Problem Solving on
Limits, Continuity & Differentiability

$\cancel{f(a^-)}$

$f'(a^+) = f'(a^-) \nearrow \infty$, $\searrow -\infty$

∞, $\cancel{-\infty}$

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9) $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1}$ (4-5) ~~repeat~~ (1998, 2M)

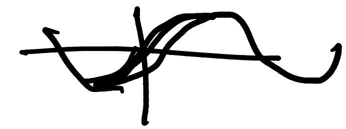
~~(a)~~ exists and it equals $\sqrt{2}$

(b) exists and it equals $-\sqrt{2}$

(c) does not exist because $x - 1 \to 0$

(d) does not exist because left hand limit is not equal to right hand limit

$\sqrt{x^2} = |x|$



$1 - \cos 2(x-1) = 2\sin^2(x-1)$

$1 - \cos 2x = 2\sin^2 x$

$\lim_{x \to 1} \frac{\sqrt{2\sin^2(x-1)}}{x-1}$

$\lim_{x \to 1} \frac{\sqrt{2}\,|\sin(x-1)|}{(x-1)}$

$\sqrt{2}$

$R \cdot H \cdot L_{1^+} = \sqrt{2} \cdot \frac{\sin(x-1)}{x-1} = \sqrt{2}$

$L \cdot H \cdot L_{1^-} = -\sqrt{2}\,\frac{\sin(x-1)}{x-1} = -\sqrt{2}$

9)

For each $x \in R$, let $[x]$ be the greatest integer less than or equal to x. Then,

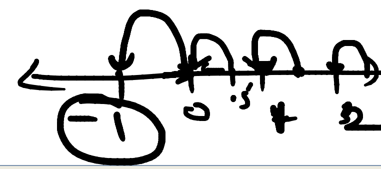
$$\lim_{x \to 0^-} \frac{x([x] + |x|) \sin [x]}{|x|} \text{ is equal to}$$

(2019 Main, 9 Jan II)

(a) 0 (b) $\sin 1$

(c) $-\sin 1$ (d) 1

$-.000001$



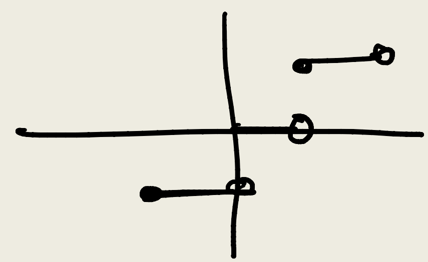
$$\lim_{x \to 0^-} \frac{x([x] + |x|) \sin [x]}{|x|}$$

$$\frac{\cancel{x}(-1 - x) \sin(-1)}{-\cancel{x}}$$

$$\lim_{x \to 0^-} -(x+1) \sin 1$$

$$-\sin 1$$

$[x]$



9) Let $p = \lim_{x \to 0^+} (1 + \tan^2 \sqrt{x})^{1/2x}$, then $\log p$ is equal to

(2016 Main)

(a) 2 $e^{1/2}$ (b) 1 (c) $\cancel{\frac{1}{2}}$ (d) $\frac{1}{4}$

1^∞

$$f(x)^{g(x)} = e^{\lim_{x \to 0^+} (\tan^2 \sqrt{x}) \cdot \frac{1}{2x}}$$

$$\boxed{e^{\lim_{x \to 0} (f(x)-1) \cdot g(x)}} = e^{\lim_{x \to 0^+} \frac{1}{2} \left(\cancel{\frac{\tan \sqrt{x}}{\sqrt{x}}}\right)^2}$$

$$\boxed{p = e^{1/2}}$$

$$\log p = \log e^{1/2}$$

$$= 1/2$$

9) For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then,

$$\lim_{x \to 1+} \frac{(1 - |x| + \sin|1 - x|)\sin\left(\frac{\pi}{2}[1 - x]\right)}{|1 - x|\,[1 - x]}$$

(2019 Main, 10 Jan I)

(a) equals 0 (b) does not exist
(c) equals – 1 (d) equals 1

$x \to 1^+$

1.0001

$|1-x| = -(1-x) = x-1$

$[1-x] = -1$

$|x| = x$

$1-1.0001$

$[-.0001]$

$$\lim_{x \to 1^+} \frac{(1-x+\sin(x-1)) \times \cancel{1}}{(x-1)\,\cancel{-1}}$$

$$\lim_{x \to 1^+} \left(\frac{-\cancel{(x-1)}}{\cancel{x-1}} + \frac{\sin(x-1)}{\cancel{\sin}(x-1)} \right) \quad 1$$

$-1 + 1 = 0$

9) $\lim_{x\to0} \dfrac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals $\left(\frac{0}{0}\right)$ (2019 Main, 8 April I)

$\sqrt{1+\cos x} = \sqrt{2\cos^2 x/2}$

(a) ~~$4\sqrt{2}$~~ (b) $\sqrt{2}$

(c) $2\sqrt{2}$ (d) 4

L' Hosptial Rule

$$\lim_{x\to0}\left(\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}\right)\times\left(\frac{\sqrt{2}+\sqrt{1+\cos x}}{\sqrt{2}+\sqrt{1+\cos x}}\right)$$

$$\lim_{x\to0}\frac{\sin^2 x\times(\sqrt{2}+\sqrt{1+\cos x})}{2-(1+\cos x)}$$

$\sin^2 x = 1-\cos^2 x$

$$2\sqrt{2}\lim_{x\to0}\frac{\sin^2 x}{1-\cos x}$$

$$2\sqrt{2}\lim_{x\to0}\frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{1-\cos x}}$$

9)

If $\lim\limits_{x \to \infty}\left(\frac{x^2+x+1}{x+1} - ax - b\right) = 4$, then ★ ★

(a) $a = 1, b = 4$ (b) $a = 1, b = -4$ ✓

(c) $a = 2, b = -3$ (d) $a = 2, b = 3$

$$\lim_{x\to\infty} \frac{x^2+x+1-(ax+b)(x+1)}{(x+1)} = 4$$

$$\frac{1}{x}$$

$$\lim_{x\to\infty} \frac{x^2(1-a)+x(1-(a+b))+1-b}{(x+1)} = 4$$

$$\lim_{x\to\infty} \frac{\cancel{x(1-a)} + (1-(a+b)) + \cancel{\frac{1-b}{x}}\,0}{1+\cancel{\frac{1}{x}}\,0} = 4$$

$1-a=0$

$\boxed{a=1}$ ✓

$1-(a+b) = 4$

$a+b=-3$

$\boxed{b=-4}$