Parabola

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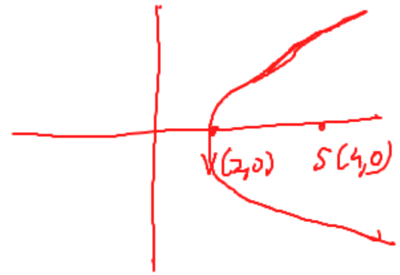
Axis of a parabola lies along X-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive X-axis, then which of the following points does not lie on it? **(2019 Main, 9 Jan, I)**

(a) $(4, -4)$ (b) $(6, 4\sqrt{2})$
(c) $(8, 6)$ (d) $(5, 2\sqrt{6})$



Soln $y^2 = 4aX$

$(y-0)^2 = 4a(x-2)$

Here $a = 2$

$\boxed{y^2 = 8(x-2)}$

'a' is the distance b/w Vertex & Focus

(4, −4) satisfies
$(6, 4\sqrt{2})$ "
(8, 6) does not satisfy
$(5, 2\sqrt{6})$ satisfy

$\Rightarrow y^2 = 4ax$

If vertex is (h, k)

$\boxed{(y-k)^2 = 4a(x-h)}$

$\boxed{y^2 = 4aX}$

$P(x_1, y_1)$

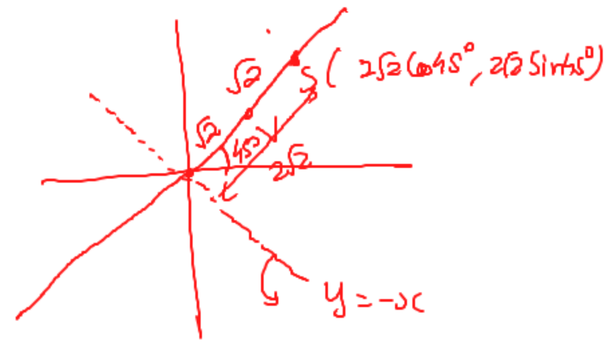
$ax+by+c=0$

$$p = \left| \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}} \right|$$

JEE Mains Problems

Axis of a parabola is $y = x$ and vertex and focus are at a distance $\sqrt{2}$ and $2\sqrt{2}$ respectively from the origin. Then, equation of the parabola is **(2006, 3M)**

(a) $(x - y)^2 = 8(x + y - 2)$ ✓
(b) $(x + y)^2 = 2(x + y - 2)$
(c) $(x - y)^2 = 4(x + y - 2)$
(d) $(x + y)^2 = 2(x - y + 2)$

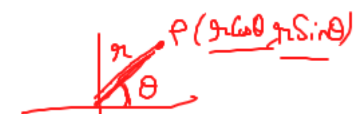


✓ Directrix $\equiv x+y=0$

$S(2\sqrt{2}\cos 45^\circ, 2\sqrt{2}\sin 45^\circ)$

✓ $S(2,2)$ $\quad P(h,k)$

$\boxed{SP = PM}$



Apply $SP = PM$ $\quad \sqrt{(h-2)^2+(k-2)^2} = \left|\frac{h+k}{\sqrt{2}}\right|$

$\Rightarrow 2(h^2+k^2-4h-4k+8) = (h+k)^2 \Rightarrow 2h^2+2k^2-8h-8k+16 = h^2+k^2+2hk$

$h^2+k^2-2hk = 8h+8k-16 \Rightarrow \boxed{(h-k)^2 = 8(h+k-2)}$

JEE Mains Problems

The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is (2001, 1M)

(a) $x = -1$ (b) $x = 1$

(c) $x = -3/2$ ✓(d) $x = 3/2$

$y^2 + 4y = -4x - 2$

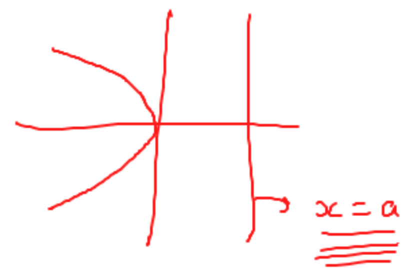
$y^2 + 4y + 4 = -4x - 2 + 4$

$(y+2)^2 = -4x + 2$

$(y+2)^2 = -4\left(x - \frac{1}{2}\right)$

$Y^2 = -4X$

$a = 1$



Eqn of Directrix, $= \boxed{X = a}$

$\left(x - \frac{1}{2}\right) = 1$

$x = \frac{1}{2} + 1 \Rightarrow x = 3/2$

JEE Mains Problems

The curve described parametrically by $x = t^2 + t + 1, y = t^2 - t + 1$ represents **(1999, 2M)**

(a) a pair of straight lines (b) an ellipse
(c) a parabola (d) a hyperbola

$x = t^2 + t + 1$ — ①

$y = t^2 - t + 1$ — ②

① + ②

$x + y = 2(t^2+1)$

$t^2 + 1 = \frac{x+y}{2}$

$t^2 = \frac{x+y}{2} - 1$

① − ②

$2t = x - y$

$t = \frac{x-y}{2}$

$\left(\frac{x-y}{2}\right)^2 = \frac{x+y-2}{2}$

$\boxed{(x-y)^2 = 2(x+y-2)}$

$x^2 + y^2 - 2xy = 2x + 2y - 4$

$x^2 + y^2 - 2xy - 2x - 2y + 4 = 0$

$a=1, b=1, h=-1, g=-1, f=-1, c=4$

$D = abc + 2fgh - af^2 - bg^2 - ch^2$

$= 4 + 2(-1) - 1\times1 - 1\times1 - 4\times1$

$= \cancel{4} - 2 - 1 - 1 - \cancel{4}$

$= -4$

$D \neq 0$ parabola or ellipse or hyp

$h^2 = 1, ab = 1$

$h^2 = ab$

$e = 1$

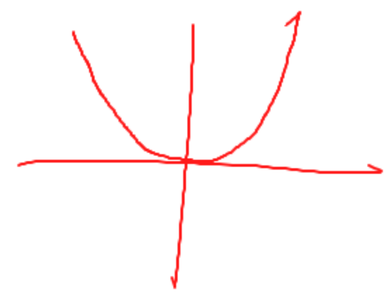
JEE Mains Problems

The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of X-axis, is

Slope of tangent = $\tan\theta$

(2019 Main, 12 Jan, II)

(a) $y = x\tan\theta - 2\cot\theta$
(b) $x = y\cot\theta + 2\tan\theta$ ✓
(c) $y = x\tan\theta + 2\cot\theta$
(d) $x = y\cot\theta - 2\tan\theta$



$x^2 = 8y \Rightarrow x^2 = 4ay$

$a = 2.$

$m = \tan\theta$

$x^2 = 4ay$

$x^2 = 4a(mx + c)$

$x^2 - 4amx - 4ac = 0$

$D = 0$

$16a^2m^2 = -16ac.$

$m^2 = -\frac{c}{a}$ $\Rightarrow \boxed{c = -am^2}$

$y = mx + c \Rightarrow \boxed{y = mx - am^2}$

$y = \tan\theta\, x - 2\tan^2\theta.$

$y = \tan\theta(x - 2\tan\theta)$

$y\cot\theta = x - 2\tan\theta$

$\boxed{x = y\cot\theta + 2\tan\theta}$

JEE Mains Problems

$y^2=4ax$ $\quad x^2=4ay$ $\quad y^2=-4ax$ $\quad x^2=-4ay$

$y=mx+\frac{a}{m}$ $\quad y=mx-am^2$ $\quad y=mx-\frac{a}{m}$ $\quad y=mx+am^2$

The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is (2014 Main)

(a) $\frac{1}{2}$ ✓ (b) $\frac{3}{2}$ (c) $\frac{1}{8}$ (d) $\frac{2}{3}$

$y^2=4x$ $\qquad x^2=-32y$

Common tangent

$\boxed{y=mx+\frac{1}{m}} \longleftrightarrow \boxed{y=mx+8m^2}$

Same.

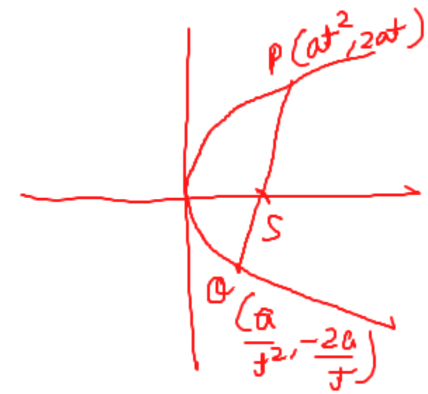
$\frac{1}{m}=8m^2 \Rightarrow m^3=\frac{1}{8} \Rightarrow m=1/2.$

JEE Mains Problems

If one end of a focal chord of the parabola, $y^2 = 16x$ is at $(1, 4)$, then the length of this focal chord is

(2019 Main, 9 April, I)

(a) 22 (b) 25 (c) 24 (d) 20



$y^2 = 16x$

$a = 4.$

$P(at^2, 2at) \equiv (4t^2, 8t) \equiv (1,4)$

$\boxed{t = 1/2.}$

$P(1,4) \quad Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right) \equiv \left(\frac{4}{1/4}, \frac{-2\times 4}{1/2}\right) \equiv (16, -16)$

$P(1,4) \quad Q(16,-16)$

$PQ = \sqrt{15^2 + 20^2} = 25$

JEE Mains Problems

If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and Y-axis, is 250 sq units, then a value of 'a' is

(2019 Main, 11 Jan, II)

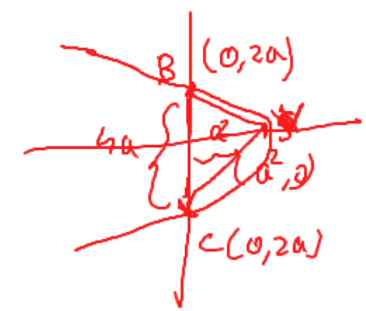
(a) $5\sqrt{5}$ (b) 5

(c) $5(2^{1/3})$ (d) $(10)^{2/3}$

$y^2 = -4(x - a^2)$

$y^2 = -4X$

Vertex is $(a^2, 0)$



Area of $\Delta YBC = 250$

$\frac{1}{2} \times \text{base} \times \text{height} = 250$

$\frac{1}{2} \times \overset{2}{\cancel{4}}a \times a^2 = 250$

$a^3 = 125$

$a = 5$

$y^2 = -4(x - a^2)$

$y^2 = 4a^2$

$y = \pm 2a$

y-axis

$x = 0$

JEE Mains Problems

$y^2 = 4ax$

$2y\frac{dy}{dx} = 4a$

$\Rightarrow \frac{dy}{dx} = \frac{2a}{y} \Rightarrow m_N = -\frac{dx}{dy} = \frac{-y}{2a} = \frac{-2at}{2a} = -t$

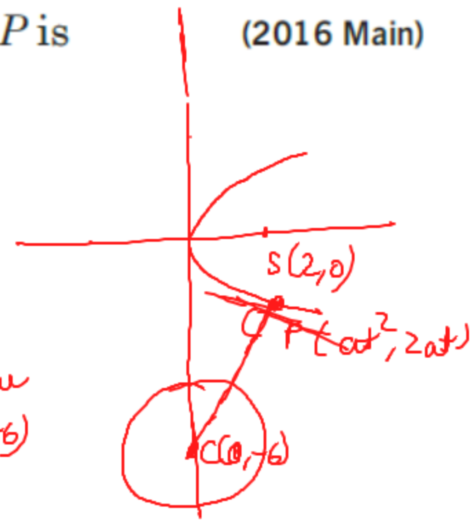
$y^2 = 4ax$ $\quad \boxed{m = -t}$

$y = mx - 2am - am^3$

$\boxed{y = -tx + 2at + at^3}$

Let P be the point on the parabola, $y^2 = 8x$, which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then, the equation of the circle, passing through C and having its centre at P is **(2016 Main)**

(a) $x^2 + y^2 - 4x + 8y + 12 = 0$
(b) $x^2 + y^2 - x + 4y - 12 = 0$
(c) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$
(d) $x^2 + y^2 - 4x + 9y + 18 = 0$



PC will be Normal for distance to be minimum.

eqn of Normal at $P(2t^2, 4t)$ is

$y = -tx + 4t + 2t^3$ passes thru $(0, -6)$

$-6 = 4t + 2t^3$

$t^3 + 2t + 3 = 0 \Rightarrow \boxed{t = -1}$

$P(2\times(-1)^2, 4(-1))$

$P(2, -4)$

radius $= PC = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$(x-2)^2 + (y+4)^2 = 8$

$x^2 + y^2 - 4x + 8y + 12 = 0$

JEE Mains Problems

$y^2=4ax$

$2y\frac{dy}{dx}=4a$

$\Rightarrow \frac{dy}{dx}=\frac{2a}{y} \Rightarrow m_N=-\frac{dx}{dy}=\frac{-y}{2a}=\frac{-2at}{2a}=-t$

$y^2=4ax$ $\quad \boxed{m=-t}$

$y=mx-2am-am^3$

$\boxed{y=-tx+2at+at^3}$

Let P be the point on the parabola, $y^2=8x$, which is at a minimum distance from the centre C of the circle, $x^2+(y+6)^2=1$. Then, the equation of the circle, passing through C and having its centre at P is **(2016 Main)**

(a) $x^2+y^2-4x+8y+12=0$

(b) $x^2+y^2-x+4y-12=0$

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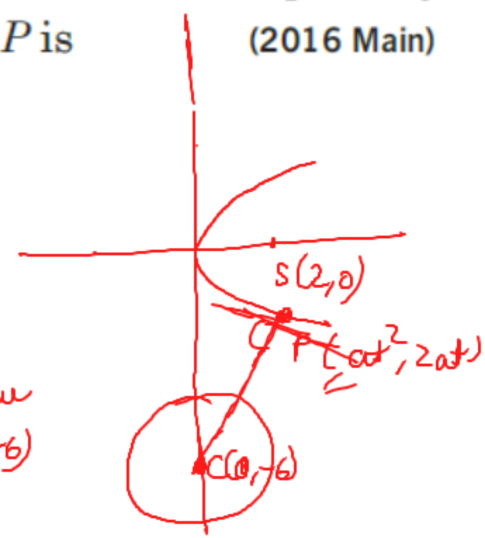
(d) $x^2+y^2-4x+9y+18=0$



PC will be Normal for distance to be minimum.

eqn of Normal at P$(2t^2, 4t)$ is

$y=-tx+4t+2t^3$ passes thru $(0,-6)$

$-6=4t+2t^3$

$t^3+2t+3=0 \Rightarrow \boxed{t=-1}$

P$(2\times(-1)^2, 4(-1))$

P$(2,-4)$

radius $=PC=\sqrt{2^2+2^2}=2\sqrt{2}$

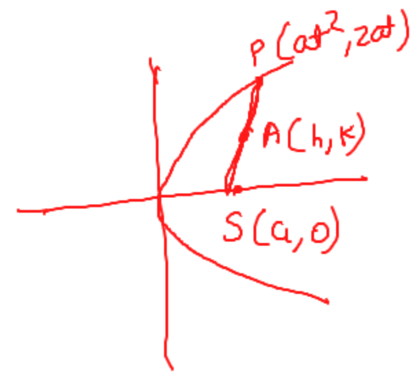
$(x-2)^2+(y+4)^2=8$

$x^2+y^2-4x+8y+12=0$

JEE Mains Problems

The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix **(2002, 1M)**

(a) $x = -a$ (b) $x = -\frac{a}{2}$ (c) $x = 0$ (d) $x = \frac{a}{2}$



$h = \frac{a+at^2}{2}$, $k = \frac{2at+0}{2}$ $\Rightarrow t = \frac{k}{a}$

$2h = a(1+t^2)$

$t^2 = \frac{2h}{a} - 1 \Rightarrow \frac{k^2}{a^2} = \frac{2h-a}{a} \Rightarrow k^2 = a(2h-a)$

$k^2 = 2a\left(h - \frac{a}{2}\right)$

$\boxed{y^2 = 2a\left(x - \frac{a}{2}\right)} = y^2 = 4\left(\frac{a}{2}\right)\left(x - \frac{a}{2}\right)$

$Y^2 = 4AX$.

Dir: $X + A = 0$

$x - \frac{a}{2} + \frac{a}{2} = 0$

$x = 0$

JEE Mains Problems

The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is (2019 Main, 10 Jan II)

(a) $8\sqrt{2}$ (b) $2\sqrt{11}$ (c) $3\sqrt{2}$ (d) $6\sqrt{3}$ ✓

Find A & B i.e p.o.i of given 2 curves

$x^2 = 4y$ $\quad \sqrt{2}y = x + 4\sqrt{2}$

$y = \frac{x}{\sqrt{2}} + 4.$ $\quad \rightarrow y_1 = \frac{x_1}{\sqrt{2}} + 4$, $\quad y_2 = \frac{x_2}{\sqrt{2}} + 4$ $\quad \Big] \; y_1 - y_2 = \frac{x_1 - x_2}{\sqrt{2}}$

$AB = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

$= \sqrt{72+36}$

$AB = 6\sqrt{3}$

$x^2 = 4\left(\frac{x}{\sqrt{2}} + 4\right)$

$x^2 = 2\sqrt{2}x + 16$

$x^2 - 2\sqrt{2}x - 16 = 0 \quad \rightarrow x_1, \; \rightarrow x_2$

$x_1 + x_2 = 2\sqrt{2}, \; x_1 x_2 = -16$

$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$

$= 8 + 64.$

$= 72.$

$(y_1 - y_2)^2 = \frac{(x_1 - x_2)^2}{2}$

$= \frac{72}{2} = 36$

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JEE Mains Problems

$\boxed{y = -ax + c}$

If the line $ax + y = c$, touches both the curves $x^2 + y^2 = 1^2$ and $y^2 = 4\sqrt{2}x$, then $|c|$ is equal to

(2019 Main, 10 April, II)

(a) $\frac{1}{\sqrt{2}}$ (b) 2 ✓(c) $\sqrt{2}$ (d) $\frac{1}{2}$

$x^2 + y^2 = a^2$

$y = mx \pm a\sqrt{1+m^2}$

$x^2 + y^2 = 1$

$\boxed{y = mx \pm 1\sqrt{1+m^2}}$

$y^2 = 4ax$

$y = mx + \frac{a}{m}$

$y^2 = 4\sqrt{2}x \quad a = \sqrt{2}$

$\boxed{y = mx + \frac{\sqrt{2}}{m}}$

$C = \frac{\sqrt{2}}{m} = \frac{\sqrt{2}}{\pm 1} = \pm\sqrt{2}$

$|c| = \sqrt{2}$

$\pm\sqrt{1+m^2} = \frac{\sqrt{2}}{m}$

$m^2(1+m^2) = 2$

$m^4 + m^2 - 2 = 0$

$m^4 + 2m^2 - m^2 - 2 = 0$

$(m^2-1)(m^2+2) = 0$

$m^2 = 1, \ m^2 = -2$ ✗

$m^2 = 1 \Rightarrow m = \pm 1$