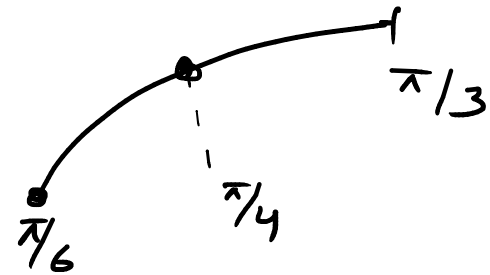
9) If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by

$$f(x) = \begin{cases} \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \neq \dfrac{\pi}{4} \\ k, & x = \dfrac{\pi}{4} \end{cases}$$ is continuous,

then k is equal to **(2019 Main, 9 April I)**

(a) $\frac{1}{2}$ (b) 2

(c) 1 (d) $\frac{1}{\sqrt{2}}$



R·H·L = L·H·L = f(a)

$$\left(\frac{0}{0}\right) \lim_{x \to \pi/4} \left(\frac{\sqrt{2}\cos x - 1}{\cot x - 1}\right) = K$$

$$\lim_{x \to \pi/4} \frac{\sqrt{2}(-\sin x)}{-\operatorname{cosec}^2 x} = \left(\frac{1}{2}\right) = K$$

9) The function $f(x) = [x]^2 - [x^2]$ (where, $[x]$ is the greatest integer less than or equal to x), is discontinuous at

(a) all integers **(1999, 2M)**

(b) all integers except 0 and 1

(c) all integers except 0

(d) all integers except 1

Repeating Nature

Test

!! I

$$f(x) = [x]^2 - [x^2]$$

$\boxed{x = 0}$

$f(0) = 0$

$f(0^-) = (-1)^2 - 0$

$= 1$

$\boxed{\text{discontinous}}$

$\boxed{x = 1}$

$f(1) = 0$

$f(1^-) = 0 - (0)$

$= 0$

$f(1^+) = (1)^2 - (1)$

$= 0$

$\boxed{\text{continous}}$

9)

If $f(x)=\begin{cases}\dfrac{\sin(p+1)x+\sin x}{x}, & x<0\\ q, & x=0\\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$

is continuous at $x=0$, then the ordered pair (p,q) is equal to **(2019 Main, 10 April I)**

(a) $\left(-\frac{3}{2},-\frac{1}{2}\right)$ (b) $\left(-\frac{1}{2},\frac{3}{2}\right)$

(c) $\left(\frac{5}{2},\frac{1}{2}\right)$ (d) $\left(-\frac{3}{2},\frac{1}{2}\right)$ ✓

$\boxed{L.H.L = R.H.L = f(a)}$

$\underline{f(0^-)} = f(0^+) = \underline{q}$

$f(0^-) = \lim_{x\to 0^-} (p+1)\dfrac{\sin(p+1)x}{(p+1)x} + \dfrac{\sin x}{x}$ $\to 1$, $\to 1$

$p+1+1$

$\underline{f(0^-) = p+2}$

$f(0^+) = \lim_{x\to 0^+} \dfrac{\sqrt{1+x}-1}{x} \times \dfrac{\sqrt{1+x}+1}{(\sqrt{1+x}+1)}$

$\dfrac{\cancel{(1}+\cancel{x})-\cancel{1}}{\cancel{x}(2)}$

$\underline{f(0^+)} = \dfrac{1}{2}$

$p+2 = 1/2$

$\boxed{p = -3/2}$ $\boxed{q = 1/2}$

8)

Let $f : [-1, 3] \to R$ be defined as

$$f(x) = \begin{cases} |x| + [x], & -1 \le x < 1 \\ x + |x|, & 1 \le x < 2 \\ x + [x], & 2 \le x \le 3, \end{cases}$$

(2019 Main, 8 April II)

★ (No. of point discontinous)

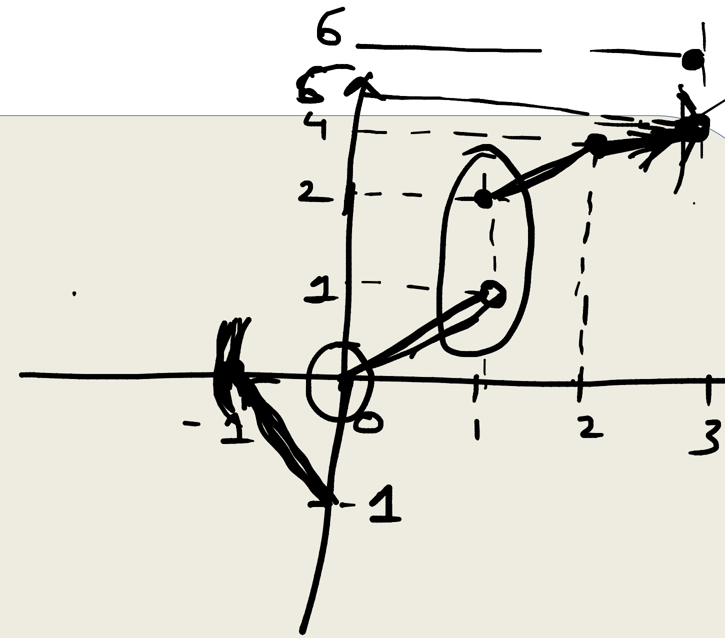
where, $[t]$ denotes the greatest integer less than or equal to t. Then, f is discontinuous at

(a) four or more points (b) only two points

(c) only three points (d) only one point

Graphical

$$f(x) = \begin{cases} (-x-1) & -1 \le x < 0 \\ (x+0) & 0 \le x < 1 \\ 2x & 1 \le x < 2 \\ x+2 & 2 \le x < 3 \\ x+3 & x = 3 \end{cases}$$



9) Let $f(x) = 15 - |x - 10|$; $x \in \mathbf{R}$. Then, the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is **(2019 Main, 9 April I)**

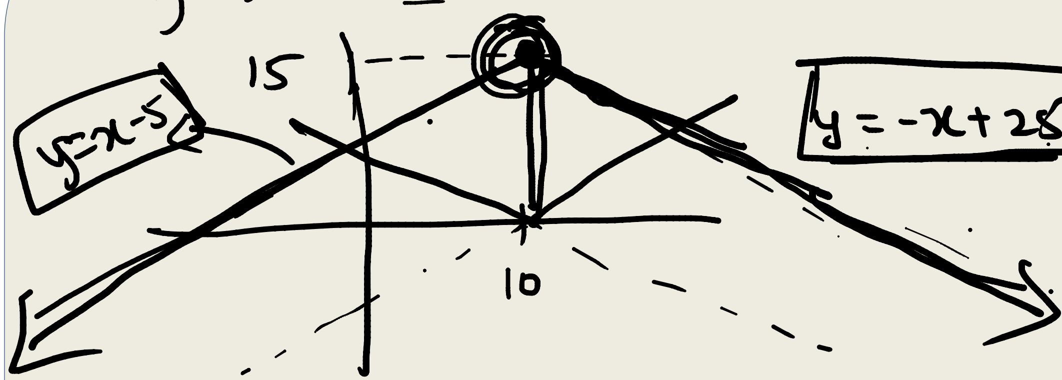
(a) $\{5, 10, 15, 20\}$ (b) $\{5, 10, 15\}$

(c) $\{10\}$ (d) $\{10, 15\}$

Graph

discontinous → Non differentiable

$f(x) = 15 - |x - 10|$



~~$g(x) = f(f(x))$~~

$f(x) \longrightarrow 1$

$f(f(x)) = 15 - \big|15 - |x - 10| - 10\big|$

$x > 10$ $= 15 - \big||25 - x| - 10\big|$

$x < 10$ $= 15 - \big||5 + x| - 10\big|$

Q9) Let S be the set of all points in $(-\pi, \pi)$ at which the function, $f(x) = \min\{\sin x, \cos x\}$ is not differentiable. Then, S is a subset of which of the following?

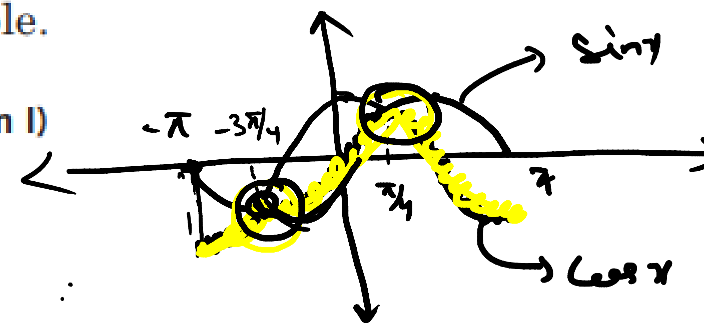
(2019 Main, 12 Jan I)

(a) $\left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$ ✗

(b) $\left\{-\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right\}$ ✗

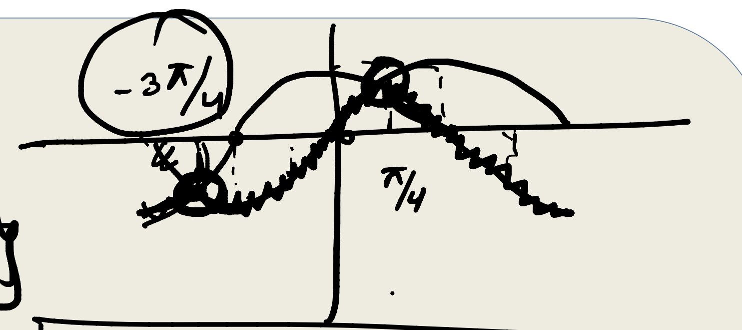
(c) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$

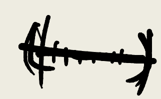
(d) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$



$f(x) = \min(\sin x, \cos x)$

$S = \left\{-\frac{3\pi}{4}, \frac{\pi}{4}\right\}$





0 $\quad 0 \le x < 1$

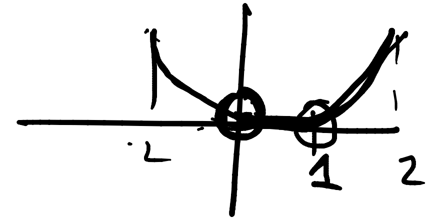
1 $\quad 1 \le x < 2$

Let $f(x)=\begin{cases} -1, & -2 \le x < 0 \\ x^2-1, & 0 \le x \le 2 \end{cases}$ and

$g(x)=|f(x)|+f(|x|)$. Then, in the interval $(-2, 2)$, g is

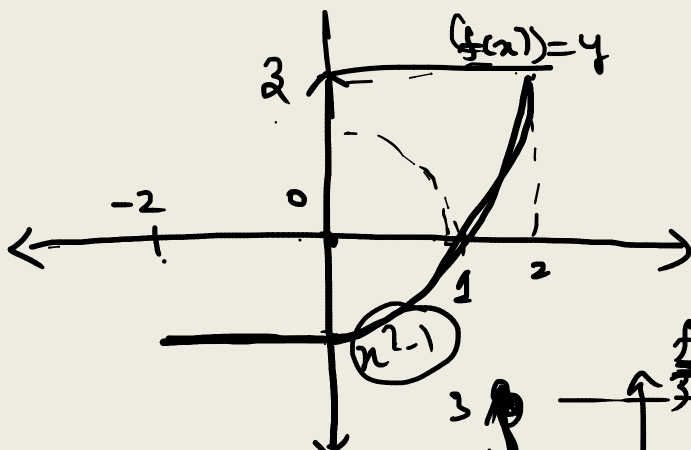
(2019 Main, 11 Jan I)

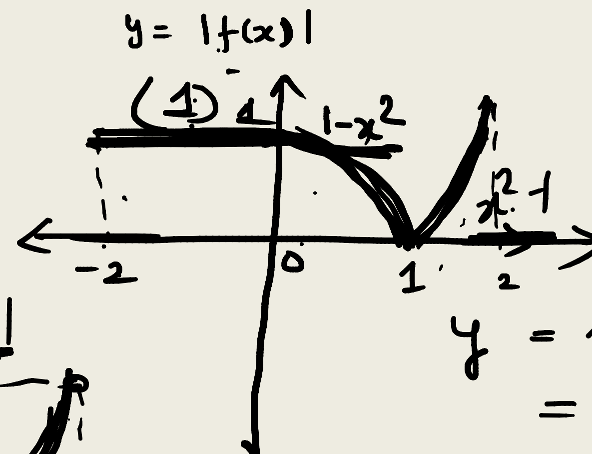
(a) not differentiable at one point
(b) not differentiable at two points
(c) differentiable at all points
(d) not continuous



$$g(x)=\begin{cases} x^2 & -2 \le x < 0 \\ 0 & 0 \le x \le 1 \\ 2(x^2-1) & 1 \le x < 2 \end{cases}$$

$g(x) = |f(x)| + f(|x|)$





$g(-1) = f(|-1|)$

$y = f(|x|)$
$= f(x)$ $\quad x>0$
$= f(-x)$ $\quad x<0$