JEE and NEET CRASH COURSE

PHYSICS

Problem Solving Class

(Momentum, Impulse, COM and Collision)

By, Ritesh Agarwal, B. Tech. IIT Bombay

P-Q730

A ball of mass m moving at a speed v collides with another ball of mass 3m at rest. The lighter block comes to rest after collision. The coefficient of restitution is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{1}{4}$ (D) none of these

$\overrightarrow{b/c}$

$$3\cancel{m}v' = \cancel{m}v \Rightarrow v' = v/3$$

$$e = \frac{v_{sep.}}{v_{app.}} = \frac{v'}{v} = \frac{1}{3}$$

P-Q730-Solution

Ans [D]

$$mv + 3m \cdot 0 = m \cdot 0 + 3u$$

$$\therefore \quad u = \frac{v}{3}$$

$$e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

$$= \frac{u-0}{v-0} = \frac{u}{v}$$

$$= \frac{1}{3}$$

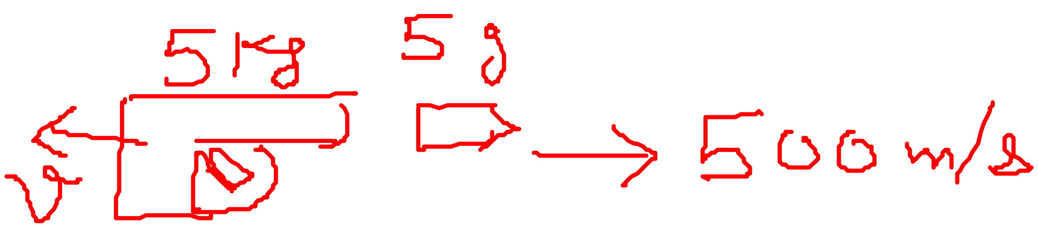
The momentum will be conserved in this situation also, only the kinetic energy of the system will change

P-Q747

A bullet of mass 5 g is shot from a gun of mass 5 kg. The muzzle velocity of the bullet is 500 m/s. The recoil velocity of the gun is

(a) 0.5 m/s (b) 0.25 m/s

(c) 1 m/s (d) Data is insufficient



$\vec{P}/C$

$\vec{p}_f = \vec{p}_i$

$$\frac{\cancel{5}}{\cancel{1000}_{2}} \times \cancel{500} - \cancel{5}v = 0$$

$$\frac{1}{2} = v \Rightarrow \boxed{v = 0.5 \text{ m/s}}$$

P-Q747-Solution

Ans [A]

$$m_B v_B = m_a v_a$$

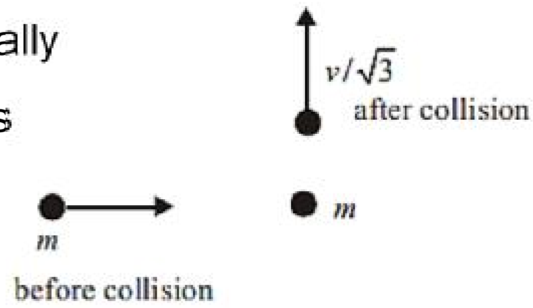
By the conservation of linear momentum

$$\Rightarrow v_G = \frac{m_B \times v_B}{m_G}$$

$$= \frac{5 \times 10^{-3} \times 500}{5} = 0.5\ m/s$$

PQ4Q279

A mass m moves with a velocity v and collides inelastically with another identical mass. After collision the first mass moves with velocity in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision **AIEEE - 2005**



(A) $\frac{2}{\sqrt{3}}v$ (B) $\frac{v}{\sqrt{3}}$

(C) v (D) $\sqrt{3}v$

$\vec{p}$/c in x dim

$mv'_x = mv \Rightarrow v'_x = v$

$\vec{p}$/c in y-dim ;

$m\frac{v}{\sqrt{3}} - mv'_y = 0$

$v'_y = v/\sqrt{3}$

$v' = \sqrt{v'^2_x + v'^2_y}$

$= v\sqrt{1+\frac{1}{3}}$

$= \frac{2}{\sqrt{3}}v$

PQ4S279

Ans [A]

Let v_1 = speed of second mass after collision

Momentum is conserved

Along X - axis, $mv_1 \cos\theta = mv$ ....(i)

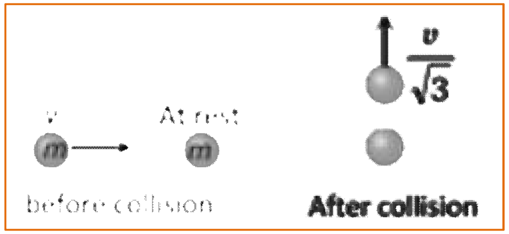
$$\text{Along Y} - \text{axis}, mv_1 \sin\theta = \frac{mv}{\sqrt{3}} \qquad \ldots \ldots (ii)$$

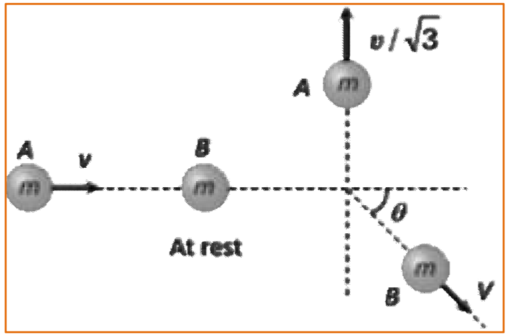
From (i) and (ii)

$$\therefore (mv_1 \cos\theta)^2 + (mv_1 \sin\theta)^2 = (mv)^2 + \left(\frac{mv}{\sqrt{3}}\right)^2$$

$$\text{or} \quad m^2 v_1^2 = \frac{4m^2v^2}{3}$$

$$\text{or} \quad v_1 = \frac{2}{\sqrt{3}} v$$





●linear momentum is conserved in horizontal,and vertical direction.

PQ4Q282

A bomb of mass 16 kg at rest explodes into two pieces of masses of 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1}. The kinetic energy of the other mass is

AIEEE - 2006

(A) 96 J (B) 144 J

(C) 288 J (D) 192 J

16 kg (rest)

$v \leftarrow$ 4 kg 12 kg $\rightarrow$ 4 m/s

$$K = \frac{p^2}{2m} = \frac{(48)^2}{2\times 4}$$

$\overrightarrow{P}/C :\rightarrow \; 12\times 4 - 4\times v = 0 \Rightarrow v = 12$ m/s

$$K.E_{4kg} = \frac{1}{2}(4)(12)^2 = 2\times 144 = 288 \text{ J}$$

PQ4S282

Ans [C]

Linear momentum is conserved

$\therefore \quad 0 = m_1 v_1 + m_2 v_2 = (12 \times 4) + (4 \times v_2)$

or $4v_2 = -48 \Rightarrow v_2 = -12 \frac{m}{s}$

$\therefore \quad$ Kinetic energy of mass $m_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \times 4 \times (-12)^2 = 288$ J.

● When there is no net external force acting on a system of particles the total momentum of the system is conserved.

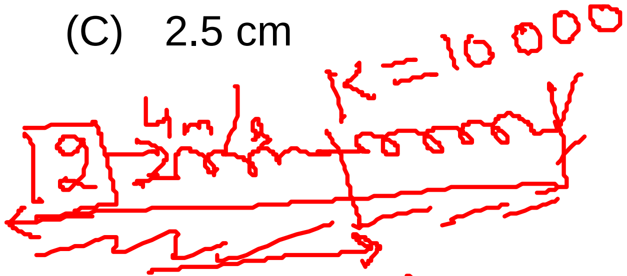
PQ4Q285

A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by

AIEEE - 2007

(A) 8.5 cm (B) 5.5 cm

(C) 2.5 cm (D) 11.0 cm



$f_k = 15N$

$$\boxed{W_{spring} = U_i - U_f}$$

$$W_{f_k} + W_{spring} = K_f - K_i$$

$$-15x + 0 - \frac{1}{2}Kx^2 = 0 - \frac{1}{2}(2)(4)^2$$

$$15x + 5000x^2 = 16$$

$$5000x^2 + 15x - 16 = 0$$

$$x = \frac{-15 + \sqrt{225 + 320000}}{2 \times 5000}$$

$$x = \frac{-15 + 400\sqrt{2}}{10000}$$

$$= \frac{-15 + 400 \times 1.4}{10000}$$

$$=$$

$$=$$

PQ4S285

Ans [B]

Let the spring be compressed by x

Initial kinetic energy of the mass = work done by spring force + work done due to friction

$$\frac{1}{2} \times 2 \times 4^2 = \frac{1}{2} \times 10000 \times x^2 + 15\,x$$

or $\quad 5000\,x^2 + 15x - 16 = 0$

or $\quad x = 0.055\text{ m} = 5.5\text{ cm.}$

● If A is the angle between force vector and displacement vector.. Then Negative work is done if:
$\pi/2 <= A <= \pi$

●Change in Kinetic energy = Work done by all forces.

PQ4Q288

A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is **AIEEE - 2008**

(A) 0.34 J (B) 1.00 J

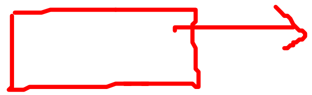
(C) 0.16 J (D) 0.67 J ✓

$\underline{\underline{\vec{p}\ /\ c}}$

2m/s rest

0.5 kg 1 kg

v



1.5 kg

$1.5\,v = 0.5 \times 2$

$v = \frac{2}{3}$ m/s

$\text{Energy loss} = K_i - K_f = \frac{1}{2}\left(\frac{1}{2}\right)(4) - \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{4}{9}\right)$

$= 1 - \frac{1}{3} = \frac{2}{3} = \underline{\underline{0.67\text{ J}}}$

PQ4S288

Ans [D]

By the law of conservation of momentum $mu = (M + m)v$

$0.50 \times 2.00 = (1 + 0.50)\,v$, $\dfrac{1.00}{1.50} = v$

Momentum is conserved if no external force acts on the body

- Loss in energy = initial energy – final energy

Initial K.E. $= \left(\dfrac{1}{2}\right) \times 0.50 \times (2.00)^2 = 1.00$ J.

Final K.E. $= \dfrac{1}{2} \times 1.50 \times \dfrac{1.00^2}{(1.50)^2} = \dfrac{1.00}{3.00} = 0.33$

$\therefore$ Loss of energy = 1.00 – 0.33 = 0.67 J.

P-Q716

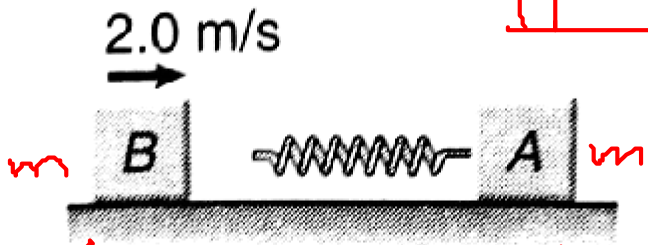
Two blocks **A** and **B** of equal mass $\boldsymbol{m = 1 \cdot 0\ kg}$ are lying on a smooth horizontal surface as shown. A spring of force constant $\boldsymbol{k = 200 N/m}$ is fixed at one end of block **A**. block **B** collides with block **A** with velocity $\boldsymbol{v_0 = 2.0 m/s}$. Find the maximum compression of the spring.

A) 1cm

B) 5cm

C) 10cm

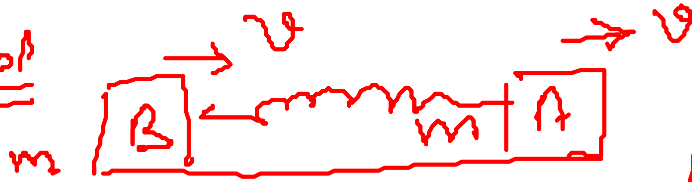
D) 20cm



$v' = 0$

Note:→ At max. compression/extension both the blocks have same velocity.

Sol



$2mv = m \times 2$ } P/C

$\boxed{v = 1 m/s}$

E/C $\quad U_i + K_i = U_f + K_f$

$0 + \frac{1}{2}(1)(4) = \frac{1}{2}kx^2 + \frac{1}{2}(2)(1)$

$4 - 2 = 200x^2$

$\Rightarrow x^2 = \frac{2}{200} = \frac{1}{100}$

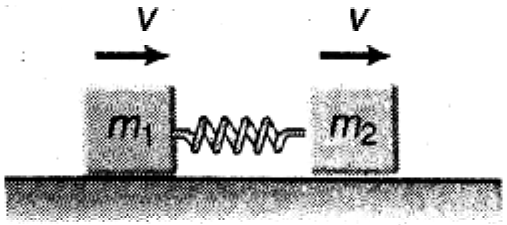
$\Rightarrow x = \frac{1}{10}\,m$

$= 10\,cm$

P-Q716-Solution

Ans [C]

At maximum compression velocity of both the blocks is same



Maximum compression $= x_m$

Conservation of momentum

$$(m_A + m_B)v = m_B v_0$$

$$(1.0 + 1.0)v = (1.0)\, v_0$$

$$v = \frac{v_0}{2} = \frac{2.0}{2} = 1.0\,\text{m/s}$$

Velocity at maximum compression

using conservation of mechanical energy

$$\frac{1}{2} m_B v_0^2 = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}kx_m^2$$

$$\frac{1}{2} \times (1) \times (2.0)^2 = \frac{1}{2} \times (1.0 + 1.0) \times (1.0)^2 + \frac{1}{2} \times (200) \times x_m^2$$

$$2 = 1.0 + 100x_m^2 \quad \text{or} \quad x_m = 0.1\,\text{m} = 10.0\,\text{cm}$$

P-Q749

A body of mass M at rest explodes into three pieces, two of which of mass M/4 each are thrown off in perpendicular directions with velocities of 3 m/s and 4 m/s respectively. The third piece will be thrown off with a velocity of

(a) 1.5 m/s

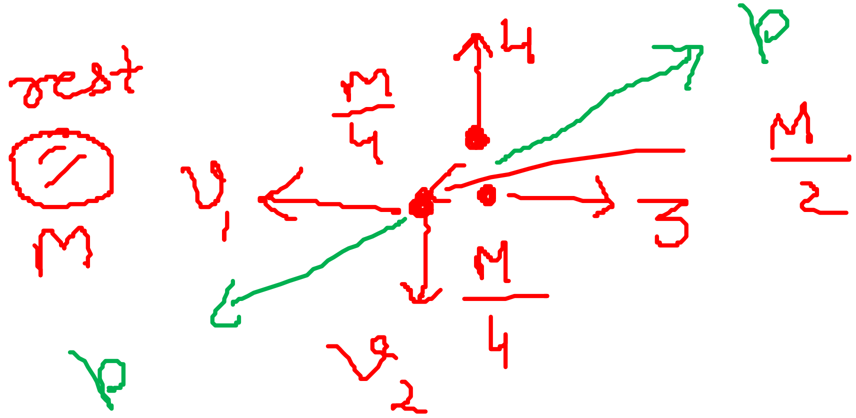
(b) 2.0 m/s

(c) 2.5 m/s

(d) 3.0 m/s

$\frac{M}{4}\sqrt{3^2+4^2}$

$= \frac{M}{4} \times 5$



$\vec{P}/C$ in x dim

$\frac{\cancel{M}}{\cancel{4}_{2}} \times 3 - \frac{\cancel{M}}{\cancel{2}} v_1 = 0$

$v_1 = \frac{3}{2}$ m/s

$\vec{P}/C$ in y-dim

$\frac{\cancel{M}}{\cancel{4}_{\cancel{2}}} \times \cancel{4}^{2} - \frac{\cancel{M}}{\cancel{2}} v_2 = 0$

$v_2 = 2$ m/s

$v = \sqrt{v_1^2 + v_2^2} = \sqrt{\frac{9}{4} + 4}$

$= \sqrt{\frac{25}{4}}$

P-Q749-Solution

Ans [C]

$$\text{Momentum of one piece } = \frac{M}{4} \times 3$$

$$\text{Momentum of the other piece } = \frac{M}{4} \times 4$$

$$\therefore \text{ Resultant momentum } = \sqrt{\frac{9M^2}{16} + M^2} = \frac{5M}{4}$$

$$\frac{5M}{4} = \frac{M}{2} \times v \text{ or } v = \frac{5}{2} = 2.5\, m / sec$$

The third piece should also have the same momentum. Let its velocity be *v*

P-Q708

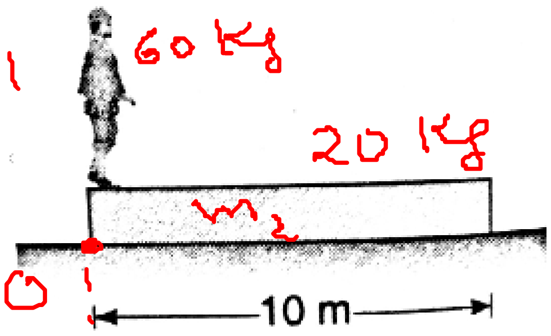
A wooden plank of mass **20kg** is resting on a smooth horizontal floor . A man of mass **60kg** starts moving from one end of the plank to the order end . The length of plank is **10m** . Find the displacement of the plank over the floor when the man reaches the other end of the plank

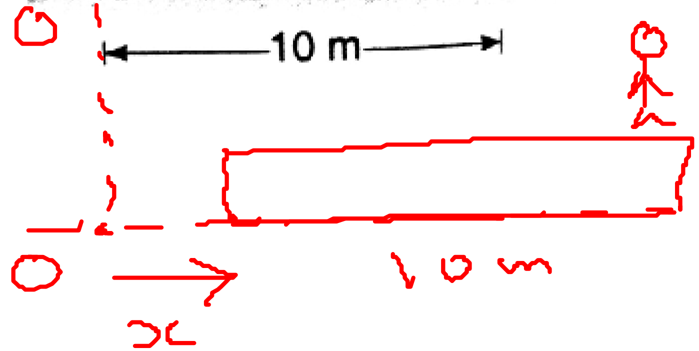
A) 1.25m

B) 4.75m

C) 10m

D) 7.5m





$\vec{x}_{cm} = \text{const}$

$\Delta x_{cm} = 0$

$m_1 \Delta x_1 + m_2 \Delta x_2 = 0$

$\boxed{\Delta x = x_f - x_i}$

$m_1 \Delta x_1 + m_2 \Delta x_2 = 0$

$\cancel{60}^{3}(x + 10 - 0) + \cancel{20}(x - 0) = 0$

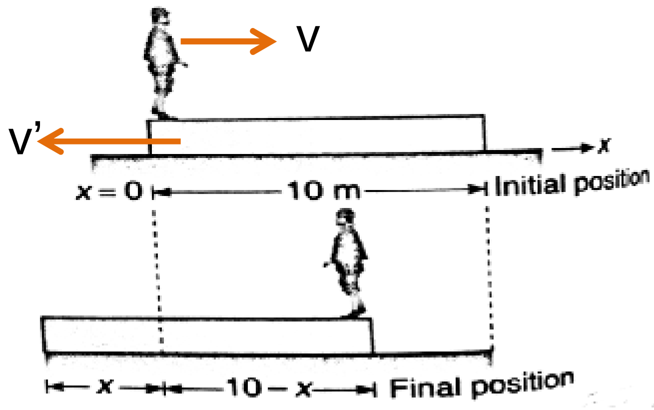
$3x + 30 + x = 0$

$\Rightarrow 4x = -30$

$\Rightarrow \boxed{x = -7.5\,m}$

P-Q708-Solution

Ans [D]



$$x_i = x_f$$

$$\frac{(60)(0)+20\left(\frac{10}{2}\right)}{60+20} = \frac{(60)(10-x)+20\left(\frac{10}{2}-x\right)}{60+20}$$

$$\frac{5}{4} = \frac{6(10-x)+2\left(\frac{10}{2}-x\right)}{8} = \frac{60-6x+10-2x}{8}$$

COM is not moving so COM also same as initial and final position of man

Centre of mass of plank lies at its centre

$$5 = 30 - 3x + 5 - x$$

$$4x = 30$$

$$x = \frac{30}{4}\text{ m}$$

$$x = 7.5\text{ m}$$

We can also solve by conservation of linear momentum

$$m(v - v') - Mv' = 0 \quad (1)$$

$$v = \frac{10}{t}$$

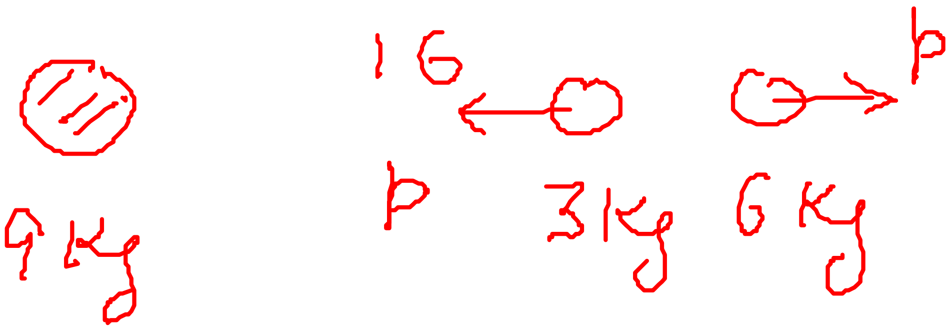
$$x_{plank} = t \times v'_{plank}$$

P-Q726

A bomb of mass 9kg explodes into two pieces of masses 3kg and 6kg. The velocity of mass 3kg is 16ms^{-1}. The kinetic energy of mass 6kg is

(A) 96J (B) 384J

(C) 192J (D) 768J



$p = 3 \times 16 = 48$

$K = \frac{p^2}{2m}$

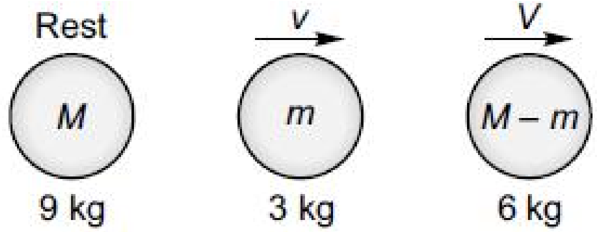
$= \frac{(48)^2}{2 \times 6}$

$= \frac{\cancel{48} \times 48}{\cancel{12}}$

$= 192 J$

P-Q726-Solution

Ans [C]



$$M \times 0 = mv + (M - m)V$$

$$\Rightarrow \qquad V = -\frac{m}{(m - m)}v$$

$$= -\frac{3}{9-3} \times 16$$

$$= -8\ \mathrm{ms^{-1}}$$

$$\therefore \text{KE of 6 kg mass} = \frac{1}{2} \times 6 \times (-8)^2$$

$$= 192\ \mathrm{J}$$

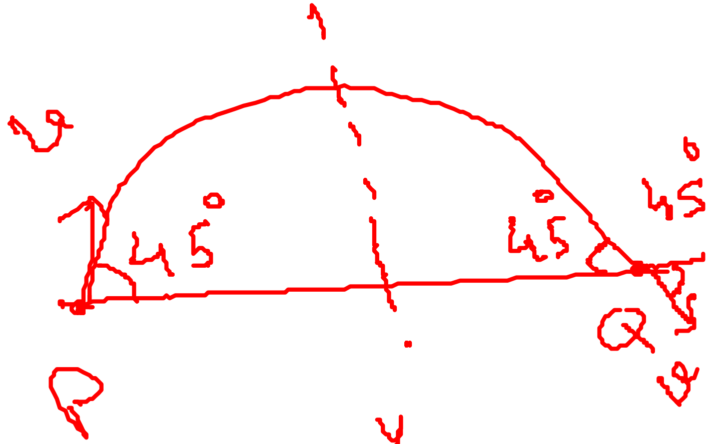
The total momentum of the system will be zero after explosion also

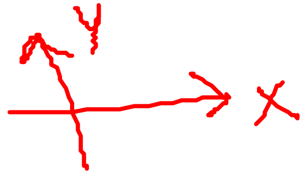
P-Q728

A projectile of mass m is fired with a velocity v from point P at an angle 45°. Neglecting air resistance, the magnitude of the change in momentum leaving the point P and arriving at Q is

(A) $mv\sqrt{2}$ ✓ (B) $2mv$

(C) $\frac{mv}{2}$ (D) $\frac{mv}{\sqrt{2}}$





$\vec{v}_i = \frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}$

$\vec{v}_f = \frac{v}{\sqrt{2}}\hat{i} - \frac{v}{\sqrt{2}}\hat{j}$

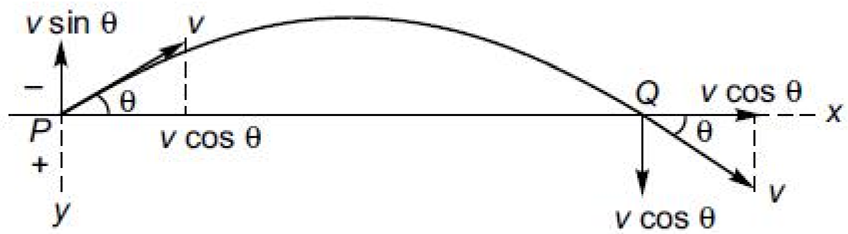
$\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i) = m\left(-\frac{2v}{\sqrt{2}}\hat{j}\right)$

$= -\sqrt{2}mv\hat{j}$

P-Q728-Solution

Ans [A]

Change in momentum along *x*-axis



$$= m\,(v\cos\theta - v\cos\theta) = 0$$

So the momentum along x-axis will be constant

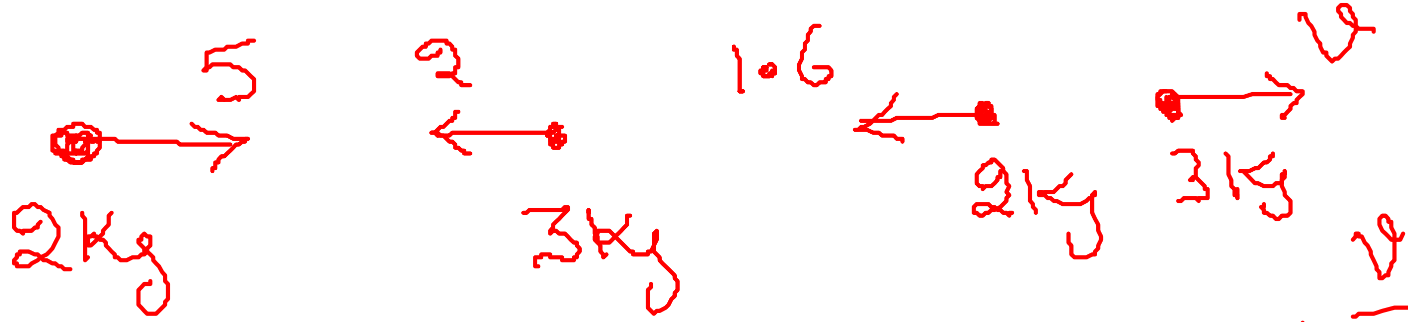
∴ Net change in momentum= Change in momentum along *y*-axis

$$= m[(+v\sin\theta) - (-v\sin\theta)]$$
$$= 2\,mv\sin\theta$$
$$= mv\sqrt{2}$$

P-Q734

A particle of mass 2kg moving with a velocity $\mathbf{5}\hat{\boldsymbol{\imath}}\ \frac{m}{s}$ collides head on with another particle of mass 3 kg moving with a velocity $-\mathbf{2}\hat{\boldsymbol{\imath}}\ \frac{m}{s}$. after the collision the first particle has speed of $\mathbf{1.6}\ \frac{m}{s}$ in negative x direction. Find coefficient of restitution

$(A)\ \frac{1}{2}$ $\qquad (B)\ \frac{3}{5}$ $\qquad$ ✓ $(C)\ \frac{4}{7}$ $\qquad (D)\ 1$



$\vec{P}/c$

$3v - 2\times1.6 = 2\times5 - 3\times2$

$3v - 3.2 = 10 - 6$

$3v = 7.2 \Rightarrow v = 2.4$

$e = \frac{v_{sep}}{v_{app.}}$

$= \frac{v+1.6}{5+2} = \frac{4}{7}$

$\boxed{e = \frac{4}{7}}$

P-Q734-Solution

Ans [C]

Conservation of momentum

$$5 \times 2 - 3 \times 2 = -2 \times 1.6 + 3 \times v_2$$

$$v_2 = 7.2/3$$

$$e = \frac{v_2 - v_1}{u_2 - u_1}$$

$$e = \frac{\frac{7.2}{3} + 1.6}{-2 - 5} = \frac{12.0}{3 \times 7}$$

$$e = \frac{4}{7}$$

Coefficient of restitution is the ratio of velocity of approach and separation

P-Q735

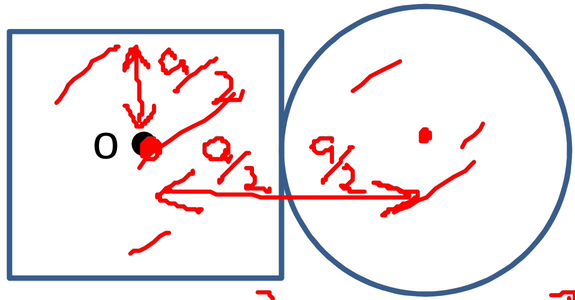
A square lamina of side a and a circular lamina of diameter a are placed touching each other as shown in figure. Find distance of their Centre of mass from point O, the Centre of square.

(A) $\frac{\pi}{\pi+4}a$

(B) $\frac{2\pi}{\pi+2}a$

(C) $\frac{2\pi}{\pi+4}a$

(D) $\frac{\pi}{\pi+2}a$



$M_1 = \sigma a^2$ $\quad M_2 = \sigma \pi \frac{a^2}{4}$

$$x_1 = \frac{M_2 a}{M_1 + M_2} = \frac{\cancel{\sigma} \frac{\pi \cancel{a^2}}{4} \cdot a}{\cancel{\sigma} \cancel{a^2}\left(1 + \frac{\pi}{4}\right)} = \frac{\pi a}{\cancel{4}} \frac{\cancel{4}}{4+\pi}$$

$$= \left(\frac{\pi}{\pi + 4}\right) a$$

P-Q735-Solution

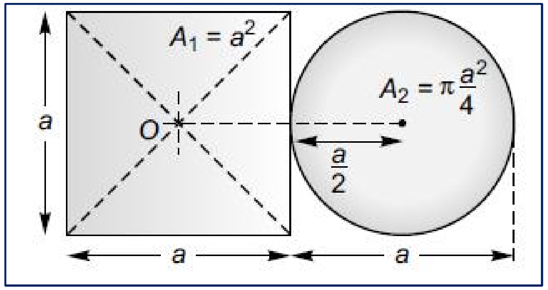
Ans [A]

$$x_{CM} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

This formula will be used for calculating the centre of mass

$$= \frac{a^2 \cdot 0 + \frac{\pi a^2}{4} \cdot (a)}{a^2 + \frac{\pi a^2}{4}}$$

$$= \frac{\pi}{4 + \pi} a$$



P-Q736

A block of mass 1kg is at $x = 10m$ and moving towards negative x-axis with velocity $6\,{}^{m}/_{s}$. Another block of mass 2kg is at $x = 12m$ and moving towards positive x-axis with velocity $4\,{}^{m}/_{s}$ at the same instant. Find position of their Centre of mass after 2s.

(A) 6.33 m (B) 12.67 m (C) 19 m (D) 25.33 m

$x = 10$ $\quad x = 12$

$6 \leftarrow$ 1kg $\quad \rightarrow v = 4\,m/s$ 2kg

$$x_{cm} = \frac{1\times 10 + 2\times 12}{1+2} = \frac{34}{3}$$

$$\vec{v}_{cm} = \frac{1\times(-6) + (2)(4)}{3} = \frac{2}{3} = \text{const.}$$

$$\vec{S}_{cm} = \vec{v}_{cm}\,t = \frac{2}{3}\times 2 = \frac{4}{3}\,m$$

$$x_f - x_i = \frac{4}{3}$$

$$x_f - \frac{34}{3} = \frac{4}{3}$$

$$x_f = \frac{38}{3} = 12.67\,m$$

P-Q736-Solution

Ans ~~[A]~~ B

$$x_{CM} = \frac{1 \times 10 + 2 \times 12}{1+2}$$

This is the position of com at $t = 0$

$$= \frac{34}{3}\,\text{m}$$

This is the velocity of com at $t = 0$

$$v_{CM} = \frac{1 \times (-6) + 2 \times (+4)}{1+2} = +\frac{2}{3}\,\text{ms}^{-1}$$

x'_{CM} (new position of CM)

$$= \frac{34}{3}\,\text{m} + 2\,\text{s}\left(\frac{2}{3}\,\text{ms}^{-1}\right)$$

Distance travelled in 2 seconds at a velocity of $\frac{2}{3}\,m/\sec$ will be $\frac{4}{3}\,m$

$$= \frac{38}{3}\,\text{m} = 12.67\,\text{m}$$