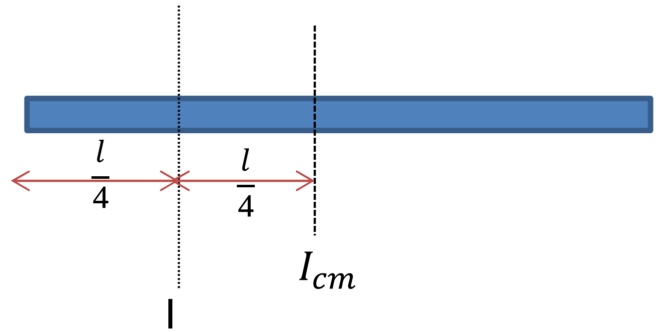
Ans [C]

$$I = I_{cm} + Md^2$$

As we know I_{cm} , we can use Parallel axis theorem.

$$\Rightarrow I = \frac{ml^2}{12} + m\left(\frac{l}{2} - \frac{l}{4}\right)^2$$

$$\Rightarrow I = ml^2\left(\frac{1}{12} + \frac{1}{16}\right) = \frac{7}{48}ml^2$$



PQ8Q7

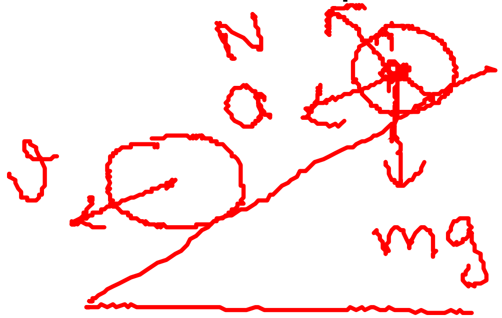
A body of mass m and radius r is released from rest along a smooth inclined plane of angle of inclination θ. The angular momentum of the body about the instantaneous point of contact after a time t from the instant of release is equal to

(A) $mgrt\cos\theta$

(B) $mgrt\sin\theta$

(C) $\left(\frac{3}{2}\right) mgrt\sin\theta$

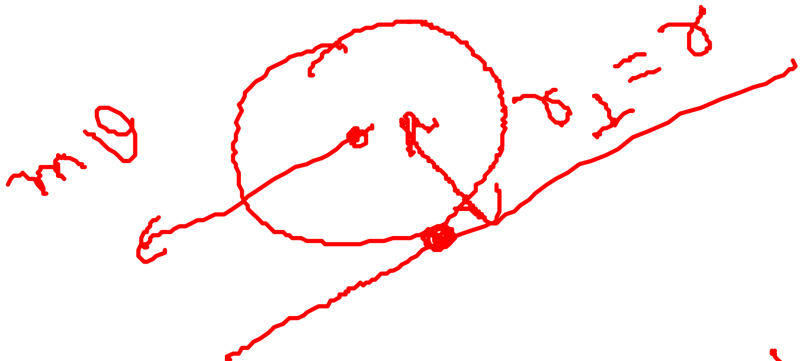
(D) None of these

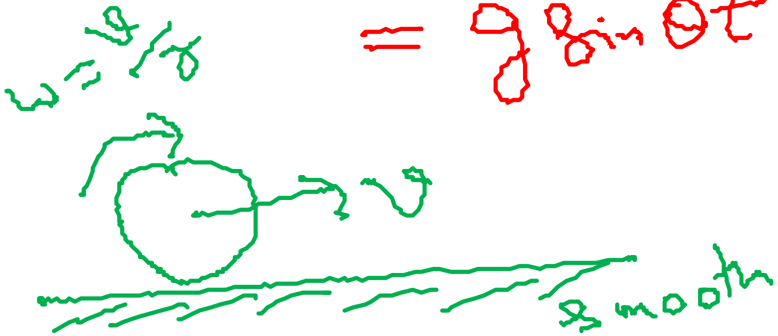


$a = g\sin\theta$

$v = u + at$

$= g\sin\theta\, t$





$L = r_\perp p = mg\sin\theta\, t \cdot r$

PQ8S7

Ans [B]

Since the surface is frictionless, the body does not rotate about its centre of mass.

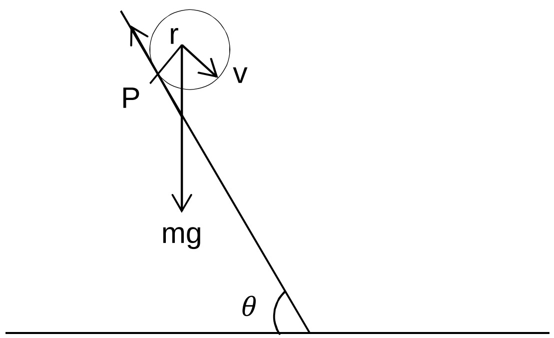
Only it slides with certain velocity v parallel to the surface

$\therefore \omega = 0$ and $v = (g \sin\theta)\, t$

$\Rightarrow$ The angular momentum after a time t is given by

L = mvr $\Rightarrow$ L = mvr

$\Rightarrow$ L = mg rt $\sin\theta$



PQ8Q443

$2/5 = 0.4$ $\quad 2/3 = 0.67$ $\quad 1$

A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling)

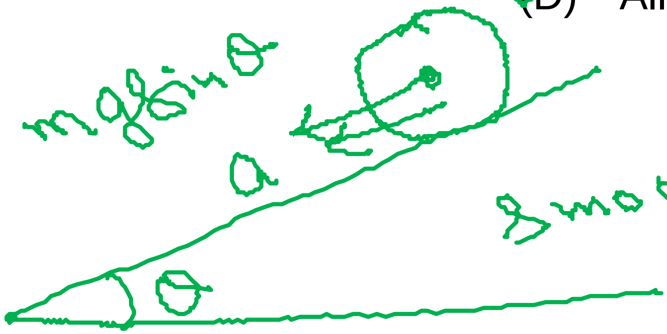
AIEEE - 2002

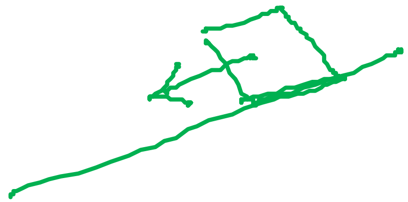
(A) Solid sphere

(B) Hollow sphere

(C) Ring

(D) All same





$a = \frac{F}{m}$

$a = g\sin\theta$

$$a = \frac{g\sin\theta}{1 + \frac{I_{cm}}{mR^2}}$$

Pure rolling

Denominator = min.

$\frac{I_{cm}}{mR^2}$ = min.

$\frac{2}{5}$;

PQ8S443

Ans [D]

The bodies slide along inclined plane. They do not roll. Acceleration for each body down the plane = $g\sin\theta$. It is the same for each body.

Acceleration down the plane(frictionless) due to gravitational force is independent of the mass and shape of the object

PQ8Q444

Moment of inertia of a circular wire of mass M and radius R about its diameter is

AIEEE - 2002

(A) $MR^2/2$ (B) MR^2

(C) $2MR^2$ (D) $MR^2/4$

$I = MR^2$

⊥ to Diameter $\to I_1 + I_2 = I$

By symmetry $I_1 = I_2 = \frac{I}{2}$

$= \frac{MR^2}{2}$

Ans.

PQ8S444

Ans [A]

A circular wire behaves like a ring M.I. about its diameter $= \dfrac{MR^2}{2}$

We know that,
By perpendicular axis theorem that states,
$I_x + I_y = I_z$
And ,$2I_x = 2I_y = I_z = Mr^2$ $(I_x = I_y,$ rotational symmetry)

PQ8Q15

A constant external torque τ acts for a very brief period Δt on a rotating system having moment of inertia I.

(A) The angular momentum of the system will change by $\tau\,\Delta t$. ✓

(B) The angular velocity of the system will change by $\frac{(\tau\,\Delta t)}{I}$ ✓

(C) If the system was initially at rest, it will acquire rotational kinetic energy $\frac{(\tau\,\Delta t)^2}{2I}$ ✓

(D) All of the above ✓

$\tau \Delta t = \Delta L \Rightarrow L = \tau \Delta t \Rightarrow I\omega = \tau \Delta t$

$\omega = \frac{\tau \Delta t}{I}$

$R.K.E. = \frac{1}{2} I \omega^2$

$= \frac{1}{2} I \frac{\tau^2 \Delta t^2}{I^2} = \frac{(\tau \Delta t)^2}{2I}$

PQ8S15

Ans [D]

Let L = angular momentum

$\Rightarrow dL = \tau dt$

For constant torque

$\Delta L = \tau \, \Delta T = I \Delta\omega$ (if $\omega_1 = 0$)

Rotational kinetic energy

$$= \frac{1}{2} I \omega^2$$

$$= \frac{(\Delta L)^2}{2I}$$

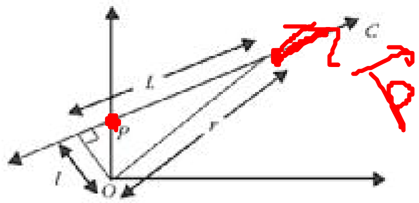
Using

$\tau = \frac{dL}{dt}$ *and*

$L = I\omega$

PQ8Q445

A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P ?



AIEEE - 2002

(A) mvL

(B) mvl

(C) mvr

(D) Zero

$L_P = 0 \because r_\perp = 0$

$L_O = \underbrace{l}_{r_\perp} \cdot \underbrace{mv}_{p}$

$r_\perp = mvl$

PQ8S445

Ans [D]

The particle moves with linear velocity v along line PC. The line of motion is through P. Hence angular momentum is zero.

L = r x p.   ,where p=mv
but here r is zero

PQ8Q446

A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius 4R is made from an iron plate of thickness t/4. Then the relation between the moment of inertia I_X and I_Y is

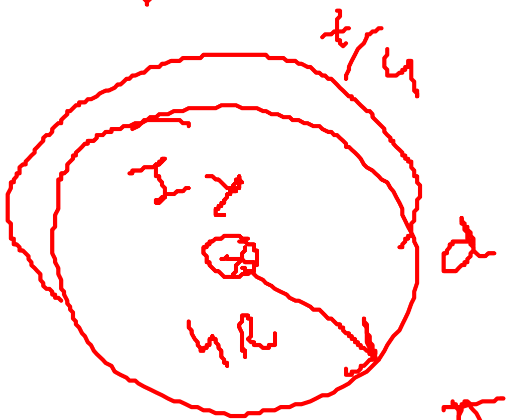
AIEEE - 2003

(A) $I_Y = 32I_X$

(B) $I_Y = 16I_X$

(C) $I_Y = I_X$

(D) $I_Y = 64I_X$



$M_1 = d\,\pi R^2 t = M$

$I_x = \frac{M_1 R^2}{2} = \frac{MR^2}{2}$

$M_2 = d \cdot \pi \times 16R^2 \frac{t}{4} = 4M$

$I_y = \frac{M_2 (4R)^2}{2} = \frac{4M \times 16R^2}{2}$

$= 64 I_x$

PQ8S446

Ans [D]

Mass of disc $X = (\pi R^2 t)\sigma$ where $\sigma =$ density

$$\therefore I_X = \frac{MR^2}{2} = \frac{(\pi R^2 t\sigma)R^2}{2} = \frac{\pi R^2 \sigma t}{2}$$

Similarly, $I_Y = \dfrac{(\text{Mass})(4R^2)^2}{2} = \dfrac{\pi(4R)^2}{2}\dfrac{t}{2}\sigma \times 16R^2$

or $I_Y = 32\pi R^2 t\sigma$

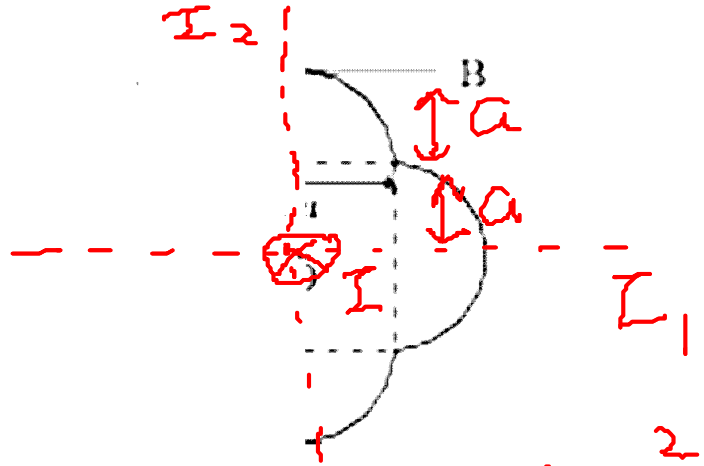
$$\therefore \frac{I_X}{I_Y} = \frac{\pi R^2 \sigma t}{2} \times \frac{1}{32\pi R^4 \sigma t} = \frac{1}{64}$$

$$\therefore I_Y = 64\, I_X$$

Here density of disc made up of same material is constant
And ,$dm = \sigma dv$
where $\sigma =$ density

PQ8Q55

A symmetric lamina of mass **M** consists of a square shape with a semicircular section over each of the edge of the square as shown in figure. The side of the square is $\mathbf{2a}$. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is $\mathbf{1.6Ma^2}$. The moment of inertia of the lamina about the tangent AB in the plane of the lamina is _____.



(A) $0.8Ma^2$ (B) $4.88Ma^2$

(C) $4.008Ma^2$ (D) $4.8Ma^2$ ✓

$I = 1.6Ma^2$

$I_1 + I_2 = I$

$I_1 = I_2 = \frac{I}{2} = 0.8Ma^2$

$I_{AB} = I_1 + M(2a)^2$

$= 0.8Ma^2 + 4Ma^2$

$= 4.8Ma^2$

PQ8S55

Ans [D]

Let ZZ' be an axis perpendicular to the lamina and passing through its centre of mass O.

Given $I_{ZZ'} = 1.6Ma^2$.

$I_{ZZ'} = I_{XX'} + I_{YY'}$ by perpendicular axis theorem as body is 2D.

By symmetry, $I_{XX'} = I_{YY'}$.

$$\Rightarrow \quad I_{XX'} = \frac{1}{2} I_{ZZ'} = 0.8Ma^2$$

$$I_{AB} = 0.8Ma^2 + M(2a)^2 = 4.8Ma^2$$

$I_{AB} = I_{XX'} + Md^2$ by parallel axis theorem

PQ8Q447

A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is

AIEEE - 2003

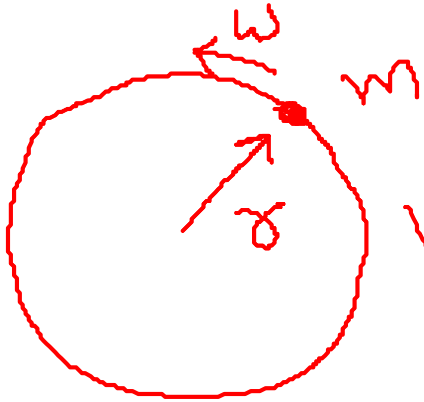
$$L' = I'\omega' = m r'^2 \omega' = m \frac{r^2}{8} \cdot 2\omega$$

$$= m r'$$

$$= L/4$$

(A) L /4 ✓ (B) 2L

(C) 4L (D) L /2



$$v = \omega r$$

$$L = I\omega = m r^2 \omega$$

$$\omega' = 2\omega$$

$$K' = \frac{1}{2} K$$

$$K.E = \frac{1}{2} m (\omega r)^2 = \frac{1}{2} m \omega^2 r^2$$

$$\frac{1}{2} m \omega'^2 r'^2 = \frac{1}{2} \cdot \frac{1}{2} m \omega^2 r^2$$

$$4 \omega^2 r'^2 = \frac{1}{2} \omega^2 r^2$$

$$\boxed{r' = \frac{r}{2\sqrt{2}}}$$

PQ8S447

Ans [A]

Angular momentum $L = I\omega$

Rotational kinetic energy $(K) = \frac{1}{2} I\omega^2$

$$\therefore \frac{L}{K} = \frac{I\omega \times 2}{I\omega^2} = \frac{2}{\omega} \Rightarrow L = \frac{2K}{\omega}$$

$$\text{or } \frac{L_1}{L_2} = \frac{K_1}{K_2} \times \frac{\omega_2}{\omega_1} = 2 \times 2 = 4$$

$$\therefore L_2 = \frac{L_1}{4} = \frac{L}{4}$$

The angular momentum and rotational the kinetic energy of a moving body are the body's properties which is very much related to their angular velocity.

PQ8Q448

Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{T}$ be the torque of this force about the origin. Then

AIEEE - 2003

(A) $\vec{r} \cdot \vec{T} = 0$ and $\vec{F} \cdot \vec{T} \neq 0$

(B) $\vec{r} \cdot \vec{T} \neq 0$ and $\vec{F} \cdot \vec{T} = 0$

(C) $\vec{r} \cdot \vec{T} \neq 0$ and $\vec{F} \cdot \vec{T} \neq 0$

(D) $\vec{r} \cdot \vec{T} = 0$ and $\vec{F} \cdot \vec{T} = 0$ ✓

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\vec{A} \cdot \vec{B} = 0 \Rightarrow \theta = 90^\circ$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} \perp \vec{r}$ and $\vec{\tau} \times \vec{F}$

$\vec{\tau} \cdot \vec{r} = 0$

$\vec{\tau} \cdot \vec{F} = 0$

PQ8S448

Ans [D]

$\because \ \vec{T} = \vec{r} \times \vec{F}$

$\therefore \ \vec{r} \cdot \vec{T} = \vec{r} \cdot \left(\vec{r} \times \vec{F}\right) = 0$

Also $\vec{F} \cdot \vec{T} = \vec{F} \cdot \left(\vec{r} \times \vec{F}\right) = 0$

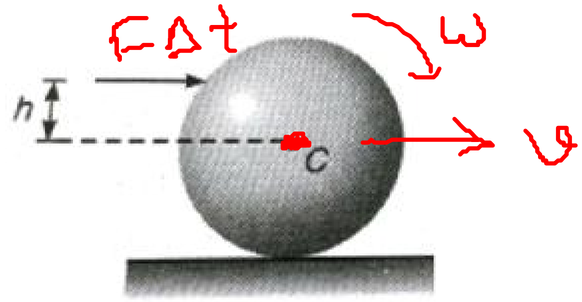
Torque is perpendicular to both r and F by definition of cross product.
And,dot product of two perpendicular vector is zero (i.e. r and T)

PQ8Q68

A solid sphere of mass M and radius R is hit by a cue at a height h above the center C. For what value of h the sphere will roll without slipping?

(A) $\frac{2}{5}R$ ✓ (B) $\frac{2}{3}R$

(C) $\frac{3}{5}R$ (D) $\frac{1}{5}R$



Linear Impulse $= \Delta\vec{p}$ — ①

$F\Delta t = mv$

Angular Impulse $= \Delta\vec{L}$ — ②

$F \cdot h \cdot \Delta t = \frac{2}{5} m R^2 \cdot \omega$

$v = \omega R$

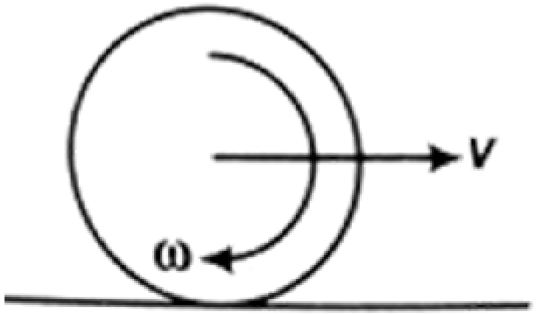
② ÷ ①

$h = \frac{\frac{2}{5}\cancel{m}\cancel{\omega} R^{\cancel{2}}}{\cancel{m}\cancel{\omega}\cancel{R}} = \frac{2}{5}R$

PQ8S68

Ans [A]

For rolling without sliding



For pure rolling:

$$v_{CM} = I\omega$$

$$v = \frac{J}{m}$$

$$v = R\omega = R = \left(\frac{\text{angular impulse}}{I}\right)$$

$$\therefore \left(\frac{J}{M}\right) = R\left(\frac{Jh}{\frac{2}{5}MR^2}\right)$$

$$\therefore h = \frac{2}{5}R$$

PQ8Q449

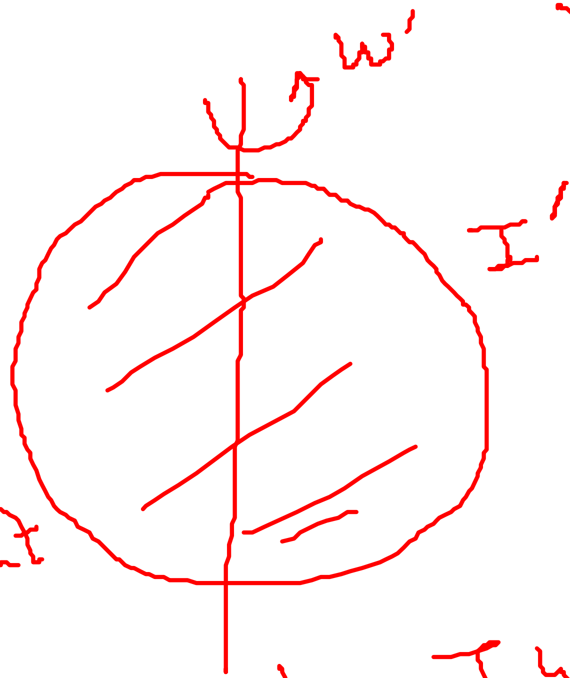
A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected? **AIEEE - 2004**

(A) Moment of inertia

(B) Angular momentum

(C) Angular velocity

(D) Rotational kinetic energy



$\vec{\tau}_{ext} = 0$

$\vec{L}$ = constant

$L = I\omega$; R.K.E. $= \frac{L^2}{2I}$

PQ8S449

Ans [B]

Free space implies that no external torque is operating on the sphere. Internal changes are responsible for increase in radius of sphere. Here the law of conservation of angular momentum applies to the system.

Law of conservation of angular momentum states that when net external torque acting on a system about a given axis is. zero , the total angular momentum of the system about that axis remains constant.
Mathematically ,
$L_i = L_f$

PQ8Q450

One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that where d_A and d_B are their densities.

AIEEE - 2004

(A) $I_A = I_B$

(B) $I_A > I_B$

(C) $I_A < I_B$

(D) $I_A/I_B = d_A/d_B$

$$I_A = \frac{2}{5} MR^2 = 0.4 MR^2$$

$$I_B = \frac{2}{3} MR^2 = 0.67 MR^2$$

PQ8S450

Ans [C]

For solid sphere, $I_A = \frac{2}{5} MR^2$

For hollow sphere, $I_B = \frac{2}{3} MR^2$

$$\therefore \frac{I_A}{I_B} = \frac{2MR^2}{5} \times \frac{3}{2MR^2} = \frac{3}{5}$$

or $I_A < I_B$.

Moment of inertia is tendency of the body to resist any change in its rotational motion. Here,hollow sphere has greater such tendency than solid sphere.

PQ8Q451

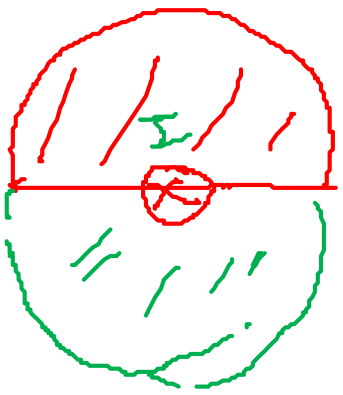
The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the center is **AIEEE - 2005**

(A) Mr^2

(B) $\frac{1}{2}Mr^2$

(C) $\frac{1}{4}Mr^2$

(D) $\frac{2}{5}Mr^2$



M, r

M, r

$I = \frac{(2M)r^2}{2} \times \frac{1}{2}$

$= \frac{Mr^2}{2}$

M

r

$I = \frac{Mr^2}{2}$

$\frac{\cancel{2}Mr^2}{\cancel{2}}$

PQ8S451

Ans [B]

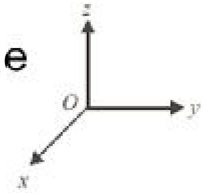
$$I = \frac{(\text{Mass of semicircular disc}) \times r^2}{2}$$

or $I = \dfrac{Mr^2}{2}$.

$I = mr^2/2$,For uniformaly distributed mass .
in case of disc mass is uniformly distributed at constant distance r about axis passing through its centre and perpendicular to its plane.

PQ8Q455

A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1, –1) is



AIEEE - 2006

(A) $-F(\hat{i}-\hat{j})$

(B) $F(\hat{i}-\hat{j})$

(C) $-F(\hat{i}+\hat{j})$

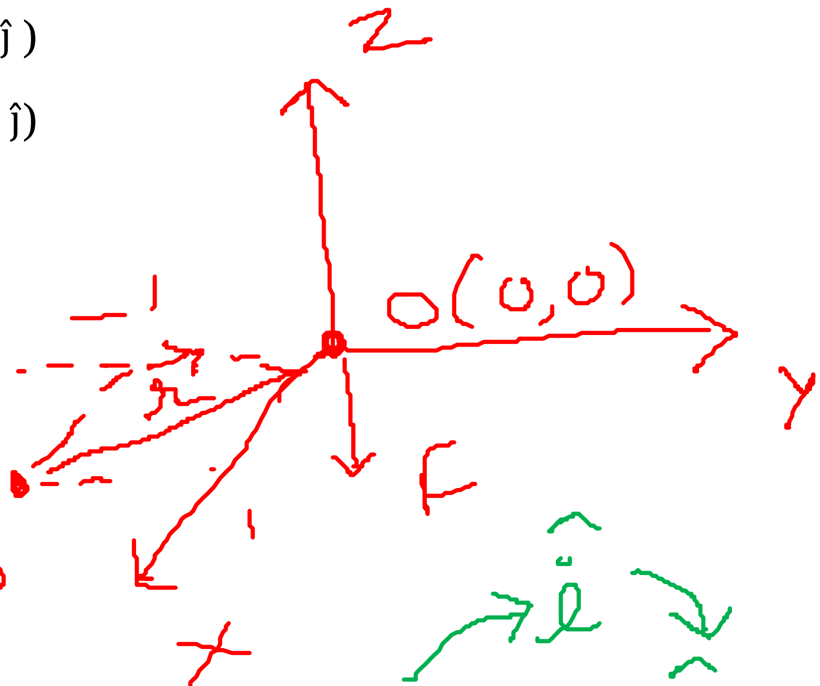
(D) $F(\hat{i}+\hat{j})$ ✓

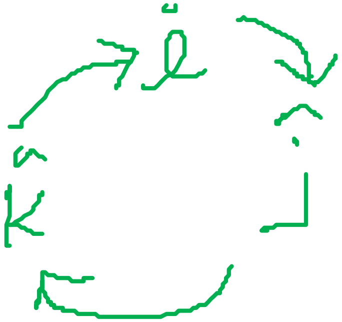
$\vec{F} = -F\hat{k}$

$\vec{r} = (0-1)\hat{i} + (0+1)\hat{j}$

$= -\hat{i} + \hat{j}$

$\vec{\tau} = \vec{r} \times \vec{F} = F\hat{j} + F\hat{i}$





PQ8S455

Ans [D]

Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{F} = -F\hat{k}$, $\vec{r} = \hat{i} - \hat{j}$

$$\therefore \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 0 & 0 & -F \end{vmatrix}$$

$$= \hat{i}F - \hat{j}(-F) = F(\hat{i} + \hat{j}).$$

here, r $^{\rightarrow}$=position vector of point –position vector of origin

Find cross product of r $^{\rightarrow}$ and F $^{\rightarrow}$ to get torque.

PQ8Q456

A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity w. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$

AIEEE - 2006

(A) $\dfrac{\omega m}{(m + 2M)}$ ✓

(B) $\dfrac{\omega(m + 2M)}{m}$

(C) $\dfrac{\omega(m - 2M)}{(m + 2M)}$

(D) $\dfrac{\omega m}{(m + 2M)}$

$L_f = L_i$

$I_f \omega_f = I_i \omega_i = mR^2 \omega$

$(mR^2 + 2MR^2)\omega_f$

$$\boxed{\omega_f = \frac{m\omega}{m + 2M}}$$

PQ8S456

Ans [A]

Angular momentum is conserved

$\therefore L_1 = L_2$

$\therefore mR^2\omega = (mR^2 + 2MR^2)\omega' = R^2(m + 2M)\omega'$

or $\omega' = \dfrac{m\omega}{m + 2M}$.

Since no net external torque acts on the system therefore total angular momentum of the system about the rotating axis remains conserved.

PQ8Q457

Four point masses, each of value m, are placed at the corners of a square ABCD of side l. The moment of inertia of this system about an axis through A and parallel to BD is

AIEEE - 2006

(A) ml^2

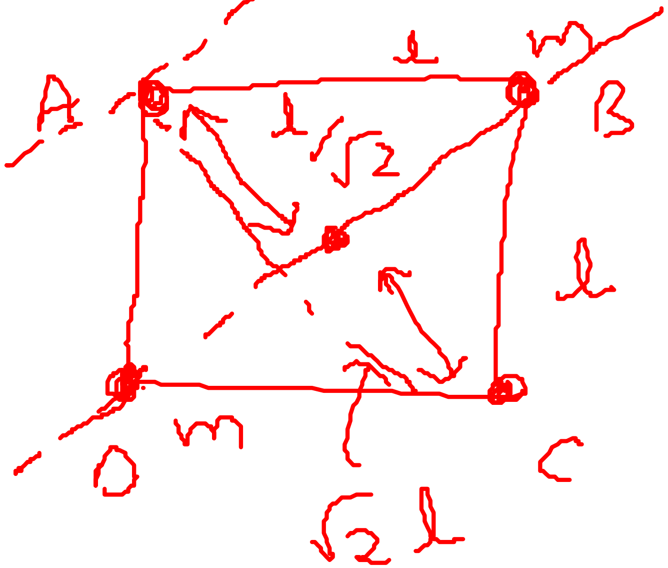
(B) $2ml^2$

(C) $\sqrt{3}ml^2$

(D) $3ml^2$

$I = ?$

I_{cm}



$I_{cm} = I_{BD} = m\left(\frac{l}{\sqrt{2}}\right)^2 \times 2 = ml^2$

$I = I_{cm} + 4m\left(\frac{l}{\sqrt{2}}\right)^2$

$= ml^2 + 4m\frac{l^2}{2} = 3ml^2$

PQ8S457

Ans [D]

$$AO\cos 45° = \frac{1}{2} \quad \therefore AO \times \frac{1}{\sqrt{2}} = \frac{l}{2}$$

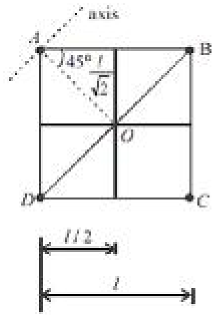
or $AO = \frac{l}{\sqrt{2}}$

$$I = I_D + I_B + I_C$$

or $I = \frac{2ml^2}{2} + m\left(\frac{2l}{\sqrt{2}}\right)^2$

$$I = \frac{2ml^2}{2} + \frac{4ml^2}{2}$$

or $I = \frac{6ml^2}{2} = 3ml^2.$



For system consisting of discrete particles
Total MI of the system $=\Sigma mr^2$
Where r is perpendicular distance of object from the axis.

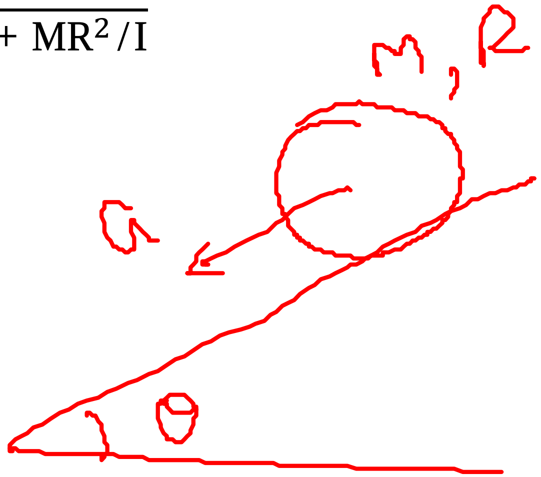
A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is **AIEEE - 2007**

(A) $\dfrac{g\sin\theta}{1 - MR^2/I}$

(B) $\dfrac{g\sin\theta}{1 + I/MR^2}$

(C) $\dfrac{g\sin\theta}{1 + MR^2/I}$

(D) $\dfrac{g\sin\theta}{1 - I/MR^2}$



$$a = \frac{g\sin\theta}{1 + I/MR^2}$$

K = Radius of gyration about axis passing through C.O.M.

$$= \frac{g\sin\theta}{1 + K^2/R^2}$$

PQ8S459

Ans [B]

Acceleration of a uniform body of radius R and mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal is given by

$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}.$$

When an object is rolling on a plane without slipping, the point of contact of the object with the plane does not move and the acceleration of centre to angular acceleration is related by a=r(angular acceleration).
Apply conservation of momentum and find the relation above mentioned.

PQ8Q460

Angular momentum of the particle rotating with a central force is constant due to

AIEEE - 2007

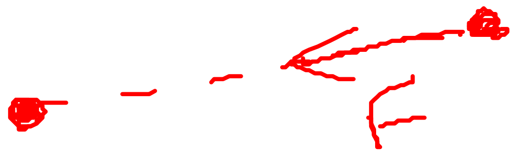
(A) Constant torque

(B) Constant force

(C) Constant linear momentum

(D) Zero torque

Force acting along the line joining the centers of two particles



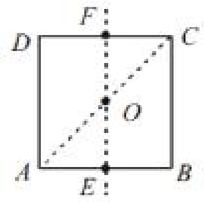
PQ8S460

Ans [D]

Central forces passes through axis of rotation so torque is zero. If no external torque is acting on a particle, the angular momentum of a particle is constant.

$\tau = r \times F$
Here r is perpendicular distance which is zero therefore torque is zero.
Thus, by law of conservation of angular momentum total momentum remains conserved.

For the given uniform square lamina ABCD, whose centre is O,



(A) $I_{AC} = \sqrt{2} I_{EF}$

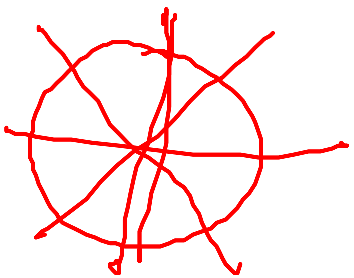
(B) $\sqrt{2} I_{AC} = I_{EF}$

(C) $I_{AD} = 3 I_{EF}$

(D) $I_{AC} = I_{EF}$

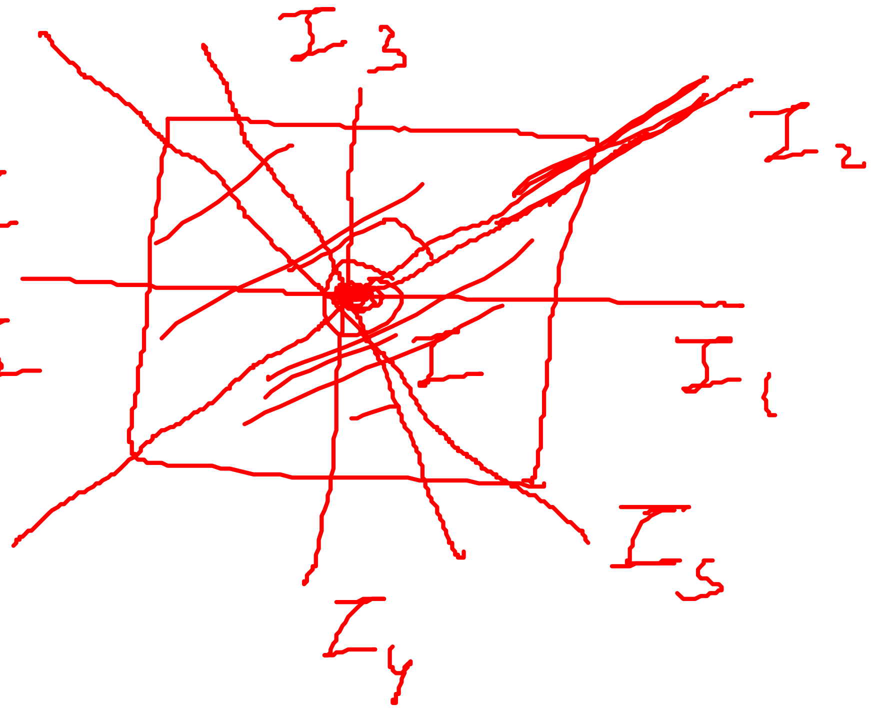
AIEEE - 2007

$I_1 = I_2 = I_3$



$I_1 + I_4 = I$

$I_2 + I_5 = I$



PQ8S461

Ans [D]

By perpendicular axes theorem,

$$I_{EF} = M\frac{a^2 + b^2}{12} = \frac{M(a^2 + a^2)}{12} = M\frac{2a^2}{12}$$

$$I_z = \frac{M(2a^2)}{12} + \frac{M(2a^2)}{12} = \frac{Ma^2}{3}$$

By perpendicular axes theorem,

$$I_{AC} + I_{BD} = I_z \Rightarrow I_{AC} = \frac{I_z}{2} = \frac{Ma^2}{6}$$

By the same theorem $I_{EF} = \frac{I_z}{2} = \frac{Ma^2}{6}$

$\therefore \; I_{AC} = I_{EF}$.

The perpendicular axis theorem states that the moment of inertia of a laminar body about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes lying in the plane of the body.

PQ8Q67

Two forces F_1 and F_2 are applied on a spool of mass M and moment of inertia/about an axis passing through its center of mass. Find the ratio $\frac{F_1}{F_2}$, so that the force of friction is zero. Given that $I < 2Mr^2$

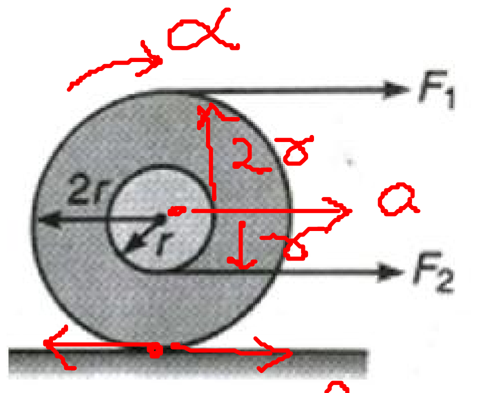
τ I

(A) $\frac{I + 2Mr^2}{Mr^2 - I}$

(B) $\frac{I + Mr^2}{Mr^2 - I}$

(C) $\frac{I + Mr^2}{4Mr^2 - I}$

(D) $\frac{I + 2Mr^2}{4Mr^2 - I}$



$\alpha \cdot 2r$

$F_{ext} = M a_{cm}$

$F_1 + F_2 = Ma$ — (1)

For no sliding

$a = 2\alpha r$

$\alpha = a/2r$

$\tau_{cm} = I_{cm}\alpha$

$F_1 \cdot 2r - F_2 \cdot r = I\alpha$

$2F_1 - F_2 = I \frac{a}{2r^2}$ — (2)

(1) ÷ (2)

$\frac{F_1 + F_2}{2F_1 - F_2} = \frac{M \cdot 2r^2}{I}$

PQ8S67

Ans [D]

$a = R\alpha$ by applied forces

For zero friction force, pure rotation

$\tau = I\alpha$

$$\therefore \frac{F_1 + F_2}{M} = 2r\left[\frac{F_1(2r) - F_2(r)}{I}\right]$$

Solving these two equations we get,

$$\frac{F_1}{F_2} = \frac{I + 2Mr^2}{4Mr^2 - I}$$

$$\frac{F_1 + F_2}{2F_1 - F_2} = \frac{2Mr^2}{I}$$

$$IF_1 + IF_2 = 4Mr^2 F_1 - 2Mr^2 F_2$$

$$F_1(I - 4Mr^2) = -F_2(I + 2Mr^2)$$

$$F_1(4Mr^2 - I) = F_2(I + 2Mr^2)$$