PHYSICS

JEE and NEET CRASH COURSE

Impulse, Momentum, Collision and Centre of Mass

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Impulse

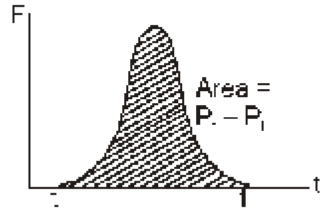
Impulse of a force $\vec{F}$ acting on a body for the time interval $t = t_1$ to $t = t_2$ is defined as :-

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}dt \quad \rightarrow \quad \vec{I} = \int \vec{F}dt = \int m\frac{d\vec{v}}{dt}dt = \int m d\vec{v}$$

$$\vec{I} = m(\vec{v}_2 - \vec{v}_1) = \Delta\vec{P} = \text{change in momentum due to force } \vec{F}$$

Also, $\vec{I}_{Res} = \int_{t_1}^{t_2} \vec{F}_{Res}\, dt = \Delta\vec{P}$ **(impulse - momentum theorem)**

Note: Impulse applied to an object in a given time interval can also be calculated from the area under force time (F-t) graph in the same time interval.



u

mg

t

$\vec{I} = \vec{F} \times \Delta t$

const.

Impulse of grav. force

$= mgt$

Momentum Conservation

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

If $\quad \vec{F}_{ext} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0 \quad ; \quad \vec{P} = \text{constant}$

When the vector sum of the external forces acting on a system is zero, the total linear momentum of the system remains constant.

$$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \ldots\ldots\ldots\ldots + \vec{P}_n = \text{constant.}$$

If $\Sigma F_x = 0$ then p_x = constant

Example

A shell is fired from a cannon with a speed of 100 m/s at an angle 60º with the horizontal (positive x-direction). At the highest point of its trajectory, the shell explodes into two equal fragments. One of the fragments moves along the negative x-direction with a speed of 50 m/s. What is the speed of the other fragment at the time of explosion.

$u_x = 100\cos 60^\circ$

$= 50$

$m/2 \quad m/2$

$50 \leftarrow \quad \rightarrow v$

$\xrightarrow{\;\;}$ +ve

$\vec{p}$/c

$\vec{p}_f = \vec{p}_i$

$\frac{m}{2}v - \frac{m}{2}\times 50$

$= m \times 50$

$v - 50 = 100$

$\boxed{v = 150 \text{ m/s}}$

Example

A block at rest explodes into three equal parts. Two parts starts moving along X and Y axes respectively with equal speeds of 10 m/s. Find the initial velocity of the third part.

Sol.

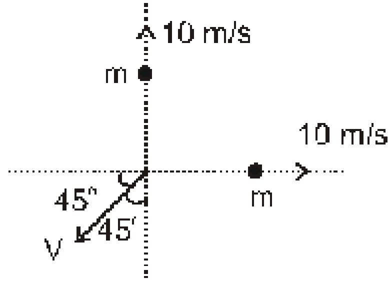
Let total mass = 3 m, initial linear momentum = 3m × 0

Let velocity of third part = $\vec{V}$

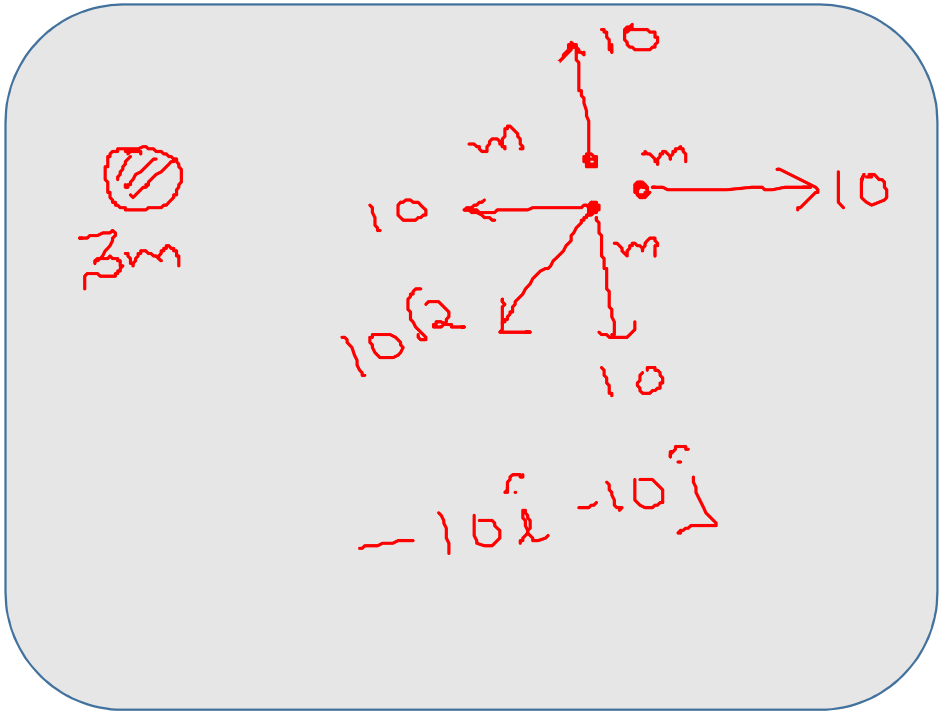
Using conservation of linear momentum :

$m \times 10\,\hat{i} + m \times 10\,\hat{j} + m\,\vec{V} = 0$

So, $\vec{V} = (-10\,\hat{i} - 10\,\hat{j})$ m/sec.



$|\vec{V}| = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2}$, making angle 135° below x-axis



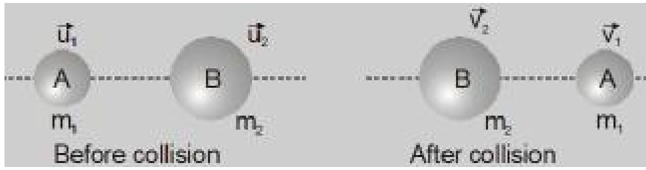
Collision

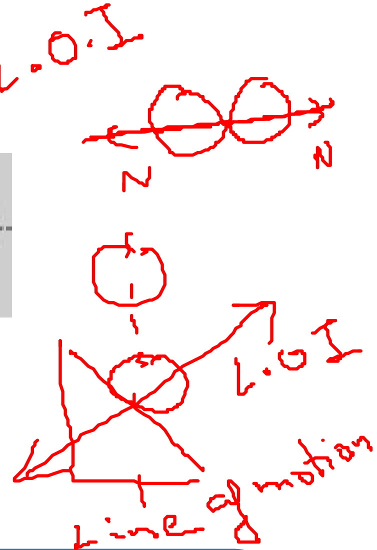
Line of Impact

A collision is said to take place when either two bodies physically collide against each other or when the path of one body is changed by the influence of the other body.
As a result of collision, the momentum and kinetic energy of the interacting bodies change.
The total momentum remains conserved in any type of collision.

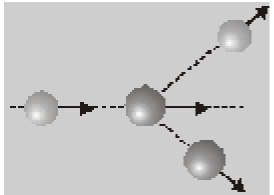
Types of collision according to the direction of collision :

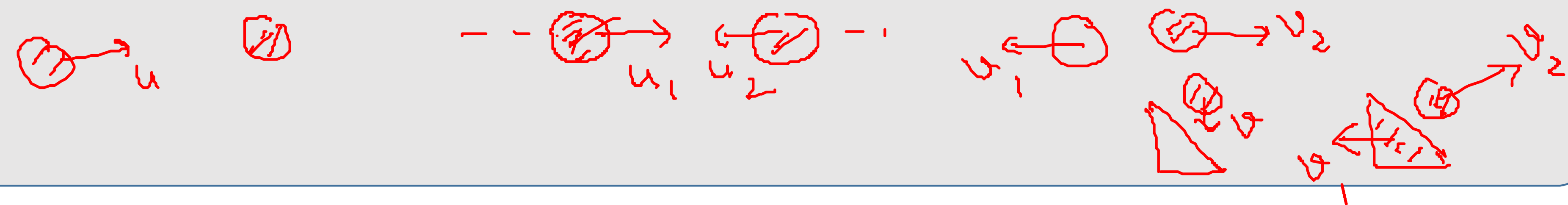
(a) Head on collision : direction of velocities of bodies is similar to the direction of collision.





(b) Oblique collision : direction of velocities of bodies is not similar to the direction of collision.





Collision

Types of collision according to the conservation law of kinetic energy :

(a) Elastic collision : kinetic energy is conserved.

$$KE_{\text{before collision}} = KE_{\text{after collision}}$$

(b) Inelastic collision : kinetic energy is not conserved. Some energy is lost in collision

$$KE_{\text{before collision}} > KE_{\text{after collision}}$$

(c) Perfect inelastic collision : Two bodies stick together after the collision. Maximum K.E. is lost by the system.

Coefficient of restitution (e)

(after collision)

$$e = \frac{\text{velocity of separation along line of impact}}{\text{velocity of approach along line of impact}}$$

(before collision)

For elastic collision

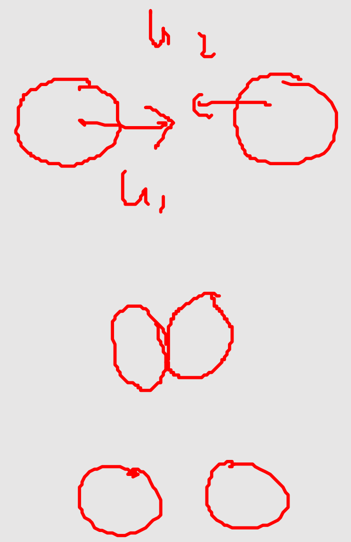
$e = 1$

For perfectly ~~elastic~~ inelastic collision

$e = 0$

For inelastic collision,

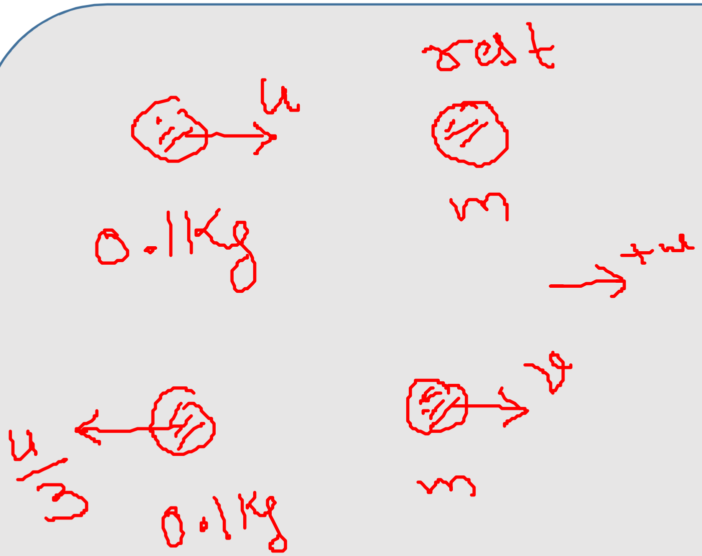
$0 < e < 1$



Example

A ball of 0.1 kg makes an elastic head on collision with a ball of unknown mass that is initially at rest. If the 0.1 kg ball rebounds at one third of its original speed. What is the mass of other ball.

Sol.



p/c

$$mv - 0.1\cdot\left(-\frac{u}{3}\right) = 0.1u$$

$$mv = 0.1 \times \frac{4u}{3} \quad -(1)$$

$$e = 1$$

$$v + \frac{u}{3} = u \Rightarrow v = \frac{2u}{3} \quad -(2)$$

from eq (1)

$$m\cancel{v} = 0.1 \times 2\cancel{v}$$

$$m = 0.2$$

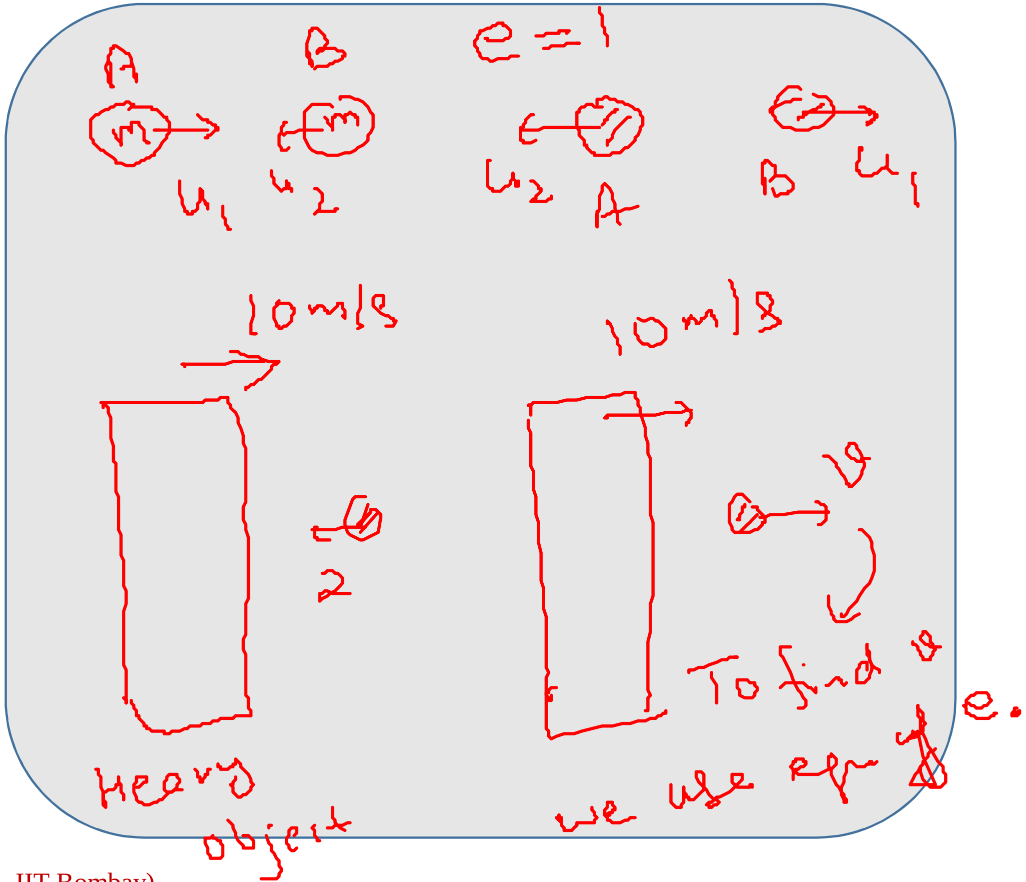
Special Cases

(a) If $m_1 = m_2 = m$ and $e = 1$

then after the collision, the bodies will exchange their velocities.

(b) If the mass of a body is negligible as compared to other

then we can neglect the change in velocity of heavier body.



Example

Two ball bearing mass 5 kg each is moving in opposite directions with equal speed 5 m/s. collides head on with each other. Find out the final velocities of the balls if collision is elastic.

Sol.

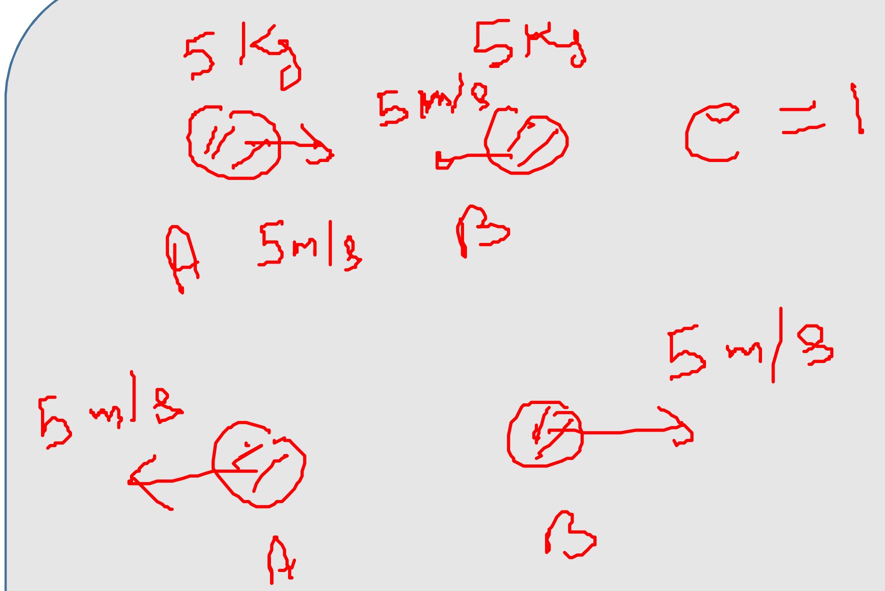
Here $m_1 = m_2 = 5\text{kg}$

$u_1 = 5 \text{ m/s} \quad u_2 = -5 \text{ m/s}$

In such type of condition velocity get interchange so

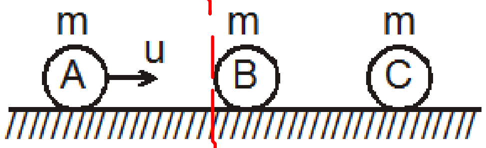
$$v_2 = u_1 = 5 \text{ m/s}$$

$$v_1 = u_2 = -5 \text{ m/s}$$



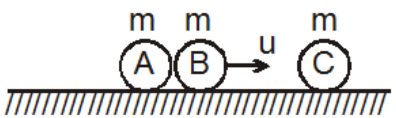
Example

Three balls A, B and C of same mass 'm' are placed on a frictionless horizontal plane in a straight line as shown. Ball A is moved with velocity u towards the middle ball B. If all the collisions are elastic then, find the final velocities of all the balls.

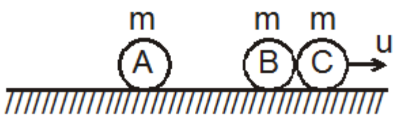


Sol.

A collides elastically with B and comes to rest but B starts moving with velocity u

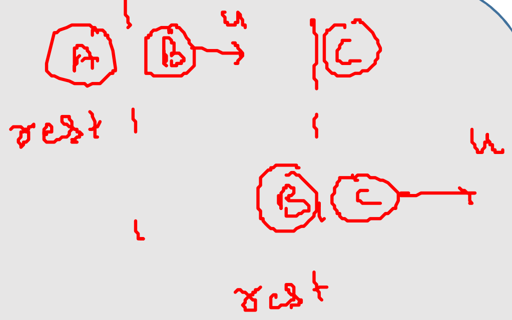


After a while B collides elastically with C and comes to rest but C starts moving with velocity u



∴ Final velocities

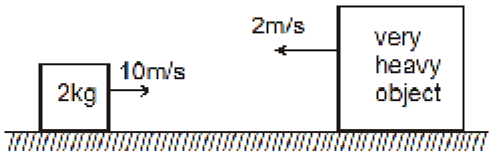
$V_A = 0$; $V_B = 0$ and $V_C = u$



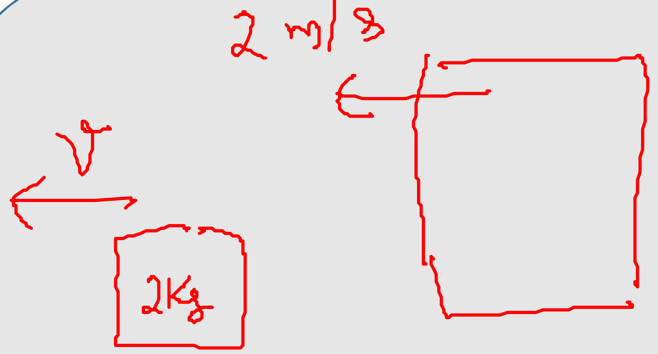
2 collision

Example

A block of mass 2 kg is pushed towards a very heavy object moving with 2 m/s closer to the block (as shown). Assuming elastic collision and frictionless surfaces, find the final velocities of the blocks.



Sol.



$$v_{sep.} = e\, v_{app.}$$

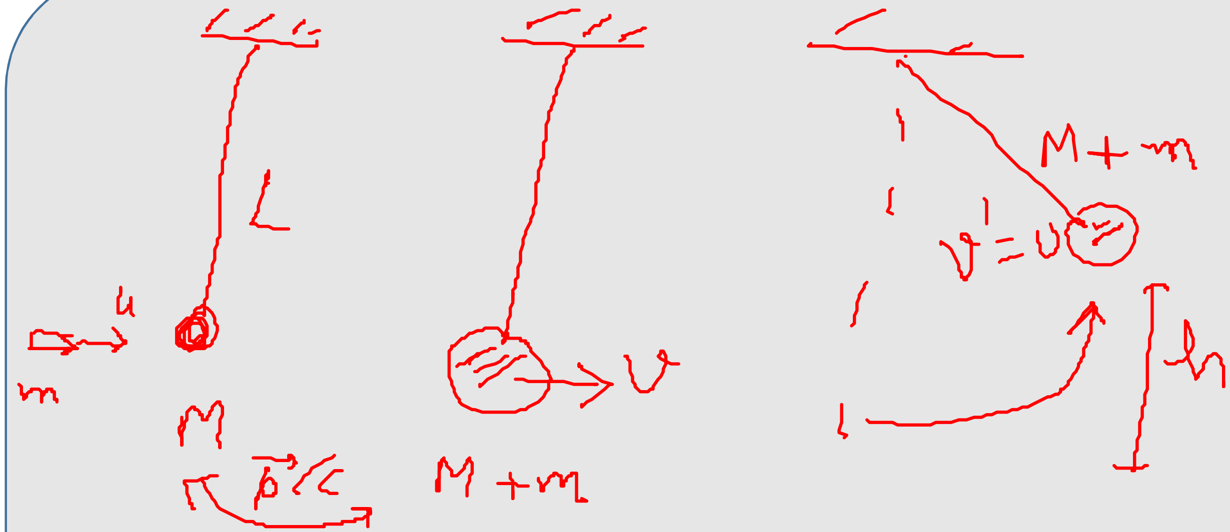
$$v - 2 = 12$$

$$\boxed{v = 14 \text{ m/s}}$$

Example

A simple pendulum of length L has a wooden bob of mass M. It is struck by a bullet of mass m moving horizontally with a speed of u. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back.

Sol.



Loss in K.E. = Gain in P.E.

$$\frac{1}{2}\cancel{(M+m)}v^2 = \cancel{(M+m)}gh$$

$$h = \frac{v^2}{2g}$$

$$= \frac{m^2u^2}{2(m+M)g}$$

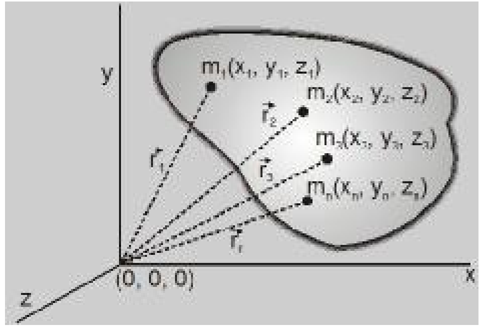
$$(M+m)v = mu \Rightarrow v = \frac{mu}{M+m}$$

Centre of Mass

It is the point at which its total mass is supposed to be concentrated.

For a discrete system of particles centre of mass is defined as

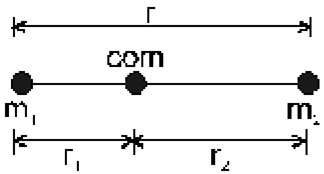
$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{1}{M}\Sigma m_i \vec{r}_i$$



$$x_{cm} = \left(\frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots\dots}{m_1 + m_2 + m_3 + \dots\dots}\right) \qquad y_{cm} = \left(\frac{m_1y_1 + m_2y_2 + m_3y_3 + \dots\dots}{m_1 + m_2 + m_3 + \dots\dots\dots}\right) \qquad z_{cm} = \left(\frac{m_1z_1 + m_2z_2 + m_3z_3 + \dots\dots}{m_1 + m_2 + m_3 + \dots\dots\dots}\right)$$

For system of two particles

$$r_1 = \left(\frac{m_2}{m_2 + m_1}\right) r \text{ and } r_2 = \left(\frac{m_1}{m_1 + m_2}\right) r$$

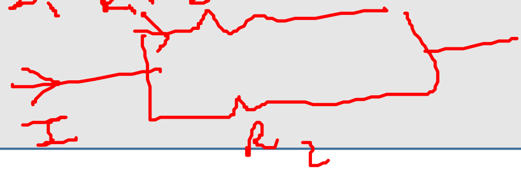


$(0,0)$ $(r,0)$

$$r_1 = \frac{m_1(0) + m_2(r)}{m_1 + m_2}$$

$$r_1 = \frac{\text{opp. mass} \times \text{dist. b/w two masses}}{\text{total mass}}$$

$$I_1 = \frac{R_2 I}{R_1 + R_2}$$



Example

The position vector of three particles of masses m_1 = 1 kg, m_2 = 2 kg and m_3 = 3 kg are $\vec{r}_1 = (\hat{i} + 4\hat{j} + \hat{k})m$, $\vec{r}_2 = (\hat{i} + \hat{j} + \hat{k})m$ and $\vec{r}_3 = (2\hat{i} - \hat{j} - 2\hat{k})m$ respectively. Find the position vector of their center of mass.

Sol.

The position vector of COM of the three particles will be given by

$$\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

Substituting the values, we get

$$\vec{r}_{COM} = \frac{(1)(\hat{i} + 4\hat{j} + \hat{k}) + (2)(\hat{i} + \hat{j} + \hat{k}) + (3)(2\hat{i} - \hat{j} - 2\hat{k})}{1 + 2 + 3}$$

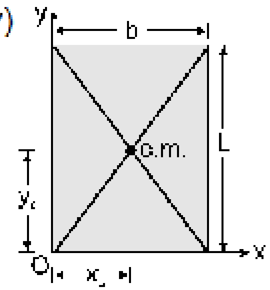
$$= \frac{1}{2}(3\hat{i} + \hat{j} - \hat{k})m$$

Center of mass of some common systems

Rectangular plate (By symmetry)

$$x_c = \frac{b}{2}$$

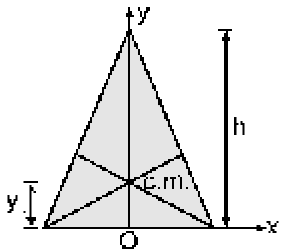
$$y_c = \frac{L}{2}$$



A triangular plate

(By qualitative argument)

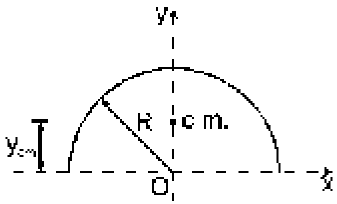
at the centroid : $y_c = \dfrac{h}{3}$



A semi-circular ring

$$y_c = \frac{2R}{\pi}$$

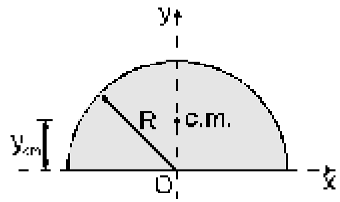
$$x_c = 0$$



A semi-circular disc

$$y_c = \frac{4R}{3\pi}$$

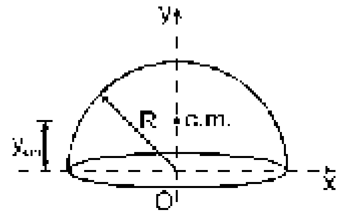
$$x_c = 0$$



A hemispherical shell

$$y_c = \frac{R}{2}$$

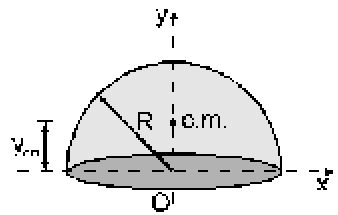
$$x_c = 0$$



A solid hemisphere

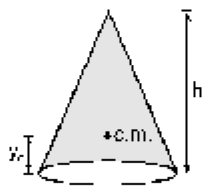
$$y_c = \frac{3R}{8}$$

$$x_c = 0$$



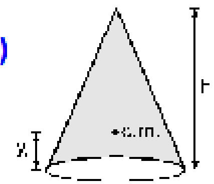
A circular cone (solid)

$$y_c = \frac{h}{4}$$



A circular cone (hollow)

$$y_c = \frac{h}{3}$$



Motion of Centre of Mass

Velocity of center of mass

$$\dot{v}_{cm} = \frac{m_1 \dot{v}_1 + m_2 \dot{v}_2 + m_3 \dot{v}_3 \ldots\ldots\ldots + m_n \dot{v}_n}{M}$$

Momentum of System

$$\vec{P}_{System} = M \vec{v}_{cm}$$

Acceleration of center of mass

$$\dot{a}_{cm} = \frac{m_1\bar{a}_1 + m_2\bar{a}_2 + m_3\bar{a}_3 \ldots\ldots\ldots + m_n\bar{a}_n}{M}$$

Net external force on a system

$$\boxed{\vec{F}_{ext} = M \vec{a}_{cm}}$$

$$\vec{v}_{cm} = \frac{d}{dt}\vec{r}_{cm}$$

$$\vec{p}_{system} = \vec{p}_1 + \vec{p}_2 + \ldots$$

$$= \left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots}{M}\right) M$$

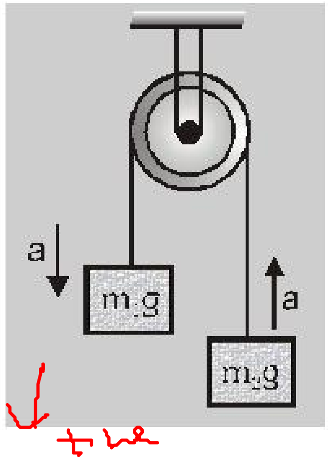
Example

Two blocks of masses m_1 and m_2 are connected by a light inextensible string passing over a smooth fixed pulley of negligible mass. Find the acceleration of the centre of mass of the system when blocks move under gravity.

Sol.

$\because$ $m_1 > m_2$ so m_1 will move downwards and m_2 upwards.

Magnitude of acc of each block $a = \dfrac{\text{net pulling force}}{\text{mass to be pulled}} = \dfrac{(m_1 - m_2)g}{m_1 + m_2}$

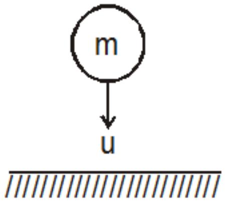


$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} = \frac{m_1 a - m_2 a}{m_1 + m_2} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)a$$ (+ve downwards and –ve upwards)

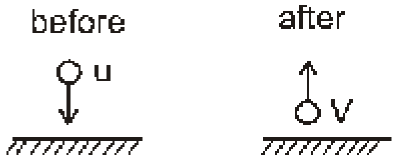
$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \times \left(\frac{m_1 - m_2}{m_1 + m_2} g\right) = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$ In the direction of acceleration of m_1, downwards

Example

A ball is approaching to ground with speed u.
If the coefficient of restitution is e then find out:
(a) the velocity just after collision.
(b) the impulse exerted by the normal due to ground on the ball.



Sol.



$$e = \frac{\text{velocity of separation}}{\text{velocity of approah}} = \frac{v}{u}$$

(a) velocity after collision = V = eu ...(1)

(b) Impulse exerted by the normal due to ground on the ball = change in momentum of ball.

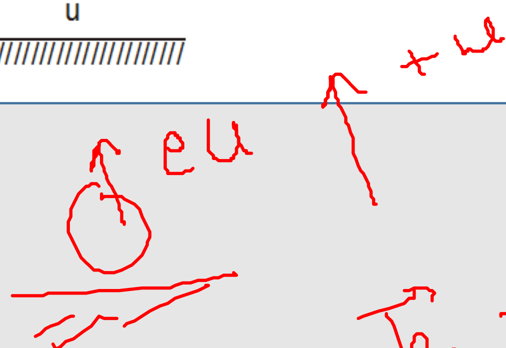
= {final momentum} – {initial momentum}

= {m v} – {– mu}

= mv + mu

= m {u + eu}

= mu {1 + e}



$$I = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

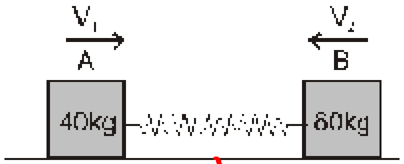
$$= meu - (-mu)$$

$$\boxed{I = mu(1+e)}$$

Example

Blocks A and B have masses 40 kg and 60 kg respectively. They are placed on a smooth surface and the spring connected between them is stretched by 1.5m. If they are released from rest, determine the speeds of both blocks at the instant the spring becomes unstretched.

Sol.



Let, both block ~~start moving with~~ some velocity V_1 and V_2 when spring is in it's natural length

Since no horizontal force on system so,
applying momentum conservation

$0 = 40\,V_1 - 60\,V_2 \qquad 2V_1 - 3V_2 \qquad \ldots(1)$

Now applying energy conservation,
Loss in potential energy = gain in kinetic energy

P.E. of spring

$$\boxed{\frac{1}{2}kx^2} = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$$

$$\frac{1}{2}\times 600\times(1.5)^2 = \frac{1}{2}\times 40\times V_1^2 + \frac{1}{2}\times 60\times V_2^2 \quad \ldots(2)$$

$$60\times\frac{9}{4} = 40v_1^2 + 60\,\frac{4}{9}v_1^2$$

$$15\times 9 = \frac{20\times 4}{3}v_1^2$$

$$\frac{9\times 9}{4} = v_1^2 \Rightarrow v_1 = \frac{9}{2} = 4.5\text{ m/s}$$

Solving equation (1) and (2) we get,
$V_1 = 4.5$ m/s, $V_2 = 3$ m/s